SOME NEW METHODS OF OPTICAL DESIGN
DEVELOPED FROM THE PROPERTIES OF
DIFFERENTIAL TRANSFER COEFFICIENTS

by

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P R E F A C E

In accordance with the Rules for the Degree of Doctor of Science, an account is given here of the work submitted in this thesis. It contains as its major part a number of papers on new methods in optical design and in addition three short publications on cytological studies in the genus Eucalyptus.

The whole of the work presented in optics is as yet unpublished. A considerable amount of this work has been submitted to the Physical Society and has been accepted for publication in their Journal, but publication has been delayed by considerations of National Security. Just lately this matter has been reconsidered and manuscripts covering the main substance of Chapters 2, 4, 5, 6, 7 and 8 have gone forward for publication. The remaining portions of this optical work will be submitted to journals as soon as possible.

Since the Rules require that all published papers must be submitted, the three cytological papers are added to the present collection. The cytological work was carried out during the years 1936 - 1939 as part of a programme for the building up of a biophysical research school within the Department of Physics. In order to establish such a school it was necessary to develop cytological techniques so that they were available, and the writer worked on these with Professor McAulay in the first stages of this work, and subsequently continued alone. The material chosen was such that, as well as establishing techniques for immediate use within the school, there would result some contribution to the knowledge of the cytology of an important Australian genus hitherto unstudied. Just prior to the outbreak of war, a general survey of the development of Eucalyptus globulus had been made, the broad lines of which are sketched in the preliminary note. War conditions necessitated the abandonment of this work.

As to the matter of collaboration, the writer became engaged in the problem of optical transfer coefficients at a time when Professor McAulay had considered already the effects of differential changes made at a single surface. From this point onwards , i.e. excluding the results of Chapter 2, sections 1 - 2, the work presented is original and independent. The paper on the transfer coefficients of the primary aberrations which appears towards the end of the thesis may ultimately be a joint publication with Professor McAulay, but what is presented is the writer's own independent solution of the problem. In the cytology of Eucalyptus the initial work was done about equally with Professor McAulay, while the subsequent work on the development of Eucalyptus globulus was unaided. This last problem remains, of course, ~~um~~finished.

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CHAPTER ONE.

INTRODUCTION.

1. Introduction.

The main attempts which have been made to develop systematic methods of optical design have resulted in various algebraic solutions of the theory of the aberrations of an optical system. The essential content of each of these is an analysis of the dependence of the aberrations on the curvatures, refractive indices, axial separation, and aperture etc., i.e. on the general parameters of the system. On account of the complexity of the the problem the common procedure has been to expand various functions in series, the expansions being carried as far as terms of a selected order, with the result that approximations of various orders have been achieved. In addition to these algebraic theories there is a trigonometrical approach based on the ray trace which does not consider orders of aberrations but employs measures of the total aberrations which are present in the image formed by the system. In the general practice of optical designing the main use of this method has been as a final test of the correction of the system. Proponents of the algebraic methods have generally considered trigonometrical ray tracing as sterile and uninstructive. For example, H. D. Taylor (1906) says 'Although the trigonometrical calculation of the course of a ray through an optical system is often highly desirable yet these are merely mechanical processes, which more especially when applied to oblique and eccentric pencils, do not lend themselves at all to analysis. They are empirical and uninstructive, or at any rate barren of enlightenment unless a large number of calculations are carried out in which certain factors such as radii or separations are varied, and the results of such variations carefully noted. All this involves much empirical work, whereas, by the aid of algebraic formulae, although they may not be quite exact, leading principles can be established and the tendencies of the corrections consequent upon the variation of any one term can always be worked out with very little trouble, and it is by the intelligent grasp of the general tendencies that an optical construction may be varied in its parts until the utmost possible perfection is realised.' Conrady (1929), however, views the algebraic and trigonometrical processes as complementary and mutually indispensable. In the preface to his treatise he speaks of 'the elegant but approximate algebraical methods' which furnish a rough solution, and 'the rigourously exact method of trigonometrical ray tracing ... (which) quickly and systematically adds the necessary finishing touches.' A little experience in optical design is sufficient to show that there is still much to be desired in the 'systematic' way in which the finishing touches are added to a rough design.

2. The Aim of the Present Thesis.

In the thesis which is now presented the immediate problem which is considered is that of the optical transfer coefficients of a lens system, and a solution of it is given based on the trigonometrical trace of the system. Broadly speaking, a transfer coefficient is a differential coefficient specifying the rate of change of some quantity associated with the final image produced by the system with one of the parameters of the system. Transfer coefficients will be developed which are applicable to tangential pencils of any width and any obliquity, and to narrow sagittal pencils of any obliquity. The writer has not yet had an opportunity to attempt to reduce the general skew trace to the same organisation as the trace of the tangential rays.

It will be shown that the development of a system of transfer coefficients provides a powerful and flexible new tool for the optical designer's use, and that, developed in this way, the trigonometrical approach makes possible a valuable new analysis of an optical system, which provides an exact measure of the dependence of each aberration on each parameter of the system, in the form of a differential transfer coefficient.
The final correction of an optical design may be undertaken by this means,

in a way which may justly be termed systematic whatever the complexity of the system.

Having developed the system of transfer coefficients in this way, some account of their general usefulness in optical design follows. We may Summarise the general features of the methods to be described as follows:-

(i) An accurate analysis is given of the dependence of the aberrations on the parameters of the system, applicable at any aperture and any obliquity, which gives a complete statement of all the tendencies or potentialities of the system. Obviously this will provide the necessary information for the subsequent correction of the system.

(ii) It is possible to develop an analysis of the origin of the aberrations, that is the contributions of the individual surfaces to the aberrations of the final image. These contributions may be computed for any aperture of the system.

(iii) A useful method is available of setting tolerances within which the various specified dimensions of the system must be controlled during production.

(iv) The organisation of the computations is made so that they may be carried out by routine computers who have no special knowledge of optics. In practice this means that a rough design may be sent to the computing room and after a few days work the designer is furnished with a complete description of the correction state and a detailed analysis of the variation of all the aberrations with each parameter of the system. This renders possible a great increase in the speed with which a design may be developed.

These matters are dealt with in the first eight chapters. In Chapter 9, an alternative analysis is given of the aberration measures which the writer has found of great use, in the design of photographic objectives especially, having features which adapted more easily to the new processes than is the case with the orthodox measures. Thereafter a number of papers prepared for publication are included which bear mainly on paraxial and primary aberration theory. At the end, a considerable portion of a computation of a photographic system is included for casual inspection.

3. The Ray Trace and Notation.

It is proposed to summarise the principal points of a standard ray trace in this section in order to introduce the basic notation. Conrady's notation is followed fairly closely, so that what is presented will link on easily with his work which forms one of the best introductions to applied optics available in English. His notation, if sometimes cumbersome, is on the whole self-explanatory. The standard equations for tracing the path of a ray through a single spherical surface are

$$(L - r) \sin U = r \sin I \tag{1.1}$$

$$\sin I' = (N/N') \sin I \tag{1.2}$$

$$U + I = U' + I' \tag{1.3}$$

$$(L' - r) \sin U' = r \sin I' \tag{1.4}$$

$$L' = (L - r) + r \tag{1.5}$$

while for a centered system the transfer formulae are

$$L_{i+1} = L'_i - d'_i \tag{1.6}$$

$$U_{i+1} = U'_i \tag{1.7}$$

Appropriate modifications hold for plane surfaces and for paraxial rays.

For an axial pencil the usual three rays, marginal (m), zonal (z), and paraxial (pxl), are traced, other rays being added in wide aperture systems if necessary. For an oblique pencil the principal ray (pr) and two rays, (a) and (b), incident on the system symmetrically about the principal ray, are traced. Sometimes rays between 'a' and 'pr' and between 'pr' and 'b' are needed, corresponding somewhat with the zonal rays of the axial pencil. These are denoted by 'za' and 'zb' respectively. In diagrams the principal ray is always drawn with a positive inclination angle, and the rays 'a' and 'b'

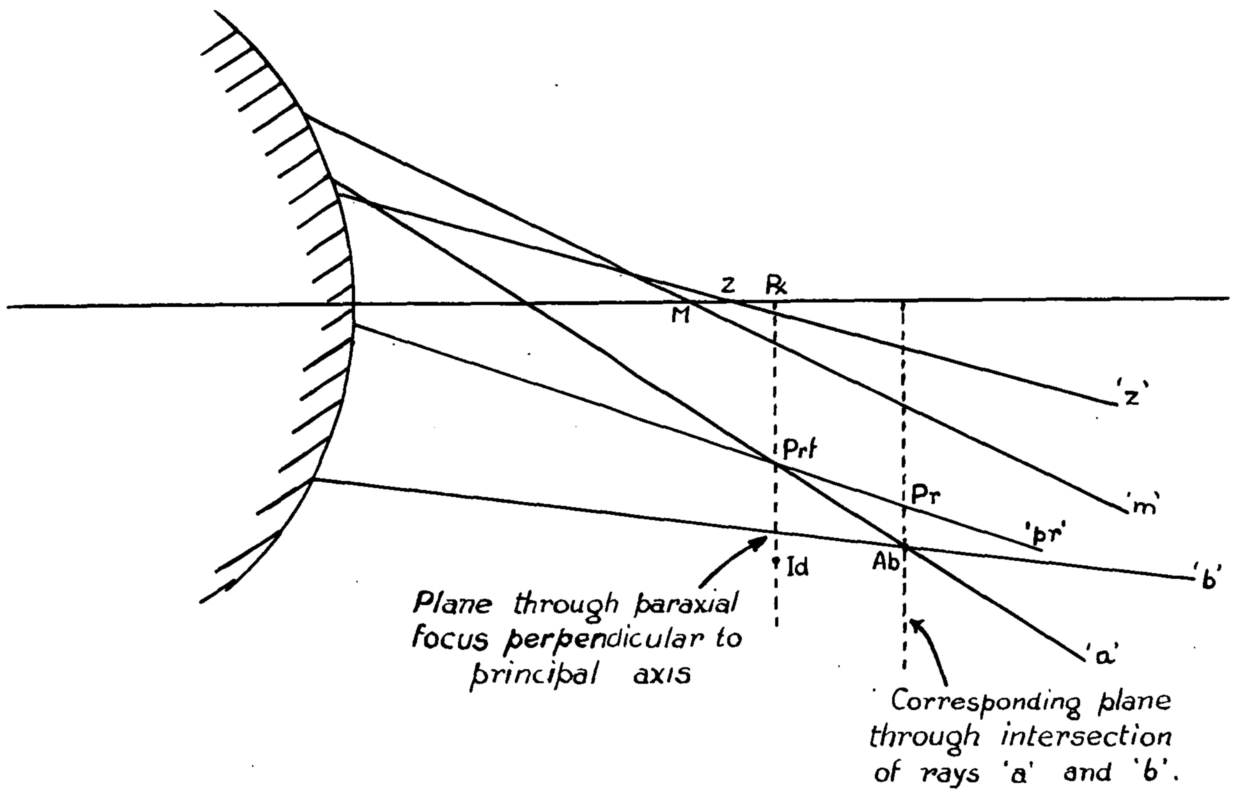

z
Px
M
'z'
'm'
Prf
Pr
'pr'
Id
Ab
'b'
'a'
Plane through paraxial focus perpendicular to principal axis
Corresponding plane through intersection of rays 'a' and 'b'.

Figure I

appear above and below it respectively. In pencils through wide aperture systems in which vignetting by the rims of the components may occur the two portions of the pencil on either side of the principal ray are of different width and we normally take the 'a' ray as the extreme ray on the narrow side of the pencil, the 'b' ray incident symmetrically with it on the other side, and denote the extreme ray on the 'b' side by the letter 'h'. When it is necessary to trace rays for different colours the subscripts 'r', 'd' and 'v' may be used. Thus the subscript 'prv' would be attached to a quantity connected with the trace of a principal ray in a colour at the violet end of the spectrum. In the absence of a colour subscript it is to be assumed that the trace is carried out using the refractive indices for some intermediate or mean wavelength in the spectral region for which the system is to be achromatised.

4. The Intersection Points and the Aberration Measures.

In figure 1.1 the paths of the rays traced through the system are shown after passing through the last surface of the system, their relative positions being exaggerated for the sake of clarity. Measures of the aberrations are defined in terms of the relative positions of certain intersection points associated with these rays, the location of each point being given by rectangluar co-ordinates relative to the pole of the last surface.

Associated with the axial pencil we have the intersection points of each ray with the principal axis of the system, viz., the points $M(L'_m, 0)$, $Z(L'_z, 0)$, and $Px(\ell', 0)$. The additional zonal rays which may be traced for colours 'r' and 'v' are not shown in the diagram, but their intersection points with the principal axis would be denoted by $Z_r(L'_{zr}, 0)$, and $Z_v(L'_{zv}, 0)$.

In an oblique pencil we are interested in the intersection point $Ab(L'_{ab}, H'_{ab})$ of the rays 'a' and 'b', and also the point $Pr(L'_{ab}, H'_{pr})$ at which the principal ray cuts the plane through Ab perpendicular to the principal axis. In the corresponding plane through the paraxial focus we have the point of penetration of the principal ray, Prf(ℓ', H'_{prf}), and the ideal image point, Id(ℓ', H'_{id}). The principal rays for colours 'r' and 'v', not shown on the diagram, would have corresponding intersections.

The aberration measures which are derivable from the results of the trace are defined in the usual way

$$LA'_m = \ell' - L'_m \tag{1.8}$$

$$LA'_z = \ell' - L'_z \tag{1.9}$$

$$Lch'_z = L'_{zr} - L'_{zv} \tag{1.10}$$

$$Tch' = H'_{prr} - H'_{prv} \tag{1.11}$$

$$Coma'_T = H'_{pr} - H'_{ab} \tag{1.12}$$

$$X'_T = L'_{ab} - L'_m \tag{1.13}$$

$$dist' = H'_{id} - H'_{prf} \tag{1.14}$$

References.

Conrady, A.E. Applied Optics and Optical Design. Oxford. (1929)

Taylor, H.D. A System of Applied Optics. 1906. p.107.

CHAPTER TWO.

THE GENERAL THEORY OF DIFFERENTIAL CHANGES AND THEIR TRANSFER COEFFICIENTS.

1. The Specification of the Changed Ray Path at a Single Spherical Refracting Surface.

Suppose that the path of a selected incident ray is traced trigonometrically through as single spherical refracting surface. The ray path is specified by the quantities (L, U,) and (L', U',), and is represented by B P B' in Figure 2.1 . Suppose that the incidence point of the ray is changed from P to P_1 , the direction of the ray remaining unchanged. The path of the ray through the surface is now represented by $B_1P_1B_1'$. We propose to specify the change in the incidence point by the perpendicular distance, dp, between the two rays, the quantity being positive when the new parallel ray lies above the original ray. The new ray will have an intersection length L + dL , compared with the intersection length, L , of the original path, and

$$dp = \sin U \cdot dL \qquad (2.1)$$

It should be noticed that dp is not the total differential of the familiar quantity L sin U . We are never interested particularly in the integral quantity p , except possibly in a formal way, but it is important to stress that relative to a given ray path, (L, U,) , which intersects the refracting surface, the quantity dp = sin U . dL , uniquely defines a neighbouring incidence point. On the other side of the surface it is natural and convenient to specify the same incidence point displacement by another quantity, dp' , which is the perpendicular distance between the original refracted ray and a line parallel to this drawn through the new incidence point. The relation between the two quantities for differential displacements of the incidence point is

$$dp' = \frac{\cos I'}{\cos I} dp$$

or,

$$\frac{\partial p'}{\partial p} = \frac{\cos I'}{\cos I} \qquad (2.2)$$

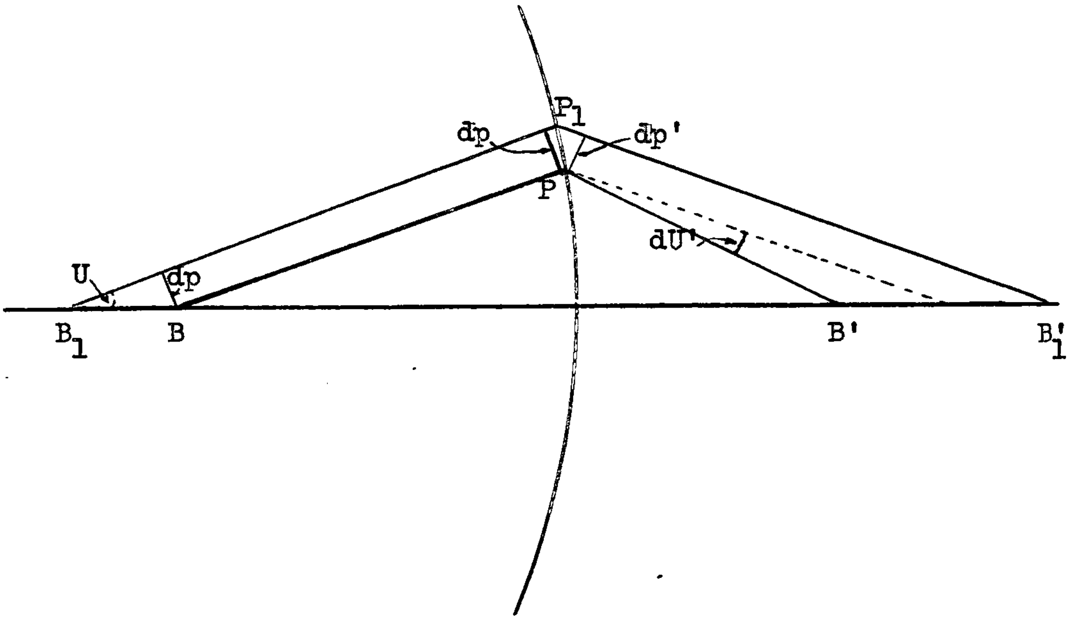


Figure 2.1

A change in the direction of the ray incident at P is specified by dU, and the resulting change in the direction of the refracted ray by dU'. It will thus be seen that any new ray path differing by differential amounts in direction and in location of incidence point from the original ray path can be specified relative to the original path by the quantities dp, dU, on the incidence side of the surface, and by the corresponding quantities dp', dU', after refraction at the surface.

2. The Effect of Differential Changes made at a Single Spherical Surface.

We consider now the effect on the direction of the refracted ray of various changes which can be made at a single surface at which an incident ray is refracted. Writing down the standard ray tracing equations in the form

$$(L - r) \sin U = r \sin I \tag{2.3}$$

$$N' \sin I' = N \sin I \tag{2.4}$$

$$U + I = U' + I' \tag{2.5}$$

we differentiate (2.4) and (2.5), putting $N/N' = n$, and obtain

$$dU' = dU - \left(n \frac{\cos I}{\cos I'} - 1 \right) dI - \frac{\sin I}{\cos I'} dn \tag{2.6}$$

If the direction of the incident ray changes from U to $U + dU$, the change in the angle of incidence is $dI \doteqdot - dU$, and hence from (2.6)

$$dU' = \frac{\partial U'}{\partial U} dU = dU - \left(n \frac{\cos I}{\cos I'} - 1\right)(- dU)$$

whence,
$$\frac{\partial U'}{\partial U} = n \frac{\cos I}{\cos I'} \tag{2.7}$$

If the position of the incidence point is changed by an amount dp, the change in I, obtained by differentiating (2.3) with U and r constant, is

$$dI = \frac{\sin U \cdot dL}{r \cos I} = \frac{dp}{r \cos I}$$

which on combination with (2.6) gives

$$\frac{\partial U'}{\partial p} = \left(1 - \frac{\partial U'}{\partial U}\right)/(r \cos I) \tag{2.8}$$

For differential variations in the relative refractive index, n , equation (2.6) gives at once

$$\frac{\partial U'}{\partial n} = -\frac{\sin I}{\cos I'} \tag{2.9}$$

If a change of differential order is made in the curvature, c , of the refracting surface the change in I , obtained by differentiating (2.3) with U constant, is

$$dI = \frac{L \sin U}{\cos I} dc$$

which on combination with (2.6) gives

$$\frac{\partial U'}{\partial c} = \left(1 - \frac{\partial U'}{\partial U}\right) \frac{L \sin U}{\cos I} \tag{2.10}$$

The equations of this section are sufficient for the complete description of the changes in the path of the refracted ray resulting from changes of differential order made at a single surface.

3. The Transfer Coefficients for dU'-Changes and dp-Changes.

We consider now the effect on the path of a ray through a centred system of k spherical refracting surfaces of differential changes made at one surface of the system. Suppose that the path of a selected ray has been traced trigonometrically through the system in the usual way. After refraction at any surface $\underline{i}$ of the system the path of the ray is described by the quantities L_i' , U_i' . Let a small change be made now at surface $\underline{i}$ such that the ray after refraction at the same incidence point on the surface takes the new direction specified by $U_i' + dU_i'$. The path of the ray through the remainder of the system will be different from the original traced path, and the ray will finally emerge from the last surface at a point displaced by some amount, dp_k' , from the emergence point of the traced ray, and in a direction inclined at an angle, dU_k' , to that of the traced ray. We may express these differential quantities in terms of dU_i'

by writing

$$dU'_k = \frac{\partial U'_k}{\partial U'_i} \, dU'_i \qquad (2.11)$$

$$dp'_k = \frac{\partial p'_k}{\partial U'_i} \, dU'_i \qquad (2.12)$$

Similarly, if a small change made at surface $\underline{i}$ results only in an incidence point displacement, dp_i, the inclination angle, U_i, of the incident ray remaining unchanged, the ray will emerge from the last surface of the system at a point and in a direction specified by some other values, dp'_k, dU'_k, relative to the emergent traced ray. For these we may write

$$dp'_k = \frac{\partial p'_k}{\partial p_i} \, dp_i \qquad (2.13)$$

$$dU'_k = \frac{\partial U'_k}{\partial p_i} \, dp_i \qquad (2.14)$$

Equations (2.11) to (2.14) define the 'transfer coefficients', $\partial U'_k / \partial U'_i$, $\partial p'_k / \partial U'_i$, $\partial p'_k / \partial p_i$, $\partial U'_k / \partial p_i$, which can be calculated for a traced ray at each surface of the system. The method of calculation depends upon relations which will be deduced in the next section. The transfer coefficients specify the effect of a dU'-change or a dp-change made at any surface on the path of the ray as it leaves the last surface of the system. These particular quantities are chosen as the fundamental transfer coefficients because any real change made in the course of an actual design can be described in terms of either a single dU'-change or a single dp-change.

4. The Relation between the Transfer Coefficients at Successive Surfaces of the System.

Denoting any two successive surfaces of the system by the subscripts $\underline{i}$ and $\underline{j}$, we follow out the effect on the ray path of a change dU'_i made at surface $\underline{i}$. From Figure 2.2 it is seen that relative to the traced

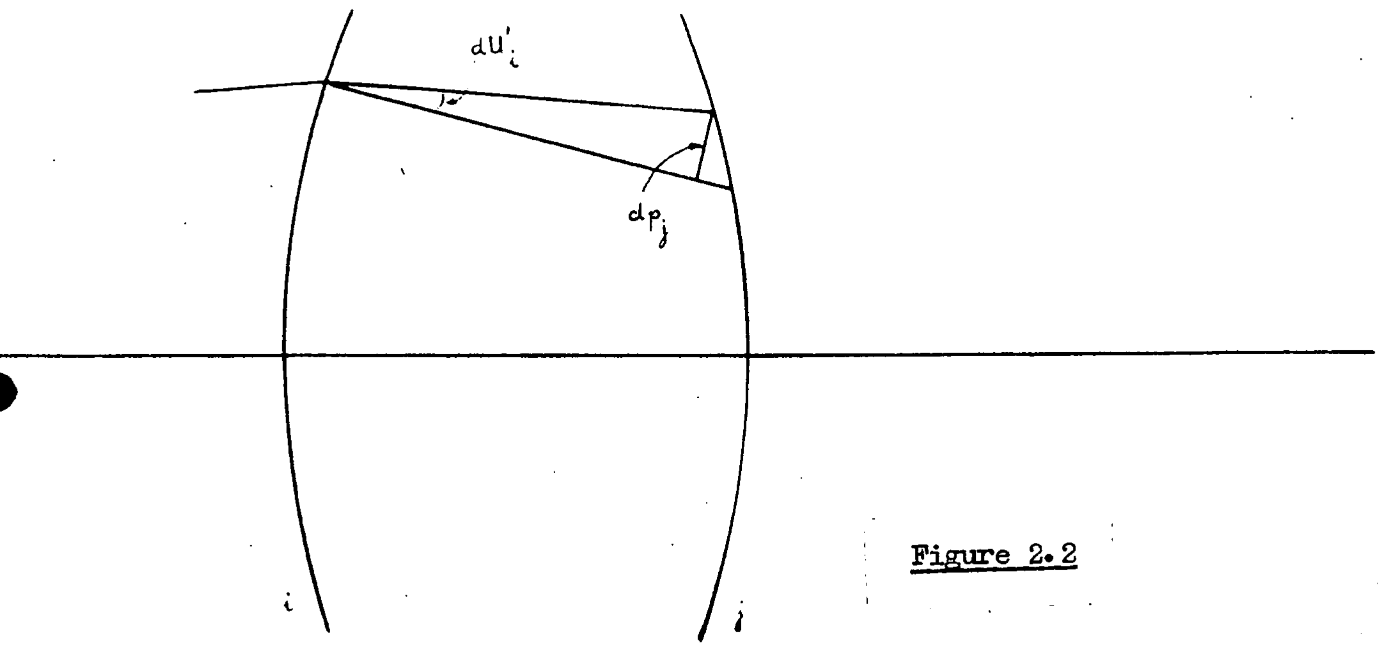


Figure 2.2

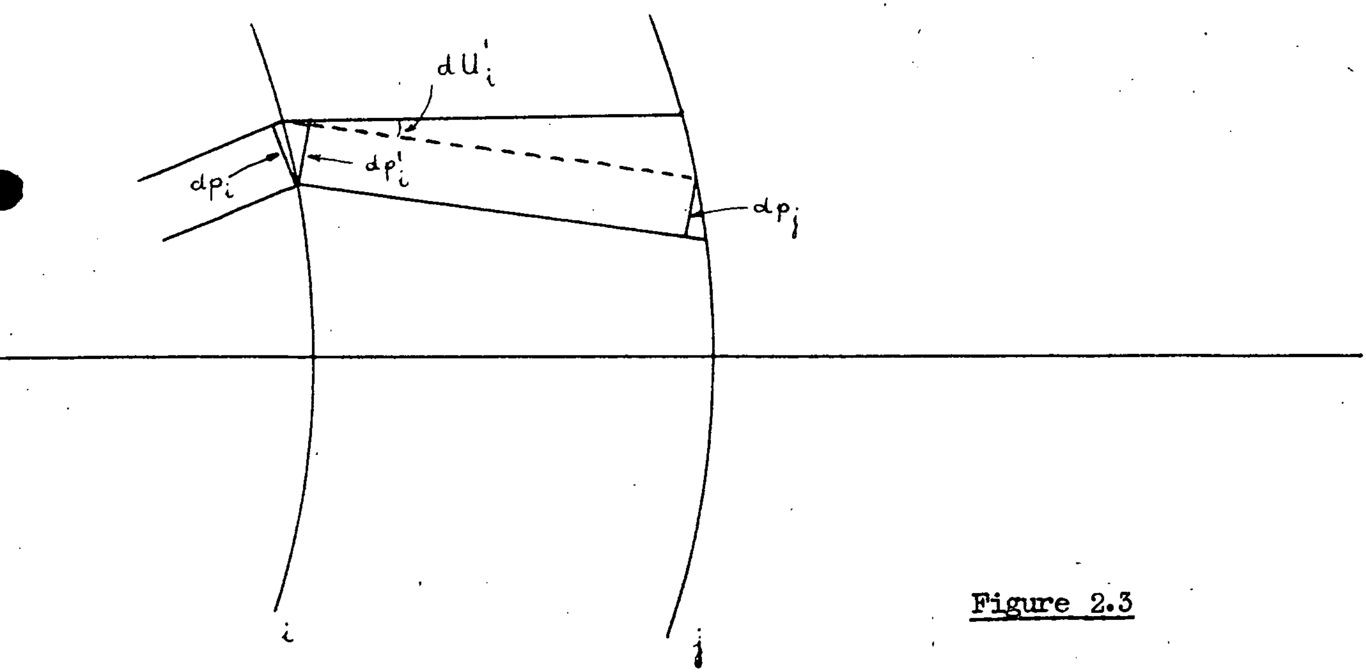


Figure 2.3

path the ray will now meet surface j with an incidence point displacement $dp_j = -D_i' \, dU_i' = D_j \, dU_i'$, and with an inclination angle change, $dU_j = dU_i'$, where $D_j = -D_i'$ is the distance between the two surfaces measured along the traced ray. Further, the change dU_j alone would produce in turn a change $dU_j' = (\partial U_j'/\partial U_j) \, dU_i'$ in the direction of the ray after refraction at surface j. Hence, the change dU_i' at surface $\underline{i}$ is equivalent in effect to the change $dp_j = D_j \, dU_i'$, together with the change $dU_j' = (\partial U_j' / \partial U_j) \, dU_i'$, at surface j. Expressing the resulting dp_k', dU_k', in terms of each of these equivalent changes, we have

$$\frac{\partial U_k'}{\partial U_i'} dU_i' = dU_k' = \frac{\partial U_k'}{\partial U_j'} dU_j' + \frac{\partial U_k'}{\partial p_j} dp_j$$

$$= \frac{\partial U_k'}{\partial U_j'} \frac{\partial U_j'}{\partial U_j} dU_i' + \frac{\partial U_k'}{\partial p_j} D_j \, dU_i'$$

whence
$$\frac{\partial U_k'}{\partial U_i'} = \frac{\partial U_k'}{\partial U_j'} \frac{\partial U_j'}{\partial U_j} + \frac{\partial U_k'}{\partial p_j} D_j \tag{2.15}$$

and similarly,
$$\frac{\partial p_k'}{\partial U_i'} = \frac{\partial p_k'}{\partial U_j'} \frac{\partial U_j'}{\partial U_j} + \frac{\partial p_k'}{\partial p_j} D_j \tag{2.16}$$

Next, we follow out the effect on the ray path of a small change of incidence point, dp_i, made at surface $\underline{i}$. It may be seen from Figure 2.3 that relative to the traced path the ray will now leave surface $\underline{i}$ after refraction with an inclination angle change, $dU_i' = (\partial U_i' / \partial p_i) \, dp_i$, and with an incidence point displacement, $dp_i' = (\partial p_i'/\partial p_i) \, dp_i$, the latter having the same effect as an equal displacement, dp_j, at surface j. Expressing the resulting dU_k', dp_k', in terms of dp_i and the equivalent set of changes dU_i', dp_j, we arrive at the relations

$$\frac{\partial U_k'}{\partial p_i} = \frac{\partial U_k'}{\partial U_i'} \frac{\partial U_i'}{\partial p_i} + \frac{\partial U_k'}{\partial p_j} \frac{\partial p_i'}{\partial p_i} \tag{2.17}$$

$$\frac{\partial p_k'}{\partial p_i} = \frac{\partial p_k'}{\partial U_i'} \frac{\partial U_i'}{\partial p_i} + \frac{\partial p_k'}{\partial p_j} \frac{\partial p_i'}{\partial p_i} \tag{2.18}$$

The four equations, (2.15) to (2.18), permit the calculation of the transfer coefficients for surface $\underline{i}$ when those for surface $\underline{j}$ are known. At the last surface of the system it is obvious that the transfer coefficients have simple values, for

$$\frac{\partial U'_k}{\partial U'_k} = 1 \qquad \text{and} \qquad \frac{\partial p'_k}{\partial U'_k} = 0$$

while $\partial U'_k/\partial p_k$ and $\partial p'_k/\partial p_k$ are calculated by equations (2.8) and (2.2) respectively. Having obtained the values of these coefficients at the last surface their values at surface (k - 1) may be calculated by use of equations (2.15) to (2.18), and so in turn for each surface of the system, the computation working from the last surface through the system until the first surface is reached. At each surface the computation involves four pairs of multiplications, there being a common factor in each pair, and four additions.

5. The Transfer Coefficients for the Intersection Points of the Normal Trace.

In the trigonometrical trace as normally applied to an optical system the various aberrations are determined from the relative positions of certain intersection points associated with the selected rays, as described in Chapter 1. If a change is made in the path of any of these typical traced rays at any surface of the system the associated intersection point, or points, in the final image space will be displaced. We now seek to develop transfer coefficients to specify such changes.

<u>The Axial Intersection Points, M, Z, and Px.</u> Let us consider first the effect of the change dU'_i on the position of the point M. Let us suppose that before the change is made in the ray path at surface $\underline{i}$ the marginal ray leaves the last surface of the system along the path PM, while after the change is made it follows the path P'M' in Figure 2.4. Relative to the

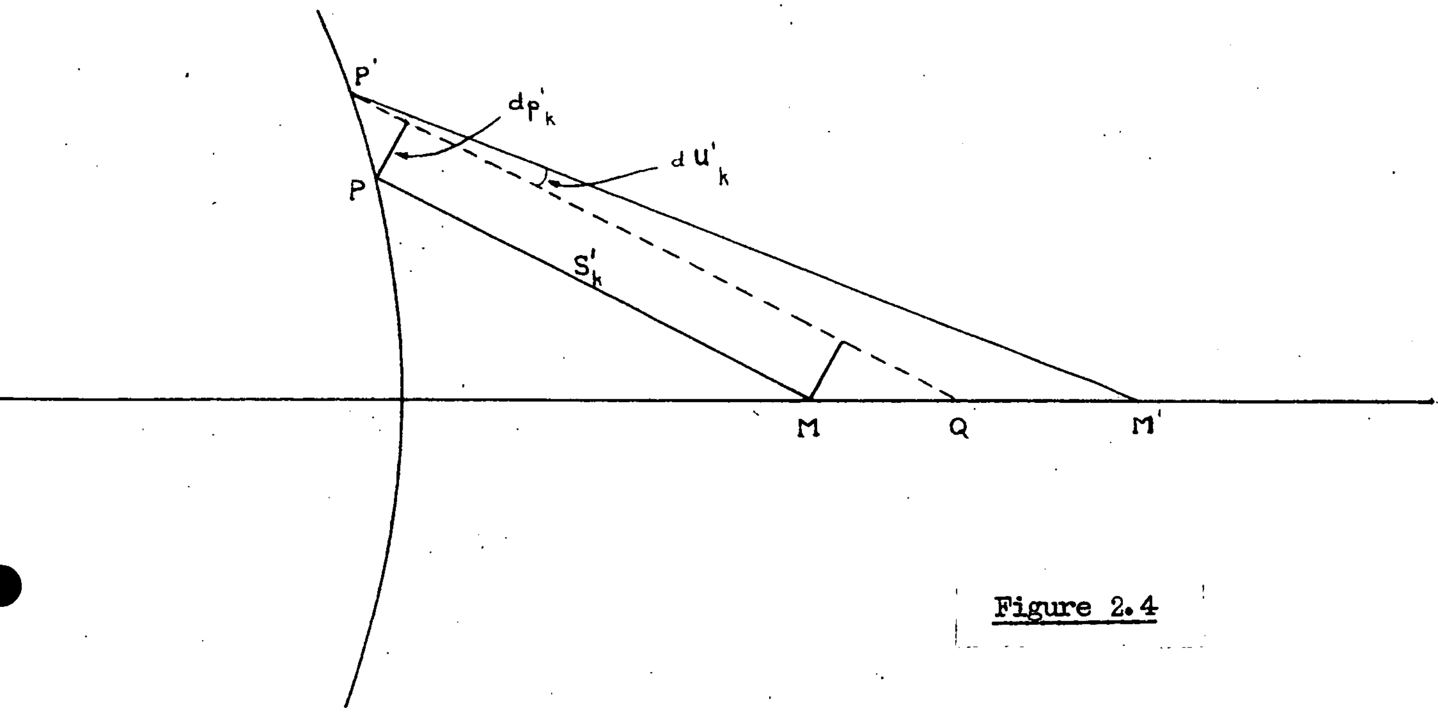


Figure 2.4

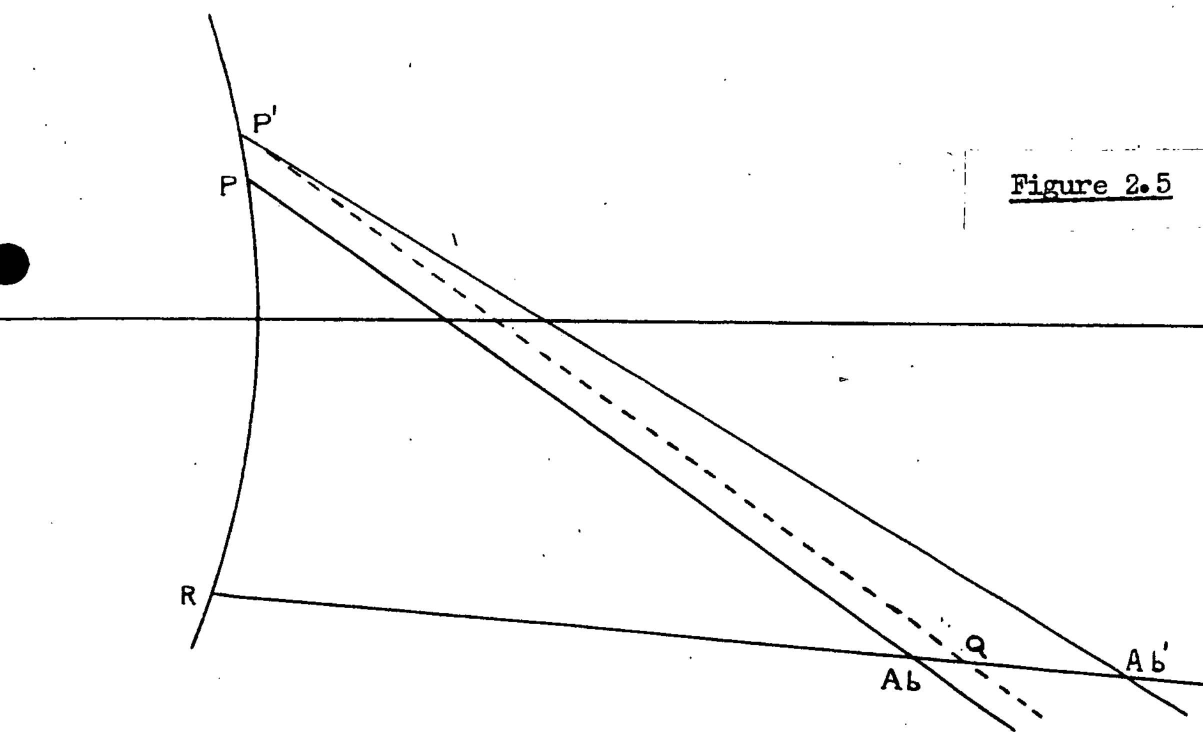


Figure 2.5

$$\frac{\partial L'_m}{\partial U'_i} = C(U'_i)_m \operatorname{cosec} U'_{mk} \tag{2.23}$$

$$\frac{\partial L'_m}{\partial p_i} = C(p_i)_m \operatorname{cosec} U'_{mk} \tag{2.24}$$

Corresponding expressions follow for the transfer coefficients for the zonal and paraxial intersection points, Z, and Px.

<u>The Intersection Point Ab</u>. For this point there are changes in two co-ordinates to be considered. We begin by considering the effect of the change dU'_{ai} in the path of the ray '<u>a</u>' at surface <u>i</u>. In Figure 2.5 P Ab and R Ab represent the emergent traced paths of the rays '<u>a</u>' and '<u>b</u>' before any change is made. As a result of the change dU'_{ai} the ray '<u>a</u>' will now leave the last surface of the system along a new path P'Ab', the point Ab' being the new position of the intersection point. Relative to the emergent traced path the new ray has an incidence point displacement $dp'_k = (\partial p'_k/\partial U'_i)_a dU'_{ai}$, and a direction change $dU'_k = (\partial U'_k/\partial U'_i)_a dU'_{ai}$. For differential changes we have, much as before,

$$\text{Ab Ab}' = \text{Ab Q} + \text{Q Ab}' = (dp'_k - S'_{ak}\, dU'_k) \operatorname{cosec}(U'_a - U'_b)_k$$

Resolving this displacement along directions parallel to and perpendicular to the principal axis, we obtain

$$dL'_{ab} = (dp'_k - S'_{ak}\, dU'_k) \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}$$

and
$$dH'_{ab} = -(dp'_k - S'_{ak}\, dU'_k) \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}$$

whence,
$$\frac{\partial L'_{ab}}{\partial U'_{ai}} = \left(\frac{\partial p'_k}{\partial U'_{ai}} - S'_{ak}\frac{\partial U'_k}{\partial U'_{ai}}\right) \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}$$

$$= C(U'_i)_a \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk} \tag{2.25}$$

and
$$\frac{\partial H'_{ab}}{\partial U'_{ai}} = -C(U'_i)_a \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk} \tag{2.26}$$

Similarly, a change dp_{ai} made in the path of ray '<u>a</u>' at surface <u>i</u> will lead to the expressions traced ray PM the displacement of the emergence point is $dp'_k = (\partial p'_k/\partial U'_i) dU'_i$ and the change in direction is $dU'_k = (\partial U'_k/\partial U'_i)\, dU'_i$, and each of these contribute to the shift of M. The effect of the displacement dp'_k alone would be to move the intersection point from M to Q, while the direction change dU'_k would, in addition, move it from Q to M'. For differential changes we then have

$$MQ = dp'_k \operatorname{cosec} U'_{mk} \qquad QM' = -S'_{mk}\, dU'_k \operatorname{cosec} U'_{mk}$$

where S'_{mk} denotes the distance PM. Hence,

$$\frac{\partial L'_m}{\partial U'_i}\, dU'_i = MM' = \left(\frac{\partial p'_k}{\partial U'_i}\, dU'_i - S'_k \frac{\partial U'_k}{\partial U'_i}\, dU'_i\right)_m \operatorname{cosec} U'_{mk}$$

whence

$$\frac{\partial L'_m}{\partial U'_i} = \left(\frac{\partial p'_k}{\partial U'_i} - S'_k \frac{\partial U'_k}{\partial U'_i}\right)_m \operatorname{cosec} U'_{mk} \qquad (2.19)$$

Similarly, the consideration of the effect of a change, dp_i, leads to

$$\frac{\partial L'_m}{\partial p_i} = \left(\frac{\partial p'_k}{\partial p_i} - S'_k \frac{\partial U'_k}{\partial p_i}\right)_m \operatorname{cosec} U'_{mk} \qquad (2.20)$$

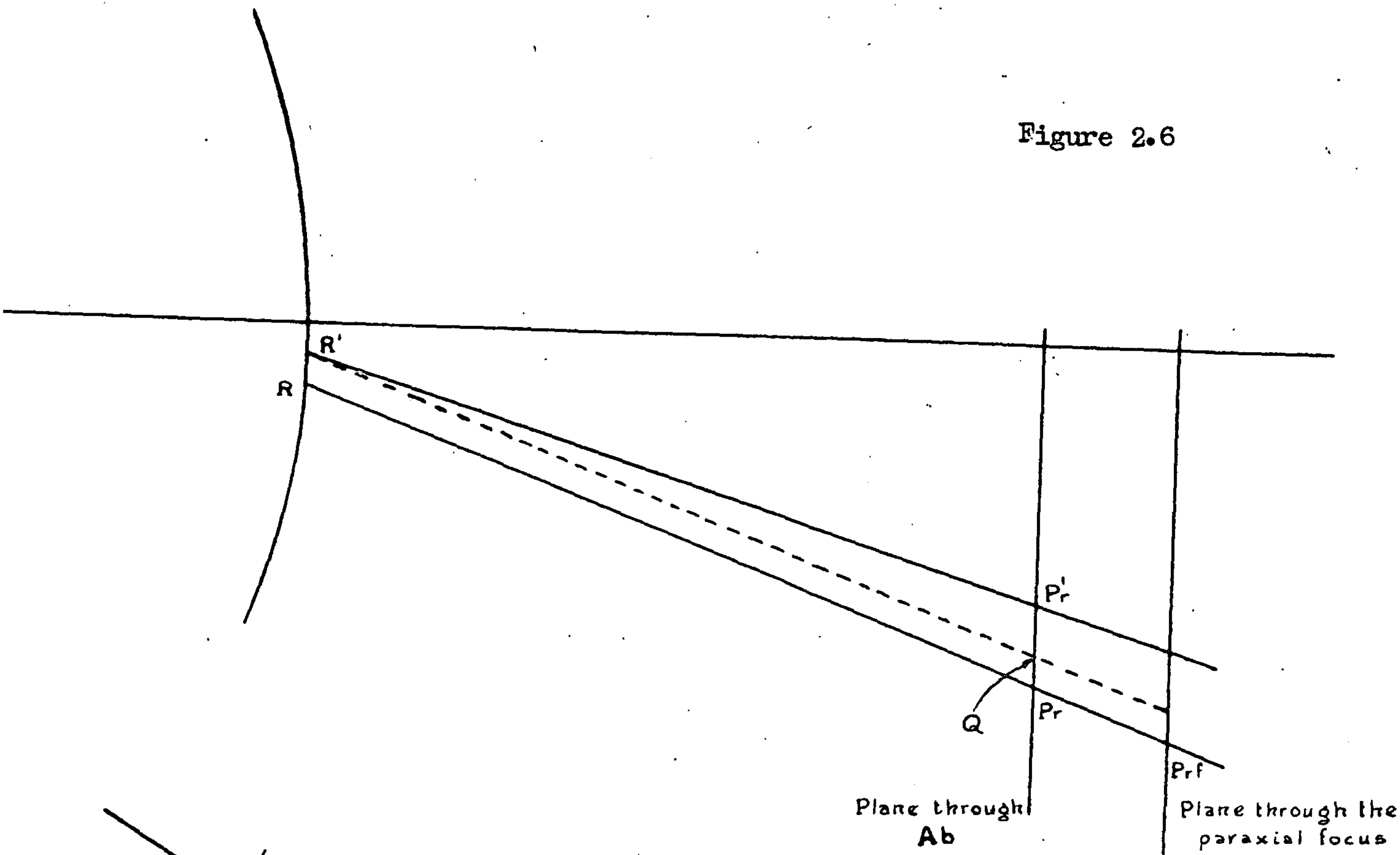
It is convenient to introduce at this stage the functional symbol C as an abbreviation, its meaning being defined by

$$C(U'_i) = \frac{\partial p'_k}{\partial U'_i} - S'_k \frac{\partial U'_k}{\partial U'_i} \qquad (2.21)$$

$$C(p_i) = \frac{\partial p'_k}{\partial p_i} - S'_k \frac{\partial U'_k}{\partial p_i} \qquad (2.22)$$

Quantities of this C-type enter characteristically into all expressions for the transfer coefficients for the intersection points, as we shall see presently. Actually, the C-quantity measures the rate of change of the intersection point with the variable concerned along a direction at right angles to the traced ray. Where it is necessary to distinguish the C-quantities of different rays we may do so by writing an appropriate suffix outside the bracket, e.g. $C(p_i)_m$. Rewriting equations (2.19) and (2.20), we have

Figure 2.6



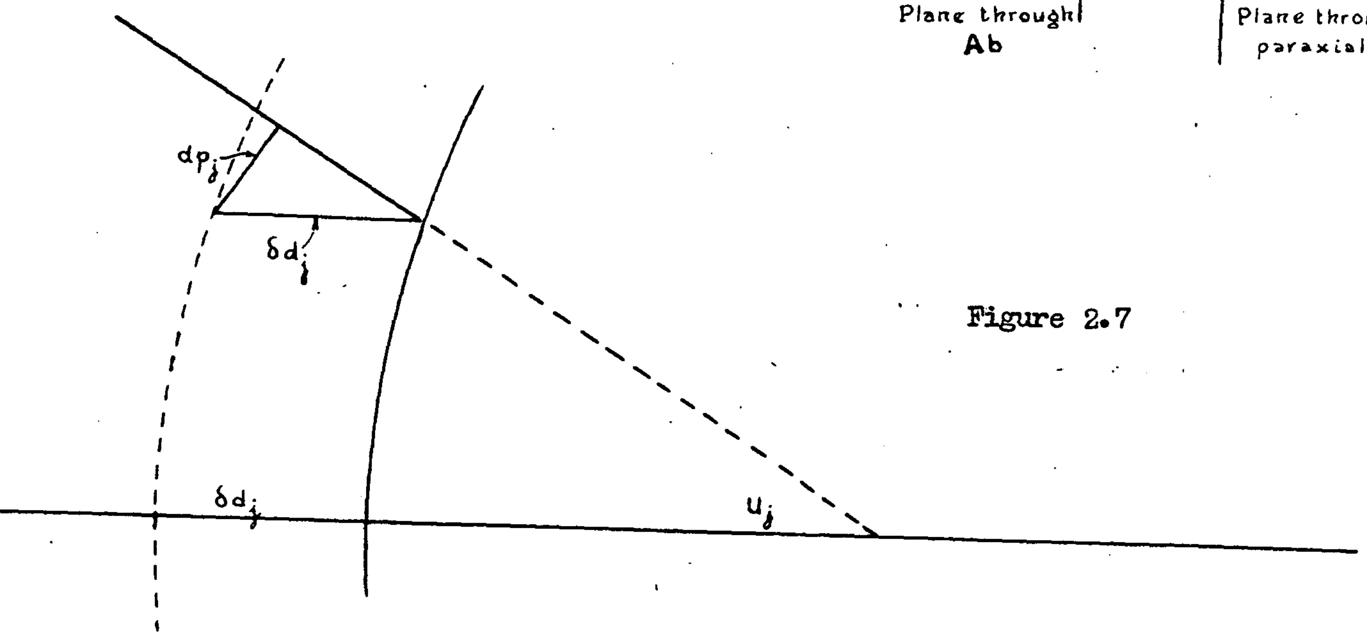


Figure 2.7

$$\frac{\partial L'_{ab}}{\partial p_{ai}} = C(p_i)_a \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk} \tag{2.27}$$

$$\frac{\partial H'_{ab}}{\partial p_{ai}} = - C(p_i)_a \operatorname{cosec} (U'_a - U'_b)_k \sin U'_{bk} \tag{2.28}$$

Turning now to the consideration of the effects of the changes dU'_{bi}, dp_{bi}, made in the path of the ray '<u>b</u>' at surface <u>i</u>, we can write down the final expressions for the transfer coefficients by inspection, a glance at Figure 2.5 being sufficient to confirm the change of sign which occurs. Thus we have,

$$\frac{\partial L'_{ab}}{\partial U'_{bi}} = - C(U'_i)_b \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{ak} \tag{2.29}$$

$$\frac{\partial H'_{ab}}{\partial U'_{bi}} = C(U'_i)_b \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{ak} \tag{2.30}$$

$$\frac{\partial L'_{ab}}{\partial p_{bi}} = - C(p_i)_b \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{ak} \tag{2.31}$$

$$\frac{\partial H'_{ab}}{\partial p_{bi}} = C(p_i)_b \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{ak} \tag{2.32}$$

<u>The Intersection Points Pr, Prf</u>. In Figure 2.6 the traced path of the principal ray after leaving the last surface of the system is represented by R Pr, and the corresponding path after a change dU'_i has been made in the ray path at surface <u>i</u> is represented by R' Pr'. Relative to the traced path the new emergent ray has an incidence point displacement $dp'_k = (\partial p'_k/\partial U'_i)\, dU'_i$, and a direction change $dU'_k = (\partial U'_k/\partial U'_i)\, dU'_i$.

Following the general lines of the previous cases we have,

$$\text{Pr Pr}' = \text{Pr Q} + \text{Q Pr}' = (dp'_k - S'_{prk}\, dU'_k) \sec U'_{prk}$$

from which it follows that

$$\frac{\partial H'_{pr}}{\partial U'_i} = \left(\frac{\partial p'_k}{\partial U'_i} - S'_k \frac{\partial U'_k}{\partial U'_i} \right)_{pr} \sec U'_{prk}$$

$$= C(U'_i)_{pr} \sec U'_{prk} \tag{2.33}$$

The corresponding expression for small dp-changes will be

$$\frac{\partial H'_{pr}}{\partial p_i} = C(p_i)_{pr} \sec U'_{prk} \tag{2.34}$$

The transfer coefficients for the point Prf with respect to the changes dU'_i, dp_i, made in the principal ray at surface $\underline{i}$ will obviously be,

$$\frac{\partial H'_{prf}}{\partial U'_i} = C(U'_i)_{prf} \sec U'_{prk} \tag{2.35}$$

$$\frac{\partial H'_{prf}}{\partial p_i} = C(p_i)_{prf} \sec U'_{prk} \tag{2.36}$$

6. The Intersection Point Transfer Coefficients for Changes of Curvature, Refractive Index, and Axial Separation.

In the actual process of completing the design of a lens system the variables at the disposal of the designer are curvature, refractive index, and the axial separation of successive surfaces. It is necessary now to develop transfer coefficients for changes in these quantities.

Change of Surface Curvature. Suppose that a differential curvature change, dc_i, is made at surface $\underline{i}$ of the system. This leaves unchanged, to first order, the incidence points of rays which strike the surface, and the total effect for any ray whose path has been traced through the original system is that after refraction the path of the ray will now be inclined to the traced path at an angle $dU'_i = (\partial U'_i/\partial c_i)\, dc_i$. Using the relations developed in the previous section we can write down the required transfer coefficients at once.

For an axial ray, taking the marginal ray as typical, we have

$$\frac{\partial L'_m}{\partial c_i} = \frac{\partial L'_m}{\partial U'_i}\frac{\partial U'_i}{\partial c_i} = \frac{\partial U'_i}{\partial c_i} C(U'_i)_m \operatorname{cosec} U'_{mk}$$

$$= C(c_i)_m \operatorname{cosec} U'_{mk} \tag{2.37}$$

For the intersection point Ab it follows that since the 'a' and 'b' rays are affected at different rates, we have

$$\frac{\partial L'_{ab}}{\partial c_i} = \frac{\partial L'_{ab}}{\partial U'_{ai}} \frac{\partial U'_{ai}}{\partial c_i} + \frac{\partial L'_{ab}}{\partial U'_{bi}} \frac{\partial U'_{bi}}{\partial c_i}$$

which, on combination with (2.25) and (2.29), gives

$$\frac{\partial L'_{ab}}{\partial c_i} = \left[C(c_i)_a \cos U'_{bk} - C(c_i)_b \cos U'_{ak}\right] \operatorname{cosec}(U'_a - U'_b)_k \tag{2.38}$$

The corresponding expression for the rate of change of the H'-coordinate is

$$\frac{\partial H'_{ab}}{\partial c_i} = -\left[C(c_i)_a \sin U'_{bk} - C(c_i)_b \sin U'_{ak}\right] \operatorname{cosec}(U'_a - U'_b)_k \tag{2.39}$$

For the intersection points Pr, Prf, we note that their positions will be affected both by the change in the path of the principal ray and by the shift along the axis of the plane through Ab and the plane through Px. For a small change, dc_i , we have

$$dH'_{pr} = \frac{\partial H'_{pr}}{\partial U'_i} \frac{\partial U'_i}{\partial c_i} dc_i - \frac{\partial L'_{ab}}{\partial c_i} dc_i \tan U'_{prk}$$

whence,

$$\frac{\partial H'_{pr}}{\partial c_i} = \frac{\partial H'_{pr}}{\partial U'_i} \frac{\partial U'_i}{\partial c_i} - \frac{\partial L'_{ab}}{\partial c_i} \tan U'_{prk}$$

$$= C(c_i)_{pr} \sec U'_{prk} - \frac{\partial L'_{ab}}{\partial c_i} \tan U'_{prk} \tag{2.40}$$

while the corresponding expression for the point Prf is

$$\frac{\partial H'_{prf}}{\partial c_i} = C(c_i)_{prf} \sec U'_{prk} - \frac{\partial l'}{\partial c_i} \tan U'_{prk} \tag{2.41}$$

Throughout the equations of this section we have used the quantity $C(c_i)$ with the meaning defined by

$$C(c_i) = C(U'_i) \frac{\partial U'_i}{\partial c_i} \tag{2.42}$$

There remains only the ideal image point to consider. For an infinitely distant object point we have

$$H'_{id} = -f' \tan U_{pr} = -(y/u'_k) \tan U_{pr}$$

$$\frac{\partial H'_{id}}{\partial c_i} = (y/u'^2_k) \tan U_{pr} \frac{\partial u'_k}{\partial u'_i} \frac{\partial u'_i}{\partial c_i} \qquad (2.43)$$

The differential coefficients in equation (2.43) are among those normally calculated for the paraxial ray, so that the transfer coefficients for the point, Id , fit conveniently into the general scheme of the computation. Corresponding expressions follow for an object point at a finite distance from the system.

Changes in Refractive Index. If the relative refractive index, $n = N/N'$, is changed by an amount dn_i at surface $\underline{i}$, the ray after refraction at this surface will undergo a direction change, $dU'_i = (\partial U'_i / \partial n_i)\, dn_i$, relative to the traced ray path. The transfer coefficients for the intersection points with respect to n-changes may therefore be written down from the corresponding expressions for the curvature coefficients by replacing $\underline{c}$ everywhere by $\underline{n}$. Writing down these relations so that they are available for reference, we have

$$\frac{\partial L'_m}{\partial n_i} = C(n_i)_m \operatorname{cosec} U'_{mk} \qquad (2.44)$$

$$\frac{\partial L'_{ab}}{\partial n_i} = \left[C(n_i)_a \cos U'_{bk} - C(n_i)_b \cos U'_{ak}\right] \operatorname{cosec}(U'_a - U'_b)_k \qquad (2.45)$$

$$\frac{\partial H'_{ab}}{\partial n_i} = -\left[C(n_i)_a \sin U'_{bk} - C(n_i)_b \sin U'_{ak}\right] \operatorname{cosec}(U'_a - U'_b)_k \qquad (2.46)$$

$$\frac{\partial H'_{pr}}{\partial n_i} = C(n_i)_{pr} \sec U'_{prk} - \frac{\partial L'_{ab}}{\partial n_i} \tan U'_{prk} \qquad (2.47)$$

$$\frac{\partial H'_{prf}}{\partial n_i} = C(n_i)_{prf} \sec U'_{prk} - \frac{\partial l'}{\partial n_i} \tan U'_{prk} \qquad (2.48)$$

$$\frac{\partial H'_{id}}{\partial n_i} = (f'/u'_k) \tan U_{pr} \frac{\partial u'_k}{\partial u'_i} \cdot \frac{\partial u'_i}{\partial n_i} \qquad (2.49)$$

In the foregoing equations the quantity, $C(n_i)$, is defined by

$$C(n_i) = C(U_i') \frac{\partial U_i'}{\partial n_i} \tag{2.50}$$

Changes in Axial Separation. The axial separation between the surfaces (i - 1) and i is specified by d_i . This quantity is essentially negative according to our sign convention, but it is more convenient than d_{i-1}' on account of the form of the final equations. In order to change the separation between these two surfaces by an amount δd_i we move the surface i and all surfaces behind it through the distance δd_i . All rays originally incident on the surface i will now meet the surface at new incidence points as a result of the change in separation. Thus if the dotted arc in Figure 2.7 represents the new position of the surface, the traced ray now meets the surface at the new incidence point specified relative to the original incidence point by $dp_i = \sin U_i \, \delta d_i$. Hence, the effect on the path of a ray of a change in axial separation may be described in terms of incidence point changes only. It follows that the expressions for the transfer coefficients for the intersection points with respect to changes of axial separation can be written down from the corresponding expressions for the curvature coefficients by replacing c everywhere by n. Gathering these together for reference, we have

$$\frac{\partial L_m'}{\partial d_i} = C(d_i)_m \operatorname{cosec} U_{mk}' \tag{2.51}$$

$$\frac{\partial L_{ab}'}{\partial d_i} = \left[C(d_i)_a \cos U_{bk}' - C(d_i)_b \cos U_{ak}'\right] \operatorname{cosec}(U_a' - U_b')_k \tag{2.52}$$

$$\frac{\partial H_{ab}'}{\partial d_i} = - \left[C(d_i)_a \sin U_{bk}' - C(d_i)_b \sin U_{ak}'\right] \operatorname{cosec}(U_a' - U_b')_k \tag{2.53}$$

$$\frac{\partial H_{pr}'}{\partial d_i} = C(d_i)_{pr} \sec U_{prk}' - \frac{\partial L_{ab}'}{\partial d_i} \tan U_{prk}' \tag{2.54}$$

$$\frac{\partial H'_{prf}}{\partial d_i} = C(d_i)_{prf} \sec U'_{prk} - \frac{\partial l'}{\partial d_i} \tan U'_{prk} \tag{2.55}$$

$$\frac{\partial H'_{id}}{\partial d_i} = (f'/u'_k) \tan U_{pr} \frac{\partial u'_k}{\partial u'_i} \cdot \frac{\partial u'_i}{\partial d_i} \tag{2.56}$$

The C-quantity occurring in these equations is given by

$$C(d_i) = C(p_i) \frac{\partial p_i}{\partial d_i} = C(p_i) \sin U_i \tag{2.57}$$

7. The Transfer Coefficients for the Aberrations of the System.

It is now obvious that we are in a position to write down expressions for the transfer coefficients for the aberrations of the system with respect to curvature, refractive index, and axial separation, or, at least for those aberrations of which measures are obtained by tracing rays in the tangential plane. Writing these down for the case of curvature changes only, we have for spherical aberration

$$\frac{\partial LA'}{\partial c_i} = \frac{\partial l'}{\partial c_i} - \frac{\partial L'}{\partial c_i} \tag{2.58}$$

for the curvature of the tangential field,

$$\frac{\partial X'_T}{\partial c_i} = \frac{\partial L'_{ab}}{\partial c_i} - \frac{\partial L'}{\partial c_i} \tag{2.59}$$

for the tangential coma,

$$\frac{\partial \, \mathrm{coma}'_T}{\partial c_i} = \frac{\partial H'_{pr}}{\partial c_i} - \frac{\partial H'_{ab}}{\partial c_i} \tag{2.60}$$

and for the distortion,

$$\frac{\partial \, \mathrm{dist}'}{\partial c_i} = \frac{\partial H'_{id}}{\partial c_i} - \frac{\partial H'_{prf}}{\partial c_i} \tag{2.61}$$

If additional rays are traced for the colours '<u>r</u>' and '<u>v</u>' for which the system is to be achromatised, the chromatic aberrations can be brought into the general scheme. Thus, if additional zonal and principal rays are traced, we have

$$\frac{\partial \, Lch'_z}{\partial c_i} = \frac{\partial L'_{zr}}{\partial c_i} - \frac{\partial L'_{zv}}{\partial c_i} \tag{2.62}$$

$$\frac{\partial \, Tch'}{\partial c_i} = \frac{\partial H'_{prr}}{\partial c_i} - \frac{\partial H'_{prv}}{\partial c_i} \tag{2.63}$$

with corresponding expressions for transfer coefficients with respect to n and d. A computation of these aberration transfer coefficients which measure the aberration change per unit change in curvature, axial separation, and refractive index for changes of differential magnitude at any surface of the system, provides the designer with complete information for the final balancing of the aberrations of the system. ~~It provides~~ It provides a very complete analysis of the tendencies of the system in as far as these can be inferred from rays in the tangential plane. H. D. Taylor (1906) writing as an exponent of algebraic methods of optical designing makes a strong case against the sterility of the trigonometrical method on the point that the latter method cannot give an analysis of 'the tendencies of the system'. In the present thesis it is contended that a complete analysis may be obtained by a systematic computation based on the ray trace, and moreover, it is an analysis which is exact for changes of differential magnitude, a feature which is shared by no algebraic method in use at present. Further, the consideration of some second order terms, as we shall see later, renders the present method capable of considerable accuracy when changes are made which are beyond differential size.

8. Transfer Coefficients for Glass Changes for the Components of a System.

In treating the refractive index as one of the variables of the system we have derived transfer coefficients which measure the aberration changes per unit change of the relative refractive index at each surface. In practice, a change of glass affects the complete component, and thus involves two surfaces. It is necessary to develop a transfer coefficient which takes account of the complete change.

Let us denote the successive components of the system by subscripts, $\underline{a}$, $\underline{b}$, ... $\underline{h}$,... using the subscript $\underline{h}$ to denote the general component. It will be sufficient to denote the first and second surfaces of the general component by subscripts 1 and 2. The refractive index of the component is N_h , while that of the medium preceding and following the component will be denoted by N_{h-1} , and N_{h+1} . If the glass of the component is now changed by an amount dN_h it is easily seen that

$$\frac{\partial n_1}{\partial N_h} = -\frac{N_{h-1}}{N_h^{\,2}} \tag{2.64}$$

$$\frac{\partial n_2}{\partial N_h} = \frac{1}{N_{h+1}} \tag{2.65}$$

<u>The Monochromatic Aberrations</u>. Excluding the chromatic aberrations which are treated separately we can now write down the change in any aberration due to a small change in the glass of the general component. Taking spherical aberration as typical of the monochromatic aberrations,

$$\frac{\partial LA'}{\partial N_h} = \frac{\partial LA'}{\partial n_2}\,\frac{\partial n_2}{\partial N_h} + \frac{\partial LA'}{\partial n_1}\,\frac{\partial n_1}{\partial N_h} \qquad ($$

$$= \frac{\partial LA'}{\partial n_2}\,\frac{1}{N_{h+1}} - \frac{\partial LA'}{\partial n_1}\,\frac{N_{h-1}}{N_h^{\,2}} \tag{2.66}$$

Corresponding expressions hold for the other aberrations.

<u>The Chromatic Aberrations</u>. It is usual to adjust the longitudinal chromatic aberration to zero for some zone of the system, often the 0.707 zone. Suppose that the system is to be achromatised for the two colours '$\underline{r}$' and '$\underline{v}$'. We write down expressions for the final intersection lengths of rays

of these colours, incident at the selected zone, in terms of the intersection length of a traced ray of an intermediate colour '$\underline{d}$' . Using first order terms only, we have

$$L'_r = L'_d + \frac{\partial L'_d}{\partial N_a}(N_r - N_d)_a + \frac{\partial L'_d}{\partial N_b}(N_r - N_d)_b + \ldots$$

the sum extending over all the components of the system. Similarly,

$$L'_v = L'_d + \frac{\partial L'_d}{\partial N_a}(N_v - N_d)_a + \frac{\partial L'_d}{\partial N_b}(N_v - N_d)_b + \ldots.$$

whence, $$L'_r - L'_v = - \frac{\partial L'_d}{\partial N_a}(N_v - N_r)_a - \frac{\partial L'_d}{\partial N_b}(N_v - N_r)_b - \ldots.$$

that is, $$Lch' = - \sum \frac{\partial L'_d}{\partial N_h} . P_h \qquad (2.67)$$

where P_h is the partial dispersion $(N_v - N_r)_h$. If we change some of the glasses of the system so that the glass constants are altered by small amounts, we have, to first order,

$$dLch' = - \sum \frac{\partial L'_d}{\partial N_h} . dP_h$$

which means that the appropriate transfer coefficients for Lch' are of the type

$$\frac{\partial Lch'}{\partial P_h} = - \frac{\partial L'_d}{\partial N_h} \qquad (2.68)$$

In a similar way we can deduce a corresponding expression for the transverse chromatic aberration together with its appropriate transfer coefficients. A measure of the transverse chromatic aberration is provided by $H'_{prr} - H'_{prv}$, the intersection points concerned being those in which the principal rays for colours '$\underline{r}$' and '$\underline{v}$' cut a fixed plane perpendicular to the axis, which is either the paraxial image plane or the plane through the intersection point Ab . Using the transfer coefficients for the traced ray of intermediate colour '$\underline{d}$' , we have

$$H'_{prv} = H'_{prd} + \left(\frac{\partial H'_{prd}}{\partial N_a}\right)_{L'} (N_v - N_d)_a + \left(\frac{\partial H'_{prd}}{\partial N_b}\right)_{L'} (N_v - N_d)_b + \dots$$

with a corresponding expression for H'_{prr}. The subscript, L' , indisates that the plane in which the intersection points occur remains fixed. This leads to

$$H'_{prr} - H'_{prv} = -\left(\frac{\partial H'_{prd}}{\partial N_a}\right)_{L'} (N_v - N_r)_a - \left(\frac{\partial H'_{prd}}{\partial N_b}\right)_{L'} (N_v - N_r)_b - \dots$$

that is,

$$Tch' = -\sum\left(\frac{\partial H'_{prd}}{\partial N_h}\right)_{L'} . P_h \tag{2.69}$$

and,

$$\frac{\partial Tch'}{\partial P_h} = -\left(\frac{\partial H'_{prd}}{\partial N_h}\right)_{L'} . \tag{2.70}$$

The expression for $\left(\frac{\partial H'_{prd}}{\partial N_h}\right)_{L'}$ is easily obtained. From (2.47) we have

$$\left(\frac{\partial H'_{pr}}{\partial n_i}\right)_{L'} = c(n_i)_{pr} \sec U'_{prk}$$

and by analogy with (2.66),

$$\left(\frac{\partial H'_{pr}}{\partial N_h}\right)_{L'} = \left(\frac{\partial H'_{pr}}{\partial n_2}\right)_{L'} . \frac{1}{N_{h+1}} - \left(\frac{\partial H'_{pr}}{\partial n_1}\right)_{L'} . \frac{N_{h-1}}{N_h^{\ 2}} \tag{2.71}$$

in which the subscripts 1, 2, refer, as before, to the first and second surfaces of the component <u>h</u>.

Equations (2.67) and (2.69) are of great value in practical designing. They provide a very rapid guide to glass changes which will improve the achromatism of the system, and permit an analysis of the distribution of the final intersection lengths of rays of different colours without the labour of separate tracings. As we shall see in a later chapter, each term in these equations gives directly the contribution of each component to the chromatic aberrations of the final image. This is a very valuable feature of these eqzations. The order of accuracy is fairly high.

It is worthwhile to consider a numerical example at this stage in order to give an idea of the degree of accuracy of the transfer coefficients. We proceed to calculate the longitudinal and transverse chromatic aberrations of a photographic system using equations (2,67) and (2.69) and to compare the results with those obtained by direct ray tracing. We will use the telephoto lens(for which a copy of the general computation is given in a later chapter) of 36 in. focal length for the purpose, calculating the longitudinal chromatic aberration for the zonal ray and the transverse chromatic aberration for the 7.5° pencil, arbitrarily selecting the lines C and F for the spectral range. From the computation on page 226 we extract the <u>chromatic coefficients</u>, $\partial L'_z / \partial N$, for the zonal ray, and from the computation on page 224 the chromatic coefficients $\left[\partial H'_{prd} / \partial N\right]_{L'}$ for the principal ray of the 7.5° pencil. Entering these along with the appropriate dispersion values in the Table, we form the products and by addition of these obtain the values of the two aberrations.

<u>TABLE.</u>

Component	$\frac{\partial L'_{zd}}{\partial N}$	$N_r - N_v$	Product	$\left[\frac{\partial H'_{prd}}{\partial N}\right]_{L'}$	Product
I	$\overline{7244.63}$	$.\overline{01108}$	80.2705	$\overline{124.48}26$	1.37927
II	4031.82	$.\overline{01933}$	$\overline{77.9351}$	63.1826	$1.\overline{22132}$
III	1947.16	$.\overline{01104}$	$\overline{21.4966}$	$\overline{100.7965}$	1.11279
IV	$\overline{1243.13}$	$.\overline{01406}$	17.4784	77.8726	$1.\overline{09475}$
		Lch'_z =	- 1.6828	Tch' =	0.17599

The full trace of the rays for the C and F lines gives

$$L'_{zC} = 461.097 \qquad H'_{prC} = -123.180$$

$$L'_{zF} = 462.765 \qquad H'_{prF} = -123.357$$

whence $\quad Lch' = 1.668 \qquad Tch' = 0.177$

The discrepancy between the two values of the Lch' is 0.0148 mm. which is 6.88 per cent of the total value, and hence of no significance. The difference between the two values of the Tch' is 0.001 mm., or 0.56 per cent of the total aberration. It is thus seen that the expressions developed for the chromatic aberrations are highly accurate, and hence the same is true of the transfer coefficients on which their computation is based. The flexibility of the method should also be noticed, for once the chromatic coefficients are calculated for a ray in one colour we may quickly ascertain the aberration of the rays of any pair of colours provided the refractive indices of the glasses are known for these colours.

9. The Transfer Coefficients for Small Changes of the Diapjragm Position.

If the procedure developed for the alteration of the axial separation of two successive surfaces is applied at the first surface of the system there is no alteration in the spacings of the surfaces of the system. It remains, then, to interpret the meaning of the values of the transfer co-efficinets for d-changes which are obtained by the formal application of the equations (2.51) to (2.57) at the first surface. Thinking of the incident traced rays in the space to the left of the first surface as fixed, a shift of the first surface will result in incidence point changes for all rays except those incident parallel to the principal axis. In particular, the original principal ray of an oblique pencil will strike the first surface at a new incidence point, and will no longer pass through the centre of the diaphragm, but will intersect the principal axis at some neighbouring point. If the diaphragm is shifted so that its centre coincides with this point, the ray in its new course will be a principal ray for the system with this new diaphragm position. The surface shift, made in the manner considered, is therefore equivalent to a change in the position of the diaphragm. The transfer coefficients associated with the surface shift must therefore give information as to the effect of the shift of the diaphragm on the aberrations.

Suppose that the diaphragm is placed between the surfaces $\underline{i}$ and $(\underline{i} + 1)$ of the system, its axial distance from the pole of the first surface being L'_{pr} . The principal ray in an oblique pencil of the original trace is aimed at the centre, O, of the entrance pupil, and passes through the centre, O', of the diaphragm. We denote the distance of O from the pole of the first surface by L_{pr} . If the point O is shifted a distance dL_{pr} the point O' will be displaced through dL'_{pr}, and the ratio of these two displacements is the longitudinal magnification. As an approximation, let us assume that the longitudinal magnification is equal to the square of the lateral magnification, as in paraxial theory, and we may write

$$\frac{dL'_{pr}}{dL_{pr}} = M'^2 = \frac{\sin^2 U_{pr}}{\sin^2 U'_{pri}}$$

A shift of the first surface, carried out in the manner prescribed, is equivalent to an equal shift of the entrance pupil. Hence the effect on any aberration, A, of a small shift of the diaphragm is given by

$$\frac{\partial A}{\partial L'_{pr}} = \frac{\partial A}{\partial d_1} \cdot \frac{\partial L_{pr}}{\partial L'_{pr}} = \frac{\partial A}{\partial d_1} \cdot \frac{\sin^2 U'_{pri}}{\sin^2 U_{pr}} \qquad (2.72)$$

defining a set of transfer coefficients for differential displacements of the diaphragm.

Another property incidental to the transfer coefficients at the first surface is worth mentioning at this point. If a small dp-change is made for any traced ray at the firts surface the resulting dp'_k and dU'_k specify the emergence point and direction of a ray close to the traced ray as it leaves the last surface of the system. The intersection of this close ray with the traced ray will obviously give the position of the focus of a narrow pencil surrounding the traced ray. In particular, if the traced ray considered is the principal ray of an oblique pencil, the focus so defined is the so-called $\underline{t}$ - focus, or focus of close tangential rays. Thus the necessity for a trace for this quantity is eliminated. It will be immediately obvious that the $\underline{t}$ - focus is given by

$$t'_k = \frac{\partial p'_k / \partial p_1}{\partial U'_k / \partial p_1} \qquad (2.73)$$

CHAPTER THREE.

THE GENERAL COMPUTATION OF THE TRANSFER COEFFICIENTS.

The General Computation.

It is proposed to consider next the problem of ordering the general computation of the transfer coefficients developed in the previous chapter. The number of equations, and their frequent complexity, may give the impression of a cumbersome analysis of which the advantages are largely nullified by the excessive labour of the computations involved. It will be shown, however, that the computation can be organised so that it can be carried out by routine computors without any special knowledge of optics. The computation, arranged for machine work, falls into three stages, the ray trace, the single surface differential coefficients, and the general transfer coefficients.

1. The Ray Trace Computation.

Since the ray trace forms the basis of the whole of the subsequent calculations it is essential that it should be carried out with care. For this reason it is wise to retain independent check formulae as a standard practice. We employ the standard ray tracing equations modified as suggested by Comrie (1940), and use the PA-formulae of Conrady (1929) for the check calculation. The latter have the advantage of being more accurate than the standard equations in the case of a long radius, while the value of PA appearing explicitly in the calculation simplifies the drawing of the path of the ray through the system, an essential accompaniment of any trace.

From the standard equation, we have

$$\begin{aligned} \sin I_i &= \frac{1}{r_i}(L_i - r_i)\sin U_i \qquad (3.1) \\ &= \frac{1}{r_i}(L'_{i-1} - d'_{i-1} - r_i)\sin U_i \\ &= \frac{1}{r_i}\left(r_{i-1} + \frac{r_{i-1}\sin I'_{i-1}}{\sin U'_{i-1}} - d'_{i-1} - r_i\right)\sin U_i \end{aligned}$$

$$= \frac{r_{i-1}}{r_i} \sin I' + \frac{r_{i-1} - d'_{i-1} - r_i}{r_i} \sin U_i$$

$$\sin I_i = \alpha_i \sin I'_{i-1} + \beta_i \sin U_i \tag{3.2}$$

where $\alpha, \beta,$ are constants of the system defined by

$$\alpha_i = r_{i-1} / r_i \tag{3.3}$$

$$\beta_i = (r_{i-1} - d'_{i-1} - r_i) / r_i \tag{3.4}$$

for all values of $\underline{i}$ from 2 to $\underline{k}$. At the first surface of the system we write (3.1) for convenience

$$\sin I_1 = \beta_1 \sin U_1 \tag{3.5}$$

where

$$\beta_1 = (L_1 - r_1) / r_1 \tag{3.6}$$

For a plane surface $\alpha = 0, \beta = -1$, so that equation (3.2) reduces to the usual form for the plane. Thus we use

$$\sin I = -\sin U$$

$$\sin U' = n \sin U$$

$$L' = L \tan U \cot U'$$

At the surface following a plane α and β become infinite. This is simply dealt with by treating this surface in a manner similar to a first surface, using equations (3.5) and (3.6).

For the ray trace, then, we have the standard equations in the following forms for machine use,

$$\sin I_i = \alpha_i \sin I'_{i-1} + \beta_i \sin U_i \tag{3.2}$$

$$\sin I' = n \sin I \tag{3.7}$$

$$U' = U + I - I' \tag{3.8}$$

$$L' = r(\sin I' + \sin U') / \sin U' \tag{3.9}$$

The tracing of a principal ray involves a right-to-left trace from the centre of the diaphragm. From a simple transformation of (3.2) it follows that if we put

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: ....

System: ....

Pencil: ....

	Surface
	L
	r
	d'
	N
	N'
$[r_{-1}/r]$	α
	$r_{-1} - d'_{-1} - r$
$[()/r]$	β
	$\alpha \sin I'_{-1}$
	$+ \beta \sin U$
	$\sin I$
$[N/N']$	n
	$\sin I'$
$[Pl. \tan U]$	$\sin I' + \sin U'$
$[Pl. \cot U']$	$r()$
$[L \tan U \cot U']$ $[()/\sin U']$	L'
	U
	$+ I$
	$U+I$
	$- I'$
	U'
	$\sin U$
	$\cos I$
	$\cos I'$
	$\sin U'$
	$\cos U'$
	$\tan U'$
	$1 - \cos(U+I)$
$[\cos I'/\cos I]$	$\partial p'/\partial p$
$[n/()]$	$\partial U'/\partial U$
	$1 - \partial U'/\partial U$
$[()/\cos I]$	e
$[e/r]$	$\partial U'/\partial p$
$[eL \sin U]$	$\partial U'/\partial c$
$[-\sin I/\cos I']$	$\partial U'/\partial n$
$[r(1-\cos(U+I))]$	X
	$d' + X_{+1} - X$
$[()/\cos U']$	D'
	$\frac{1}{2}(I - U)$
	$\cos \frac{1}{2}(I - U)$
	$\frac{1}{2}(I' - U')$
	$\cos \frac{1}{2}(I' - U')$
	$L \sin U$
$[()/\cos \frac{1}{2}(I - U)]$	PA
	$PA \cos \frac{1}{2}(I' - U')$
$[()/\sin U']$	L'

$$\alpha_i' = r_{i+1}/r_i \tag{3.10}$$

$$\beta_i' = (r_{i+1} + d_i' - r_i)/\, r_i \tag{3.11}$$

for values of $\underline{i}$ from 1 to $(k - 1)$, and write $n' = N'/N$, the equations for the right-to-left trace are

$$\sin I_{i-1}' = \alpha_i' \sin I_i + \beta_i' \sin U_i' \tag{3.12}$$

$$\sin I = n' \sin I' \tag{3.13}$$

$$U = U' + I' - I \tag{3.14}$$

$$L = r(\sin I + \sin U)/\sin U \tag{3.15}$$

At the k^{th} surface, of course,

$$\sin I_k' = \frac{1}{r_k}(L_k' - r_k) \sin U_k'$$

$$= \beta_k' \sin U_k' \tag{3.16}$$

The arrangement of the computation is shown on the accompanying computing sheet. The left-to-right ray trace form is set out in the upper part of the sheet and its PA-check appears in the lowest block on the page. Below the angle register of the trace there is set conveniently a block of functions of the angles which are required for the later stages of the computation. It is frequently advantageous to complete this part as the trace proceeds, particularly if the Tables used have the cosines tabulated alongside the sines. On the extreme left of the quantities printed in the margin of the computing sheet certain functions appear in square brackets. These are intended as reminders to the computer to avoid the necessity of looking up the form of the expressions when in doubt. In these bracketed expressions the symbol () refers to the quantity computed on the preceding line. The subscript -1 is used as an abbreviation for $(i - 1)$. Thus in the sixth line of the computation the computer is reminded that the constant $\underline{\alpha}$ is r_{-1} / r, and in the eighth line that the constant $\underline{\beta}$ is $(r_{-1} - d'_{-1} - r)/r$. In lines fourteen to sixteen provision is made for calculating L' for either a spherical surface or a plane. For a spherical surface the entries in these lines are $(\sin I' + \sin U')$, $r(\sin I' + \sin U')$, and $L' = r(\sin I' + \sin U') / \sin U'$ respectively. In the case of a plane the entries are $\tan U$, $\cot U'$, and $L' = L \tan U \cot U'$ respectively. The only Tables which have been readily available in Australia during the war in sufficient numbers for a computing team are the six figure tables of P. F. Adams : Tables of Sines and Cosines at intervals of 10 seconds, with Proportional Parts (Government Printer, Sydney). The great disadvantage of not using the decimal subdivision of the degree is offset to some extent for our purpose in these tables by the tabulation of the cosines alongside the sines, an arrangement which has proved useful for the block of functions appearing below the angle register. Latterly, we have been able to obtain some copies of Peters' Seven Figure Tables, and these are being used regularly now although six figure tables would be adequate for most purposes. Full ray traces are given in the detailed computations appearing later in the paper.

2. The Computation of the Single Surface Coefficients.

These comprise the differential coefficients $\partial p'/\partial p$, $\partial U'/\partial U$, $\partial U'/\partial p$, $\partial U'/\partial c$, $\partial U'/\partial n$ defined in Chapter 2, Section 2. We begin by calculating $\partial p'/\partial p$ from equation (2.2). Then from equations (2.7) and (2.2) we have

$$\frac{\partial U'}{\partial U} = \frac{n}{\partial p'/\partial p} \qquad (3.17)$$

Again, writing

$$e = \left(1 - \frac{\partial U'}{\partial U}\right) / \cos I \qquad (3.18)$$

we calculate $\underline{e}$, and then from equations (2.8) and (2.10) respectively

$$\frac{\partial U'}{\partial p} = \frac{e}{r} \qquad (3.19)$$

and

$$\frac{\partial U'}{\partial c} = e \cdot L \sin U \qquad (3.20)$$

of which the quantity $L \sin U$ appears explicitly in the PA-check.

Fundamental Transfer Coefficients.

Surface	
$\partial U'_+/\partial U_+$	
D_+	
$(\partial U'_k/\partial U'_+)(\partial U'_+/\partial U_+)$	
$(\partial U'_k/\partial \mu_+)\,D_+$	
$\partial U'_k/\partial U'$	
$(\partial \mu'_k/\partial U'_+)(\partial U'_+/\partial U_+)$	
$(\partial \mu'_k/\partial \mu_+)\,D_+$	
$\partial \mu'_k/\partial U'$	
$\partial U'/\partial \mu$	
$\partial \mu'/\partial \mu$	
$(\partial U'_k/\partial U')(\partial U'/\partial \mu)$	
$(\partial U'_k/\partial \mu_+)(\partial \mu'/\partial \mu)$	
$\partial U'_k/\partial \mu$	
$(\partial \mu'_k/\partial U')(\partial U'/\partial \mu)$	
$(\partial \mu'_k/\partial \mu_+)(\partial \mu'/\partial \mu)$	
$\partial \mu'_k/\partial \mu$	

Finally the coefficient $\partial U'/\partial n$ is calculated directly from equation (2.9). The length, D', of the ray path intercepted between successive surfaces is needed for the calculation of the transfer coefficients and is most conveniently computed here. The best procedure for our purpose is to calculate X from the relation

$$X = r(1 - \cos \overline{U + I}) \tag{3.21}$$

and then D' is given by

$$D'_i = (d'_i + X_{i+1} - X_i) / \cos U'_i \tag{3.22}$$

The relation (3.21) is chosen as it only requires one consultation of the tables and one multiplication. The difference of the cosine from unity is, of course, taken as the table is being consulted.

The first stage in the computation of this section of the coefficients is to look up the cosines of I, I', U', and $U + I$, thus completing the the block of functions of the angles of the trace. The remainder of the computation is set conveniently on the ray tracing sheet below this block. All the entries required are then immediately at hand without the necessity of any copying out. The whole calculation requires only nine machine operations for each surface.

3. The Computation of the Fundamental Transfer Coefficients.

We consider now the computation of the fundamental transfer coefficients, $\partial U'_k / \partial U'_i$, $\partial p'_k / \partial U'_i$, $\partial U'_k / \partial p_i$, and $\partial p'_k / \partial p_i$. These are calculated directly from the four equations (2.15) to (2.18), the calculation beginning at the last surface of the system and progressing surface by surface through the system to the first surface. The arrangement of the computation is shown on the opposite page. The general subscript $\underline{i}$ is suppressed as unnecessary and the subscript + is used to denote the next surface, i.e. the surface (i + 1) The entries in lines 1, 2, 9, and 10 are copied up from the previous calculation, and the values of the fundamental transfer coefficients for the last surface are entered, these having the simple values noted in section 4 of Chapter 2. The calculation for the surface (k - 1) is then commenced by putting on the keyboard of the machine the value of $\partial U'_+/\partial U_+$ and multiplying it in turn by $\partial U'_k/\partial U'_+$ and $\partial p'_k/\partial U'_+$, entering the products in lines 3 and 6 respectively. Next, D_+ is multiplied in succession by $\partial U'_k/\partial p_+$ and $\partial p'_k/\partial p_+$, the products being entered in lines 4 and 7. The two additions are then made giving the entries in lines 5 and 8. The remainder of the calculation follows in a similar manner. It will then be seen that there is an orderly swing about the computation which makes it easy and rapid in routine use. Thus for any surface, excluding the last, the entry in line 1 is multiplied by the <u>latest</u> entries in each of lines 5 and 8 and then the entry in line 2 is multiplied in turn by the latest entries in each of lines 13 and 16, after which the two additions are made. The process is now repeated, entry 9 being multiplied in turn by the latest entries in lines 5 and 8, followed by entry 10 being multiplied in turn by the latest entries in lines 13 and 16. The two additions complete the calculation for the surface. The orderliness of this procedure was one of the factors which favoured the selection of the pair of differentials $\delta U'$ and δp for the specification of the changed ray path relative to the traced path. The use of a computing form in which the entries of lines 5, 8, 13, and 16 appear between red lines is a great help in picking up the correct multiplier each time.

4. The Paraxial Ray Trace and Coefficients.

In the case of a paraxial ray there is, as usual, a considerable simplification in the computations. For the sake of completeness we write down the modified forms of the ray tracing equations which are

Computer:

Date:

PARAXIAL RAY TRACE.

System:

	Surface
	y
	l
	r
	d'
	N
	N'
$[r_{-1}/r]$	a
	$r_{-1}—d'_{-1}—r$
$[()/r]$	β
	$a\ .\ i'_{-1}$
	$\beta\ .\ u$
	i
$[N/N']$	n
	i'
	$r(u' + i')$
$[()/u']$	l'
	u
	$u + i$
	u'
	lu
$[()/u']$	l'
	$1 — n$
$[()/r]$	$\partial u'/\partial p$
$[(1 — n)lu]$	$\partial u'/\partial c$

Fundamental Transfer Coefficients.

$\partial u'_+/\partial u_+$
d_+
$(\partial u'_k/\partial u'_+)(\partial u'_+/\partial u_+)$
$(\partial u'_k/\partial p_+)\, d_+$
$\partial u'_k/\partial u'$
$(\partial p'_k/\partial u'_+)(\partial u'_+/\partial u_+)$
$(\partial p'_k/\partial p_+)\, d_+$
$\partial p'_k/\partial u'$
$\partial u'/\partial p$
$(\partial u'_k/\partial u')(\partial u'/\partial p)$
$(\partial u'_k/\partial p_+)$
$\partial u'_k/\partial p$
$(\partial p'_k/\partial u')(\partial u'/\partial p)$
$(\partial p'_k/\partial p_+)$
$\partial p'_k/\partial p$

$$i = \alpha\, i'_{-1} + \beta\, u \qquad (3.23)$$

$$i' = n\, i \qquad (3.24)$$

$$u' = u + i - i' \qquad (3.25)$$

$$l' = r(u' + i') \,/\, u' \qquad (3.26)$$

The expressions for the single surface differential coefficients are greatly simplified also, mainly on account of the cosines of the paraxial angles being unity. Thus we have

$$\partial p'/\partial p = 1 \qquad (3.27)$$

$$\partial U'/\partial U = n \qquad (3.28)$$

$$\partial U'/\partial p = (1 - n)/r \qquad (3.29)$$

$$\partial U'/\partial c = (1 - n).l u \qquad (3.30)$$

$$\partial U'/\partial n = - i \qquad (3.31)$$

while the length of the ray path intercepted between successive surfaces is, of course, simply the axial separation, d'.

The arrangement of the computation is shown on the accompanying paraxial computing form. The single surface coefficients only require three operations at each surface. The simplified form of the computation for the fundamental transfer coefficients is also shown. The first half of the computation is not modified, but as a result of equation (3.27) two multiplications are eliminated in the second half.

5. The Transfer Coefficients for the Intersection Points and the Aberrations.

The considerations of this section bring us to that part of the computations which require the most thorough organisation for successful use of the present methods. It is proposed to consider in some detail the calculations involved in each of the types of rays traced through the system. In the first place we will consider the transfer coefficients for the

Ray....... TRANSFER COEFFICIENTS. Axial Pencil.

	Surface	
	$\partial U'_+ / \partial U_+$	copy
	D_+	copy
	$(\partial U'_k / \partial U'_+)(\partial U'_+ / \partial U_+)$	
	$(\partial U'_k / \partial p_+) D_+$	
	$\partial U'_k / \partial U'$	
	$(\partial p'_k / \partial U'_+)(\partial U'_+ / \partial U_+)$	
	$(\partial p'_k / \partial p_+) D_+$	
	$\partial p'_k / \partial U'$	
	$\partial U' / \partial p$	copy
	$\partial p' / \partial p$	copy
	$(\partial U'_k / \partial U')(\partial U' / \partial p)$	
	$(\partial p'_k / \partial p_+)(\partial p' / \partial p)$	
	$\partial U'_k / \partial p$	
	$(\partial p'_k / \partial U')(\partial U' / \partial p)$	
	$(\partial p'_k / \partial p_+)(\partial p' / \partial p)$	
	$\partial p'_k / \partial p$	
	$- \; S'_k \; \partial U'_k / \partial U'$	
	$C(U')$	
	$C(U') \operatorname{cosec} U'_k$	
	$- \; S'_k \; \partial U'_k / \partial p$	
	$C(p)$	
	$C(p) \operatorname{cosec} U'_k$	
	$\partial U' / \partial c$	copy
$C(c) \operatorname{cosec} U'_k]$	$\partial L'_k / \partial c$	
	$\partial U' / \partial n$	copy
$C(n) \operatorname{cosec} U'_k]$	$\partial L'_k / \partial n$	
$\sin U]$	$\partial p / \partial d$	copy
$C(d) \operatorname{cosec} U'_k]$	$\partial L'_k / \partial d$	

intersection points M, Z, and Px, associated with the axial pencil. The computations for the marginal and zonal rays are identical in form, equations (2.37), (2.44), and (2.51), together with the subsidiary equations (2.21), (2.22), (2.42), (2.50), and (2.57) providing the computing relations. The complete form of the computation for such a ray is set out on the interleaved computing sheet. The first sixteen lines comprise the calculation of the fundamental transfer coefficients as already described. In the next block of the work we compute $C(U')$ from equation (2.21) and $C(p)$ from equation (2.22) and form the products $C(U')$ cosec U'_k and $C(p)$ cosec U'_k. In the next stage the first of these products is multiplied in turn by $\partial U'/\partial c$ and $\partial U'/\partial n$, and the second is multiplied by $\partial p/\partial d$ (= sin U) giving the final values of the required transfer coefficients $\partial L'/\partial c$, $\partial L'/\partial n$ and $\partial L'/\partial d$ respectively. On the computing form the word 'copy' is inserted in a number of lines indicating that the entries appearing in these lines are copied in from previous computations. It will be seen that there are twenty-one machine operations per surface in the complete calculation for an axial ray in this arrangement.

The computation for the paraxial ray follows precisely similar lines leading to the transfer coefficients $\partial l'/\partial c$, $\partial l'/\partial n$, and $\partial l'/\partial d$. On the paraxial computing sheet shown this occupies twenty-seven lines, involving nineteen machine operations per surface. The remaining fifteen lines shown on the paraxial form are placed there for convenience and will be discussed further on.

Coming now to the computations for the rays of an oblique pencil it is necessary to consider each ray separately as the arrangements of the calculation differ considerably. In connection with the rays 'a' and 'b' we require the transfer coefficients for the intersection point Ab. Equations (2.38), Ray 'a'. TRANSFER COEFFICIENTS. Oblique Pencil.

Surface	
$\partial v'_+ / \partial v_+$	copy
D_+	copy
$(\partial v'_k / \partial v'_+)(\partial v'_+ / \partial v_+)$	
$(\partial v'_k / \partial p_+)\ D_+$	
$\partial v'_k / \partial v'$	
$(\partial p'_k / \partial v'_+)(\partial v'_+ / \partial v_+)$	
$(\partial p'_k / \partial p_+)\ D_+$	
$\partial p'_k / \partial v'$	
$\partial v' / \partial p$	copy
$\partial p' / \partial p$	copy
$(\partial v'_k / \partial v')(\partial v' / \partial p)$	
$(\partial v'_k / \partial p_+)(\partial p' / \partial p)$	
$\partial v'_k / \partial p$	
$(\partial p'_k / \partial v')(\partial v' / \partial p)$	
$(\partial p'_k / \partial p_+)(\partial p' / \partial p)$	
$\partial p'_k / \partial p$	
$-S'_k \quad \partial v'_k / \partial v'$	
$C(v')$	
$C(v')\ \mathrm{cosec}()\ \cos v'_{bk}$	
$-C(v')\ \mathrm{cosec}()\ \sin v'_{bk}$	
$-S'_k \quad \partial v'_k / \partial p$	
$C(p)$	
$C(p)\ \mathrm{cosec}()\ \cos v'_{bk}$	
$-C(p)\ \mathrm{cosec}()\ \sin v'_{bk}$	
$\partial v' / \partial c$	copy
$C(c)_a\ \mathrm{cosec}()\ \cos v'_{bk}$	
$\partial v' / \partial n$	copy
$C(n)_a\ \mathrm{cosec}()\ \cos v'_{bk}$	
$\partial p / \partial d$	copy
$C(d)_a\ \mathrm{cosec}()\ \cos v'_{bk}$	
$-C(c)_a\ \mathrm{cosec}()\ \sin v'_{bk}$	
$-C(n)_a\ \mathrm{cosec}()\ \sin v'_{bk}$	
$-C(d)_a\ \mathrm{cosec}()\ \sin v'_{bk}$	
v'_{ak}	
$-\ v'_{bk}$	
$(v'_a - v'_b)_k$	
$\mathrm{Cosec}\ (v'_a - v'_b)_k$	
$\cos v'_{bk}$	
$\sin v'_{bk}$	
$\mathrm{Cosec}\ ()\ \cos v'_{bk}$	
$\mathrm{Cosec}\ ()\ \sin v'_{bk}$	

Paraxial Ray. TRANSFER COEFFICIENTS. Axial Pencil.

Surface	
$\partial u'_+/\partial u_+$	copy
d_+	copy
$(\partial u'_k/\partial u'_+)(\partial u'_+/\partial u_+)$	
$(\partial u'_k/\partial p_+)\ d_+$	
$\partial u'_k/\partial u'$	
$(\partial p'_k/\partial u'_+)(\partial u'_+/\partial u_+)$	
$(\partial p'_k/\partial p_+)\ d_+$	
$\partial p'_k/\partial u'$	
$\partial u'/\partial p$	copy
$(\partial u'_k/\partial u')(\partial u'/\partial p)$	
$(\partial u'_k/\partial p_+)$	copy
$\partial u'_k/\partial p$	
$(\partial p'_k/\partial u')(\partial u'/\partial p)$	
$(\partial p'_k/\partial p_+)$	copy
$\partial p'_k/\partial p$	
$-\ l'_k\ \ \partial u'_k/\partial u'$	
$c(U')$	
$\partial l'_k/\partial u'$	
$-\ l'_k\ \ \partial u'_k/\partial p$	
$c(p)$	
$\partial l'_k/\partial p$	
$\partial u'/\partial c$	copy
$\partial l'_k/\partial c$	
$\partial u'/\partial n$	copy
$\partial l'_k/\partial n$	
$\partial p/\partial d$	copy
$\partial l'_k/\partial d$	
$C(c)_{prf}\ \sec U'_{prk}$	copy
$-\ \partial l'_k/\partial c\ \tan U'_{prk}$	
$\partial H'_{prf}/\partial c$	
$C(n)_{prf}\ \sec U'_{prk}$	copy
$-\ \partial l'/\partial n\ \tan U'_{prk}$	
$\partial H'_{prf}/\partial n$	
$C(d)_{prf}\ \sec U'_{prk}$	copy
$-\ \partial l'/\partial d\ \tan U'_{prk}$	
$\partial H'_{prf}/\partial d$	
$\partial u'_k/\partial c$	
$u'_k \tan U_{pr} \cdot \partial u'_k/\partial c]$ $\partial H'_{id}/\partial c$	
$\partial u'_k/\partial n$	
$\partial H'_{id}/\partial n$	
$\partial u'_k/\partial d$	
$\partial H'_{id}/\partial d$	

Surface.	
$\partial U'_+/\partial U_+$	copy
D_+	copy
$(\partial U'_k/\partial U'_+)(\partial U'_+/\partial U_+)$	
$(\partial U'_k/\partial h_+)\,D_+$	
$\partial U'_k/\partial U'$	
$(\partial h'_k/\partial U'_+)(\partial U'_+/\partial U_+)$	
$(\partial h'_k/\partial h_+)\,D_+$	
$\partial h'_k/\partial U'$	
$\partial U'/\partial h$	copy
$\partial h'/\partial h$	copy
$(\partial U'_k/\partial U')(\partial U'/\partial h)$	
$(\partial U'_k/\partial h_+)(\partial h'/\partial h)$	
$\partial U'_k/\partial h$	
$(\partial h'_k/\partial U')(\partial U'/\partial h)$	
$(\partial h'_k/\partial h_+)(\partial h'/\partial h)$	
$\partial h'_k/\partial h$	
$-S'_k \;\; \partial U'_k/\partial U'$	
$C(U')$	
$C(U')\ \sec U'_k$	
$-S'_k \cdot \partial U'_k/\partial h$	
$C(h)$	
$C(h)\ \sec U'_k$	
$\partial U'/\partial c$	copy
$C(c)\ \sec U'_k$	
$-\partial L'_{ab}/\partial c \;\; \tan U'_k$	
$\partial H'_{pr}/\partial c$	
$\partial U'/\partial n$	copy
$C(n)\ \sec U'_k$	
$-\partial L'_{ab}/\partial n \;\; \tan U'_k$	
$\partial H'_{pr}/\partial n$	
$\partial h/\partial d$	copy
$C(d)\ \sec U'_k$	
$-\partial L'_{ab}/\partial d \;\; \tan U'_k$	
$\partial H'_{pr}/\partial d$	
$-S'_{prfk} \;\; \partial U'_k/\partial U'$	
$C(U')_{prf}$	
$C(U')_{prf}\ \sec U'_k$	
$-S'_{prfk} \;\; \partial U'_k/\partial h$	
$C(h)_{prf}$	
$C(h)_{prf}\ \sec U'_k$	
$C(c)_{prf}\ \sec U'_k$	
$C(n)_{prf}\ \sec U'_k$	
$C(d)_{prf}\ \sec U'_k$	

Surface	
$\partial U'_+ / \partial U_+$	copy
D_+	copy
$(\partial U'_k / \partial U'_+)(\partial U'_+ / \partial U_+)$	
$(\partial U'_k / \partial h_+)\ D_+$	
$\partial U'_k / \partial U'$	
$(\partial h'_k / \partial U'_+)(\partial U'_+ / \partial U_+)$	
$(\partial h'_k / \partial h_+)\ D_+$	
$\partial h'_k / \partial U'$	
$\partial U' / \partial h$	copy
$\partial h' / \partial h$	copy
$(\partial U'_k / \partial U')(\partial U' / \partial h)$	
$(\partial U'_k / \partial h_+)(\partial h' / \partial h)$	
$\partial U'_k / \partial h$	
$(\partial h'_k / \partial U')(\partial U' / \partial h)$	
$(\partial h'_k / \partial h_+)(\partial h' / \partial h)$	
$\partial h'_k / \partial h$	
$-S'_k \quad \partial U'_k / \partial U'$	
$C(U')$	
$-C(U')$ cosec() $\cos U'_{ak}$	
$C(U')$ cosec() $\sin U'_{ak}$	
$-S'_k \quad \partial U'_k / \partial h$	
$C(h)$	
$-C(h)$ cosec() $\cos U'_{ak}$	
$C(h)$ cosec() $\sin U'_{ak}$	
$\partial U' / \partial c$	copy
$-C(c)_b$ cosec() $\cos U'_{ak}$	
$C(c)_a$ cosec() $\cos U'_{bk}$	copy
$\partial L'_{ab} / \partial c$	
$\partial U' / \partial n$	copy
$-C(n)_b$ cosec() $\cos U'_{ak}$	
$C(n)_a$ cosec() $\cos U'_{bk}$	copy
$\partial L'_{ab} / \partial n$	
$\partial h / \partial d$	copy
$-C(d)_b$ cosec() $\cos U'_{ak}$	
$C(d)_a$ cosec() $\cos U'_{bk}$	copy
$\partial L'_{ab} / \partial d$	
$C(c)_b$ cosec() $\sin U'_{ak}$	
$-C(c)_a$ cosec() $\sin U'_{bk}$	copy
$\partial H'_{ab} / \partial c$	
$C(n)_b$ cosec() $\sin U'_{ak}$	
$-C(n)_a$ cosec() $\sin U'_{bk}$	copy
$\partial H'_{ab} / \partial n$	
$C(d)_b$ cosec() $\sin U'_{ak}$	
$-C(d)_a$ cosec() $\sin U'_{bk}$	copy
$\partial H'_{ab} / \partial d$	

(2.39), (2.45), (2.46), (2.52) and (2.53) furnish the main computing relations. The interleaved computing form for the 'a' ray shows the general arrangement for the computation from this ray. The fundamental transfer coefficients occupy the first block of the calculation after which the quantities $C(U')_a$ and $C(p)_a$ are computed. From these the products $C(U')\operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}$, $-C(U')\operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}$, $C(p)\operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}$, and $-C(p)\operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}$ are formed. Multiplication of the first two of these products in turn by $\partial U'/\partial c$ and $\partial U'/\partial n$ gives the quantities

$$C(c)_a \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}, \quad -C(c)_a \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}$$

$$C(n)_a \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}, \quad -C(n)_a \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}.$$

Multiplication of the second pair of products by $\partial p/\partial d$ gives the quantities $C(d)_a \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{bk}$ and $-C(d)_a \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk}$. These last six rather complex quantities are later carried over to the computation sheet of the 'b' ray for combination with the corresponding quantities computed for that ray. The complete computation for the 'a' ray involves twenty-six operations per surface. The computation for the 'b' ray follows exactly corresponding lines and a perusal of the computing form is sufficient explanation in itself. In addition, the combination of the main terms from the 'a' and 'b' computations is made on this sheet giving finally the six transfer coefficients for the co-ordinates of the point Ab.

In connection with the ray 'pr' we require the transfer coefficients for the points Pr and Prf. Equations (2.40), (2.41), (2.47), (2.48), (2.54), and (2.55) supply the necessary computing expressions. The computation sheet for this ray shows a simple arrangement for the computation which is self-explanatory. The transfer coefficients for the point Pr are calculated on this sheet, but it is more convenient to carry the values of the quantities $C(c)_{prf} \sec U'_{prk}$, $C(n)_{prf} \sec U'_{prk}$, and $C(d)_{prf} \sec U'_{prk}$ to the paraxial computation sheet for combination with the paraxial terms $\partial l'/\partial c \tan U'_{prk}$, etc.

to give the transfer coefficients for the point Prf.

The remaining point to consider is the ideal image point, Id . For the transfer coefficients for this point equations (2.43) , (2.49) , and (2.56) furnish the necessary relations for the computations. These are conveniently set out on the bottom of the paraxial computation sheet, and involve only six machine operations for each surface. We have treated here only the case of an infinitely distant object, but the modifications for close objects call for no comment.

A little collecting of terms in accordance with equations (2.58) to (2.61) and the corresponding relations for refractive index and axial separation changes, leads to the transfer coefficients for the aberrations.

In employing the methods of the present paper the writer has used printed forms of the five types just described and found that they provide an entirely satisfactory outline for the computing team. In a few days his team of computers can furnish a complete analysis of, say, a photographic objective together with a description of the 'tendencies' of the system in terms of the transfer coefficients for the tangential aberrations.

CHAPTER FOUR.

A SECOND ORDER CORRECTION TERM.

A Second Order Correction Term.

In the development given in the preceding chapters we have considered only first order terms, a procedure which is adequate for our general purpose as long as small changes only are made in the system. In the course of correcting a lens system it is frequently necessary to make curvature changes of the order of $0.001\ \text{mm.}^{-1}$, and thickness changes of the order of one millimetre. When such changes are made, especially in systems of large aperture, it is found that the shifts of the various intersection points in the final image space as predicted by the use of the transfer coefficients are often in error by as much as 12 per cent. There is one second order term associated with the final stage of the calculation of the transfer coefficients the use of which will improve considerably the order of accuracy of such changes. It arises through the neglecting of the effect of the small change, dU'_k , in the final direction of the emergent ray, on the calculation of the transfer coefficients for the intersection points from the corresponding C - quantities. It would seem that the wider problem of second order terms affecting the fundamental transfer coefficients is such that the amount of computation involved is incommensurate with the results obtained therefrom.

1. The Correction Term for Single Changes in the System.

In Figure 4.1 we represent by R_1M a traced ray in the final image space which intercepts the optical axis of the system at M. As a result of a change made within the system, let us say a curvature change, dc_i , at surface $\underline{i}$, the emergent ray now follows the path R_2M'. The lines MP and MQ are drawn perpendicular to R_1M and R_2M' respectively. Then,

$$MP \quad = \quad dp'_k \; - \; S'_k\, dU'_k \quad = \quad C(c_i)dc_i$$

$$MQ \quad = \quad MP \cos dU'_k$$

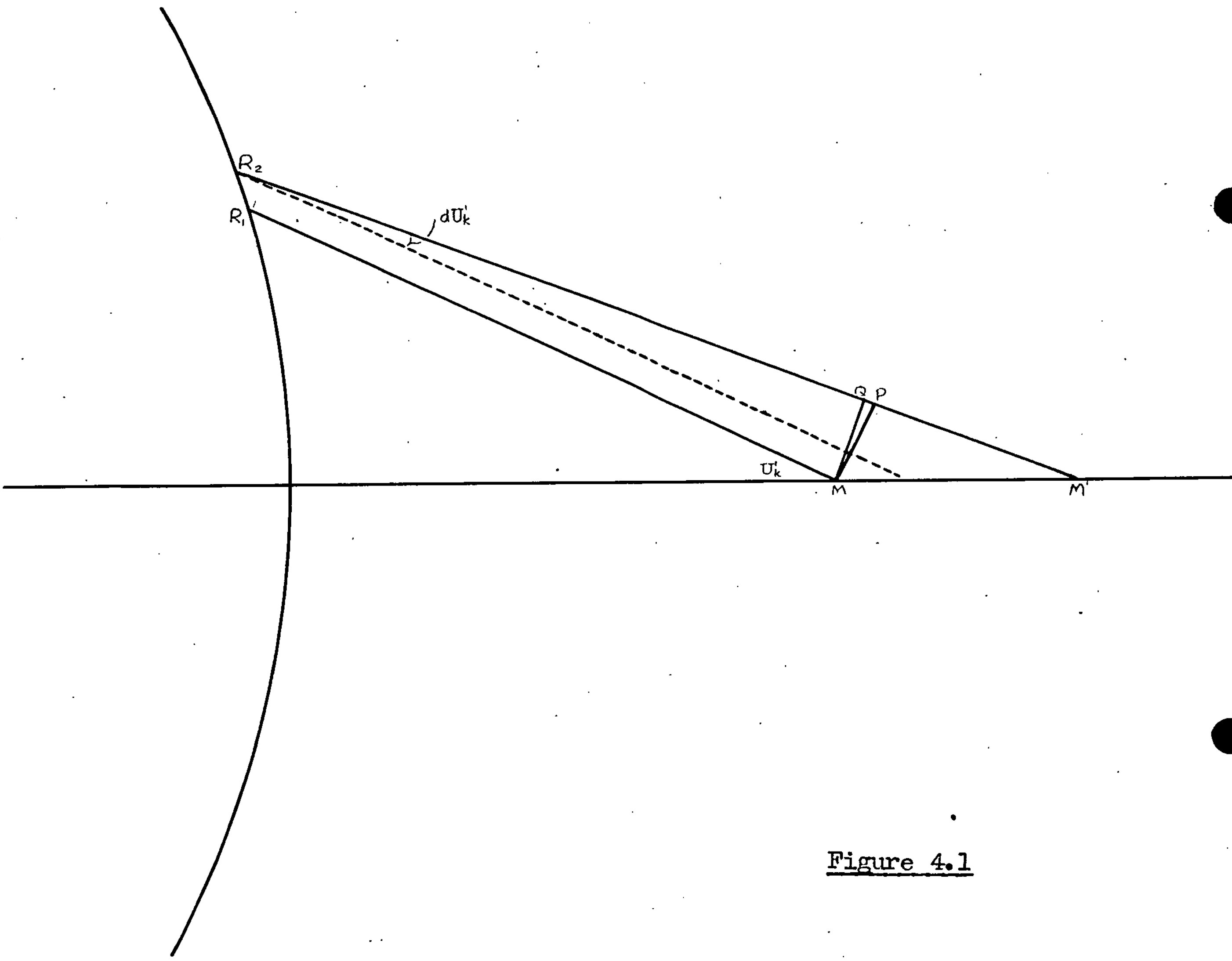


Figure 4.1

$$MM' = MQ \operatorname{cosec}(U_k' + dU_k')$$

$$= C(c_i)dc_i \cos dU_k' \operatorname{cosec}(U_k' + dU_k')$$

$$= C(c_i)dc_i \cos dU_k' \left[\operatorname{cosec} U_k' + \frac{\partial}{\partial U_k'}(\operatorname{cosec} U_k')\, dU_k'\right]$$

It would be unusual for dU_k' to be greater than about 2° so that we may safely put $\cos dU_k' = 1$. Writing $MM' = \delta L'$, the last equation becomes

$$\delta L' = C(c_i)dc_i \operatorname{cosec} U_k' \,(1 - \cot U_k'\, dU_k')$$

$$= C(c_i)\operatorname{cosec} U_k'\, dc_i - C(c_i)\operatorname{cosec} U_k' \cot U_k' \frac{\partial U_k'}{\partial c_i}\,(dc_i)^2 \qquad (4.1)$$

The first term on the right hand side of (4.1) is the first order term which we have used all along, while the second term now appears as a correction term to be added to it. We write this correction term as $\lambda'(c_i)(dc_i)^2$ where

$$\lambda'(c_i) = -\,C(c_i) \operatorname{cosec} U_k' \cot U_k' \frac{\partial U_k'}{\partial c_i} \qquad (4.2)$$

In the next place we consider the intersection of a ray with a fixed plane at right angles to the optical axis. Proceeding ina a manner similar to the foregoing it is easily seen that the change in the H' co-ordinate of the intersection point is given by

$$\delta H' = C(c_i)dc_i \cos dU_k' \sec(U_k' + dU_k')$$

$$= C(c_i)dc_i \cos dU_k' \left[\sec U_k' + \frac{\partial}{\partial U_k'}(\sec U_k')\, dU_k'\right]$$

$$= C(c_i)dc_i \cos dU_k' \sec U_k' \,(1 + \tan U_k'\, dU_k')$$

which on putting $\cos dU' = 1$, and rearranging, becomes

$$\delta H' = C(c_i)\sec U_k'\, dc_i + C(c_i)\sec U_k' \tan U_k' \frac{\partial U_k'}{\partial c_i}\,(dc_i)^2 \qquad (4.3)$$

The first term on the right hand side is the familiar first order of the previous chapters, while the second is the correction term to be applied where greater accuracy is desired. Writing this in the form $\nu'(c_i)(dc_i)^2$

we have $$\nu'(c_i) = C(c_i) \sec U'_k \tan U'_k \frac{\partial U'_k}{\partial c_i} \tag{4.4}$$

There remains yet one important case to consider, that in which the ray intersects a plane at right angles to the optical axis, the plane changing its position when a change is made in the system. This plane may be, for example, the plane of the Ab intersection for some pencil or the paraxial image plane. Having in mind the use to be made of intersections with the paraxial image plane in our subsequent work, we will consider this case in detail. From equation (2.41) it follows that for any ray intersecting the paraxial image plane

$$\frac{\partial H'}{\partial c_i} = C(c_i) \sec U'_k - \frac{\partial l'}{\partial c_i} \tan U'_k$$

Extending this expression to take account of the effect of the change in U'_k it is fairly obvious that the change in the H' co-ordinate of the intersection point resulting from the curvature change, dc_i, will be given by

$$\begin{aligned} \delta H' &= C(c_i) dc_i \sec(U'_k + dU'_k) \\ &\quad + - \left[\frac{\partial l'}{\partial c_i} dc_i + \lambda'_x(c_i)(dc_i)^2\right] \tan(U'_k + dU'_k) \end{aligned} \tag{4.5}$$

where $\lambda'_x(c_i)$ is the λ' correction term for the paraxial ray, which is given by

$$\lambda'_x(c_i) = - \frac{\partial l'}{\partial c_i} \frac{1}{u'_k} \frac{\partial u'_k}{\partial c_i} \tag{4.6}$$

Developing the terms of equation (4.5) we obtain

$$\begin{aligned} \delta H' &= C(c_i)\, dc_i \sec U'_k \,(1 + \tan U'_k\, dU'_k) \\ &\quad - \left[\frac{\partial l'}{\partial c_i} dc_i + \lambda'_x(c_i)(dc_i)^2\right] \tan U'_k (1 + \operatorname{cosec} U'_k \sec U'_k\, dU'_k) \end{aligned}$$

Multiplying out the bracketed expressions and neglecting the third order term which occurs, we obtain

$$\begin{aligned}\delta H' &= \left[C(c_i)\sec U'_k - \frac{\partial l'}{\partial c_i}\tan U'_k\right] dc_i \\ &+ C(c_i)\sec U'_k \tan U'_k \frac{\partial U'_k}{\partial c_i}(dc_i)^2 \\ &- \frac{\partial l'}{\partial c_i}\tan U'_k \operatorname{cosec} U'_k \sec U'_k \frac{\partial U'_k}{\partial c_i}(dc_i)^2 \\ &- \lambda'_x(c_i)\tan U'_k (dc_i)^2 \qquad (4.7)\\ &= \text{first order terms} + [\nu'(c_i) + \gamma'(c_i) + \rho'(c_i)]\,(dc_i)^2\end{aligned}$$

where we have written

$$\gamma'(c_i) = -\frac{\partial l'}{\partial c_i}\tan U'_k \operatorname{cosec} U'_k \sec U'_k \frac{\partial U'_k}{\partial c_i} \qquad (4.8)$$

and

$$\rho'(c_i) = -\lambda'_x(c_i)\tan U'_k \qquad (4.9)$$

$$= \frac{\partial l'}{\partial c_i}\tan U'_k \frac{1}{u'_k}\frac{\partial u'_k}{\partial c_i} \qquad (4.10)$$

If we write now

$$\mu'(c_i) = \nu'(c_i) + \gamma'(c_i) + \rho'(c_i) \qquad (4.11)$$

the correction term to be applied for intersection points in the paraxial image plane is $\mu'(c_i)(dc_i)^2$. In spite of the apparent complexity of the preceding expressions they are computed very easily on account of the fact that portion of each term have already been computed at an earlier stage in the general computation.

Some examples taken at random from a computation are set out below, the system being an f/1.5 camera lens of focal length 106.5 mm. These examples will afford some idea of the effect of the use of the correction term for single changes made within the system.

Example 1. The extreme marginal ray of the axial pencil of this lens had the following coefficients for curvature changes at the first surface:

$$\frac{\partial L'_m}{\partial c_1} = -9913.0 \qquad \lambda'(c_1) = 878530$$

On making a change of curvature at the first surface of amount 0.001 mm.$^{-1}$

the radius thereby changing from + 72.84 mm. to + 67.89 mm., the following results are obtained:

Expected change in L'_m by first order coefficient	=	- 9.913 mm.
Expected change corrected by second order term	=	- 9.034 mm.
Change as calculated by trace	=	- 8.838 mm.

The use of the correction term in this case reduces the error of the predicted change from 12.16 per cent to 2.22 per cent.

Example 2. A paraxial ray traced through the same lens had the following coefficients at the first surface:

$$\frac{\partial l'}{\partial c_1} = -7447.2 \qquad \lambda'(c_1) = 446350$$

For the same change in curvature at the first surface of the system we find the following values:

Expected change in l' by first order coefficient	=	- 7.447 mm.
Expected change, corrected,	=	- 7.001 mm.
Change as calculated by trace	=	- 7.031 mm.

In this case the use of the correction term reduces the error of the predicted change from 6.35 per cent to 0.43 per cent.

Example 3. The principal ray of an oblique pencil incident on the same lens at an angle of 17° to the principal axis had the following coefficients:

$$\frac{\partial H'}{\partial c_1} = 1764.8 \qquad \mu'(c_1) = -117550$$

For the same curvature change made at the first surface of the lens we find

Expected change in H' by first order coefficients	=	1.765 mm.
Expected change, corrected	=	1.647 mm.
Change as calculated by trace	=	1.637 mm.

The error in the prediction is thus reduced from 7.82 to 0.61 per cent.

2. The Correction Term for Several Simultaneous Changes.

In the course of an actual design it is usual to make several changes in the system at the same time and then to check the result of these by a new trace of the system, either in full or in part. If the changes are such that a considerable alteration has been made, or rather that a considerable alteration is expected, it will generally be desirable to use the dU_k' correction term to improve the accuracy of the expected changes. In order to do this we must alter the form of the expressions for these correction terms. In applying the first order theory to several changes made simultaneously we treat the change in the position of an intersection point as a total differential. Thus, suppose that changes are made in the curvatures of surfaces $i, j, \ldots r$, then for an intersection point on the principal axis

$$\begin{aligned} dL' &= \frac{\partial L'}{\partial c_i} dc_i + \frac{\partial L'}{\partial c_j} dc_j + \ldots + \frac{\partial L'}{\partial c_r} dc_r \\ &= \sum_i^r \left(\frac{\partial p_k'}{\partial c_i} - S_k' \frac{\partial U_k'}{\partial c_i}\right) \operatorname{cosec} U_k' \; dc_i \\ &= \left(\sum dp_k' - S_k' \sum dU_k'\right) \operatorname{cosec} U_k' \end{aligned} \tag{4.12}$$

Primarily, then, we must regard the summation of the effects of the several differential changes as made at the last surface so that the ray under consideration as a result of the changes will emerge from the last surface with an incidence point displacement, $\Sigma dp_k'$, and a direction change, $\Sigma dU_k'$, relative to the traced ray. For changes that are not differential in size we still obtain a very useful tool for design by assuming that the summation property still holds, and the order of accuracy can be improved by applying the dU_k' corrections. The value of the dU_k' for which we correct is, of course, the sum of the various dU_k' due to the several changes. The correction terms must therefore be written as factors multiplying the resultant angle changes, $\Sigma dU_k'$, and not as factors of $(dc_i)^2$.. e as in the previous section. Anticipating the needs of subsequent methods we consider two main cases, the intersection point of the paraxial ray with the principal axis and the intersection point of any ray with the paraxial image plane.

By analogy with equations (4.1) it is easily seen that

$$\delta l' = \Sigma \frac{\partial l'}{\partial c_i} dc_i (1 - \cot u'_k \, \Sigma du'_k)$$

$$= \Sigma \frac{\partial l'}{\partial c_i} dc_i - \left(\Sigma \frac{\partial l'}{\partial c_i} \frac{1}{u'_k} dc_i\right) (\Sigma du'_k) \qquad (4.13)$$

Hence the correction term to be applied for the paraxial intersection point is

$$(\Sigma \lambda_i (c_i) \, dc_i)(\Sigma du'_k)$$

where
$$\lambda (c_i) = - \frac{\partial l'}{\partial c_i} \frac{1}{u'_k} \qquad (4.14)$$

and
$$\Sigma du'_k = \Sigma \frac{\partial u'_k}{\partial c_i} dc_i \qquad (4.15)$$

For the intersection point of any ray with the paraxial image plane we have by analogy with equation (4.5)

$$\delta H' = [\Sigma C(c_i) \, dc_i] \sec (U'_k + \Sigma dU'_k)$$

$$- \left[\Sigma \frac{\partial l'}{\partial c_i} dc_i + \Sigma \lambda(c_i) \, dc_i . \Sigma du'_k\right] \tan(U'_k + \Sigma dU'_k)$$

$$= \Sigma [C(c_i) \sec U'_k - \frac{\partial l'}{\partial c_i} \tan U'_k] dc_i$$

$$+ \Sigma C(c_i) \sec U'_k \, dc_i \tan U'_k \cdot \Sigma dU'_k$$

$$- \Sigma \frac{\partial l'}{\partial c_i} \tan U'_k \, dc_i \, \mathrm{cosec}\, U'_k \sec U'_k \cdot \Sigma dU'_k$$

$$+ \Sigma \frac{\partial l'}{\partial c_i} \tan U'_k \, dc_i \frac{1}{u'_k} \cdot \Sigma du'_k \qquad (4.16)$$

$$= \text{first order term} + \Sigma \nu(c_i) dc_i \cdot \Sigma dU'_k$$

$$+ \Sigma \gamma (c_i) dc_i \cdot \Sigma dU'_k + \Sigma \rho (c_i) dc_i \cdot \Sigma du'_k$$

where
$$\nu(c_i) = C(c_i) \sec U'_k \tan U'_k \tag{4.17}$$

$$\gamma(c_i) = -\frac{\partial U'}{\partial c_i} \tan U'_k \sec U'_k \operatorname{cosec} U'_k \tag{4.18}$$

$$\rho(c_i) = \frac{\partial U'}{\partial c_i} \tan U'_k \frac{1}{u'_k} \tag{4.19}$$

Combining the firt two of these terms by writing

$$\pi(c_i) = \nu(c_i) + \gamma(c_i) \tag{4.20}$$

the correction term becomes

$$\Sigma\, \pi(c_i)dc_i \cdot \Sigma\, dU'_k + \Sigma\, \rho(c_i)dc_i \cdot \Sigma\, du'_k$$

It will have been noticed that the quantities corresponding to $\nu(c_i)$, $\gamma(c_i)$, $\rho(c_i)$, and $\lambda(c_i)$ developed in the first section are distinguished from those of the present section by the addition of a dash. This serves to remind the computor that they relate to single changes only.

Although the expressions arrived at for the correction term look rather formidable they are easily computed at the end of the general computation because considerable portions of them appear explicitly in the general computation. We have not considered any corrections for d-changes or n-changes, but these are precisely similar to those developed for curvature changes and can be written down by analogy with the foregoing.

CHAPTER V.

THE DIFFERENTIAL CORRECTION OF AN OPTICAL SYSTEM.

The Differential Correction of an Optical System.

In the trigonometrical correction of an optical system the usual procedure has required, in the first place, the selection of a surface of the system at which it is known on general theoretical grounds that small changes in some datum associated with the surface will produce aberration changes of the desired type. The appropriate datum is then altered by a small amount, typical rays are re-traced through the altered system, and the aberration changes noted. One or more additional changes in the same datum may be made, followed by fresh ray tracings, and the amount of the required datum change deduced by interpolation or from a rough plot of the aberration changes against the datum changes. In the notes of his lectures given in 1919, Conrady says ' In arriving at a new design by the method of trigonometrical trials - the only one available for the deep curvatures of microscope objectives - it will be found necessary and also quickest to proceed systematically, changing one datum at a time so as to be able to interpolate in a simple and straightforward way for the desired correction. The temptation will often be strong to superpose a second change, but students may be assured that the process does not pay.'

In contrast to this procedure we will consider now the immense advance in designing power that is obtained by the use of the differential transfer coefficients. This consideration will be simplified by the use of specific examples. In the first place the correction state of the system at the given stage of the development of the design is analysed by ray traces of a number of pencils - an axial pencil, and perhaps three or more oblique pencils at intervals of 7 - 10 degrees across the field. The number of pencils and the angular intervals between them, as also the particular rays traced for the pencil, will depend on the type and requirements of the system. Following this the general computation of the transfer coefficients for the intersection points and the aberrations is carried out along the lines suggested in Chapter III. This results in a complete analysis of the tendencies of the system in its present state, and the work up to this stage may be carried out by routine computing assistants, leaving the designer completely free for other matters. As an example we consider a portion of the analysis of a wide angle photographic objective of the Ross Xpres type working at an aperture of f/4. On the first of the accompanying computing sheets the transfer coefficients for the intersection points are collected for three pencils only. This sheet contains an immense amount of information, giving accurate measures of the rate of change of thirteen quantities associated with the traced rays in the final image space with the twenty-five to thirty constructional parameters of the system. On the second sheet the transfer coefficients for the aberrations sampled by the pencils are set out, the array of numbers being considerably reduced. This sheet summarises very completely the tendencies of the system. A study of the transfer coefficients at each surface shows exactly what may be achieved in the way of correction by alterations made at that surface. Thus, for example, if we consider the transfer coefficients for d-changes at the sixth surface, that is, for changes which alter the central airspace of the system, it is seen that considerable correction of curvature of field may be obtained by this means, accompanied by very small coma changes, small distortion changes except at the edge of the field, and quite small spherical aberration changes. Similarly, it is seen that a shift of surface five admits of considerable changes in the distortion of the system. Curvature changes at surface four will admit a considerable amount of coma, together with

TABLE I.

Transfer Coefficients for the Intersection Points. Wide Angle Lens.

		Axial Pencil.			22° Oblique Pencil.					32° Oblique Pencil.				
		L'_m	L'_z	l'	L'_{ab}	H'_{ab}	H'_{pr}	H'_{id}	H'_{prf}	L'_{ab}	H'_{ab}	H'_{pr}	H'_{id}	H'_{prf}
1.	∂/∂c	15376·	12198·	10319·	11195·	3745·	3726	3456·	3374·	10899·	5426·	5420·	5346·	5061·
	∂/∂d				·0242	·0122	·0099		·0001	0991	0849	0440		0172
	∂/∂n	997·2	890·1	835·7	933·0	409·2	403·9	279·9	364·8	991·9	665·4	661·5	433·0	565·0
2.	∂/∂c	1159·	937·0	812·7	1347·	476·6	587·6	294·6	372·9	2224·	1224·	1272·	455·6	399·1
	∂/∂d	5·681	4·432	3·739	3·306	·3442	·3765	·6377	·5506	4·886	3·992	4·479	·9862	·6688
	∂/∂n	551·0	463·1	414·3	593·3	155·2	159·7	150·2	87·76	841·8	364·6	373·6	232·3	109·3
3.	∂/∂c	9469·	7609·	6434·	7176·	2700·	2616·	2362·	2318	6938·	3912·	3813·	3654·	3502·
	∂/∂d	6·243	4·856	4·087	3·958	·5137	·5565	·7153	·6082	3·677	·4313	·4542	1·106	·7073
	∂/∂n	397·7	365·6	349·2	393·5	184·2	181·0	128·2	163·2	426·3	305·6	302·5	198·3	254·8
4.	∂/∂c	7362·	6624·	6114·	7332·	2865·	2754·	2309·	2265·	8298·	4935·	4754·	3541·	3404·
	∂/∂d	·0875	·0500	·0216	3·107	1·464	1·367	·0790	·1263	8·557	5·827	5·534	·1221	·2568
	∂/∂n	624·9	610·6	609·4	733·3	372·2	365·6	230·1	315·8	922·8	705·1	693·4	355·9	499·6
5.	∂/∂c	5807·	5701·	5700·	7091·	2805·	2733·	2210·	2173·	9035·	5469·	5333·	3418·	3270·
	∂/∂d	2·001	1·901	1·888	2·243	·2136	·2171	·1756	·0742	2·131	·0197	·0236	·2715	·1266
	∂/∂n	108·3	107·3	108·7	137·0	93·04	92·05	42·16	80·69	190·1	184·8	183·0	65·21	132·7
6.	∂/∂c	5701·	5442·	5284·	6628·	2725·	2794·	2225·	2253·	7947·	5080·	5207·	3442·	3560·
	∂/∂d	·3815	·3408	·3333	1·109	·1343	·1436	·1589	·1683	2·149	·8323	·8289	·2457	·2937
	∂/∂n	321·5	315·9	314·6	423·9	84·92	87·44	132·5	43·46	567·7	201·0	206·0	204·9	49·48
7.	∂/∂c	6493·	5844·	5367·	6181·	2675·	2746·	2376·	2418·	6278·	4289·	4395·	3674·	3832
	∂/∂d	·0224	·0256	·0231	·2407	·0540	·0448	·0737	·1509	·9651	·1748	·2131	·1140	·3979
	∂/∂n	266·5	256·8	251·0	312·1	96·08	98·37	111·1	73·79	379·9	184·7	189·0	171·8	109·2
8.	∂/∂c	9290·	6846·	5407·	5902·	2649·	2683·	2454·	2484·	6174·	3716·	3751·	3795·	3895·
	∂/∂d	1·941	1·561	1·335	3·906	·8957	·9998	0034	·0338	6·796	3·111	3·290	·0052	·0864
	∂/∂n	936·5	840·6	782·0	902·1	312·9	318·3	354·9	270·0	971·6	520·7	528·5	548·9	411·2
9.	∂/∂c	856·5	732·2	666·2	1004·	465·7	456·7	306·7	320·9	2076·	1399·	1409·	474·3	537·5
	∂/∂d	1·637	1·008	·6497	·0903	·4887	·4189	·6364	·7165	·8724	·4196	·3191	·9842	1·260
	∂/∂n	298·4	245·2	215·6	280·0	193·8	191·2	99·25	165·3	379·0	386·7	382·4	153·5	281·4
10.	∂/∂c	10616·	8366·	6980·	7343·	3649·	3687·	3429·	3491·	6767·	5405·	5383·	5303·	5515·
	∂/∂d	1·466	·8937	·5686	·2239	·3842	·3194	·5708	·6381	1·142	·1383	·0476	·8827	1·106
	∂/∂n	344·0	301·4	275·9	316·1	117·3	118·6	135·6	102·5	338·1	194·3	196·0	209·6	157·6

TABLE II.

Transfer Coefficients for the Aberrations. Wide Angle Lens.

		Axial Pencil		22° Pencil.			32° Pencil.		
		LA'_m	LA'_z	X'_T	$Coma'_T$	$Dist'$	X'_T	$Coma'_T$	$Dist'$
1.	$\partial/\partial c$	5058.	1879.	1003.	$\overline{19.19}$	82.68	1299.	$\overline{6.24}$	284.6
	$\partial/\partial d$			.0242	$\overline{.0023}$	$\overline{.0001}$	.0991	.0409	.0172
	$\partial/\partial n$	$\overline{161.4}$	$\overline{54.37}$	42.91	5.283	84.83	101.8	3.809	132.0
2.	$\partial/\partial c$	$\overline{346.8}$	$\overline{124.3}$	409.6	$\overline{111.0}$	78.36	1288.	$\overline{47.94}$	$\overline{56.48}$
	$\partial/\partial d$	$\overline{1.942}$	$\overline{.6928}$	$\overline{1.126}$	$\overline{.0324}$	$\overline{.0871}$	$\overline{9.018}$	.4872	$\overline{.3175}$
	$\partial/\partial n$	136.7	48.73	$\overline{130.2}$	4.517	62.43	$\overline{378.7}$	8.953	123.0
3.	$\partial/\partial c$	$\overline{3035.}$	$\overline{1175.}$	$\overline{432.2}$	83.56	$\overline{44.38}$	$\overline{670.7}$	99.31	$\overline{151.9}$
	$\partial/\partial d$	$\overline{2.156}$	$\overline{.7691}$	$\overline{.8976}$	$\overline{.0428}$	$\overline{.1070}$	$\overline{1.178}$	$\overline{.0229}$	$\overline{.3989}$
	$\partial/\partial n$	$\overline{48.51}$	$\overline{16.40}$	27.97	3.207	34.99	60.72	3.158	56.50
4.	$\partial/\partial c$	1248.	510.0	$\overline{707.9}$	$\overline{110.3}$	4430	$\overline{1674.}$	$\overline{181.3}$	167.0
	$\partial/\partial d$	$\overline{.0659}$	$\overline{.0284}$	3.057	.0976	.0473	8.507	.2930	.1347
	$\partial/\partial n$	$\overline{15.53}$	$\overline{1.237}$	122.7	6.620	85.66	313.4	11.71	143.7
5.	$\partial/\partial c$	$\overline{107.3}$	$\overline{1.10}$	1390.	72.02	$\overline{36.82}$	3334.	136.8	$\overline{148.0}$
	$\partial/\partial d$	$\overline{.1136}$	$\overline{.0135}$	.3417	$\overline{.0085}$	$\overline{.1013}$	.2294	.0039	$\overline{.3981}$
	$\partial/\partial n$	0.379	1.420	29.66	.9846	38.53	82.78	1.849	67.48
6.	$\partial/\partial c$	417.6	158.1	$\overline{1186}$	68.84	$\overline{27.76}$	$\overline{2506.}$	126.9	$\overline{117.8}$
	$\partial/\partial d$	$\overline{.0482}$	$\overline{.0075}$	.7681	$\overline{.0093}$	$\overline{.0094}$	1.808	.0034	$\overline{.0480}$
	$\partial/\partial n$	6.983	1.332	$\overline{108.0}$	2.519	89.02	$\overline{251.8}$	4.996	155.4
7.	$\partial/\partial c$	$\overline{1126.}$	$\overline{477.5}$	336.8	$\overline{70.11}$	42.70	433.4	$\overline{106.1}$	158.1
	$\partial/\partial d$	.0007	$\overline{.0025}$	$\overline{.2663}$	.0091	.0772	$\overline{.9908}$	.0383	.2839
	$\partial/\partial n$	15.56	5.801	$\overline{55.33}$	2.287	37.29	$\overline{123.2}$	4.211	62.58
8.	$\partial/\partial c$	3883.	1439.	943.4	33.85	$\overline{30.12}$	1671.	35.55	$\overline{99.46}$
	$\partial/\partial d$	.6057	.2261	2.344	$\overline{.1041}$	$\overline{.0304}$	5.234	$\overline{.1793}$	.0812
	$\partial/\partial n$	154.6	58.64	$\overline{61.46}$	5.342	84.93	$\overline{131.0}$	7.815	$\overline{137.7}$
9.	$\partial/\partial c$	190.4	66.03	$\overline{271.7}$	$\overline{9.00}$	$\overline{14.21}$	$\overline{1344.}$	10.25	$\overline{63.22}$
	$\partial/\partial d$	$\overline{.9870}$	.3579	$\overline{7.098}$	.0698	.0801	$\overline{1.880}$	.1005	.2760
	$\partial/\partial n$	$\overline{82.76}$	$\overline{29.58}$	34.79	2.544	66.09	133.8	4.214	127.9
10.	$\partial/\partial c$	3636.	$\overline{1386.}$	$\overline{1022.}$	11.94	61.97	$\overline{1599.}$	21.91	212.0
	$\partial/\partial d$	$\overline{.8978}$	$\overline{.3252}$	$\overline{1.118}$	.0648	.0673	$\overline{2.036}$	.0906	.2228
	$\partial/\partial n$	68.02	25.44	$\overline{14.68}$	1.271	33.07	$\overline{36.76}$	1.777	52.06

TABLE III.

Aberration Transfer Coefficients for Glass Changes.

	LA'_m	LA'_z	X'_T	$Coma'_T$	Dist'
$\partial/\partial N_a$	150.13	52.302	-100.48	0.891	7.780
$\partial/\partial N_b$	-140.51	-49.202	115.59	0.167	-7.029
$\partial/\partial N_c$	7.11	1.96	- 23.54	-1.887	1.382
$\partial/\partial N_d$	12.53	5.22	- 8.48	1.195	-1.322
$\partial/\partial N_e$	-115.64	-42.75	47.16	-0.651	5.529
$\partial/\partial N_f$	117.16	43.00	-35.34	-0.240	-6.180

TABLE IV.

The Chromatic Coefficients of the Axial and 22° Oblique Pencils.

	L'_z	l'	$C(n)\sec U'_{pr}$
$\partial/\partial N_a$	- 640.16	- 587.86	257.93
$\partial/\partial N_b$	677.23	628.03	- 288.52
$\partial/\partial N_c$	- 157.51	- 155.56	66.51
$\partial/\partial N_d$	- 119.76	- 114.53	60.45
$\partial/\partial N_e$	502.11	459.35	- 251.03
$\partial/\partial N_f$	- 446.98	- 403.98	232.18

curvature, spherical aberration, and distortion. Comparing curvature changes at surface five with those at surface four, we notice that the same strong coma and distortion effects are present, but very much heavier curvature changes and much smaller spherical aberration changes. In this general way the detailed correcting properites of all the surfaces are studied carefully in view of selecting a combination of suitable changes which will improve the correction state of the system.

In making definite changes during the correction of the system it is to be remembered that the transfer coefficients are accurate for changes of differential magnitude only. This does not make them of no value, but in predicting the effect of finite changes by their use some attention must be paid to the magnitude of the changes produced in the image space. Thus suppose that a curvature change of 0.001 $mm.^{-1}$ were contemplated at surface one in our example. According to the value of $\partial LA'/\partial c$ for the marginal ray, the value of LA'_m should change by 5.06 mm. A glance at the first sheet of transfer coefficients, however, shows that the intersection point M will shift through a distance of 15.38 mm. These changes are far from differential size, and hence it cannot be expected that the predicted values of the aberration changes will be accurately achieved when displacements of this order are being made in the image space. Again, the same change of curvature will presumably cause a coma change of only 0.0062 in the 32^{o} pencil, whereas, in actuality, the intersection points Ab and Pr are each displaced laterally through distances of the order of 5.4 mm. What is to be stressed is that some discretion must be used when finite changes of considerable size are contemplated, particularly when the coefficients employed are derived as a sum or difference of two other coefficients each of which may be large. The amounts of the absolute displacements in the image space resulting from any change in the parameters should always be examined.

2. The Analysis of the Chromatic Correction of the System.

The real significance of the transfer coefficients for n-changes which appear in Tables I and II will be understood from the information which is presented in Tables III and IV. In these are collected the transfer coefficients of the aberrations for glass changes at any component and the chromatic coefficients respectively. In the material of Table IV there is a basis for the analysis of the colour correction state of the system, and we proceed to discuss this next. Since the lens is based on the symmetrical type, and hence the glasses in the front and rear halves of the system are similar, we may combine the chromatic coefficients for the components which have similar glasses since in the evaluation of the chromatic aberration for any spectral range these coefficients would each be multiplied by the same dispersion value. These combined chromatic coefficients appear in the second columns of Tables VI to VIII. In the rough design of the system at the present stage the components a and f were to be of DBC 615553, components b and e of LF 549467, and components c and d of HC 519604. The dispersions of these glasses are given in Table V.

TABLE V.

Dispersions of the Glasses.

	DBC	LF	HC
$N_b - N_D$	- .00517	- .00539	- .00406
$N_C - N_D$	- .00328	- .00342	- .00256
$N_e - N_D$	.00274	.00291	.00211
$N_F - N_D$	.00785	.00835	.00603
$N_{G'} - N_D$	.01422	.01529	.01089

TABLE VI.

Contributions to Lch'_z for Different Spectral Ranges.

Comp.	Chr. Coeff.	Range b - D	Range C - D	Range e - D	Range F - D	Range G'- D
a, f	-1087.14	5.6205	3.5658	-2.9788	-8.5340	-15.4591
b, e	+1179.33	-6.3566	-4.0333	3.4319	9.8478	18.0320
d, c	-277.27	1.1257	0.7098	-0.5850	-1.6719	- 3.0195
Lch'_z		0.3896	0.2423	-0.1319	-0.3585	-0.4466

TABLE VII.

Contributions to lch' for Different Spectral Ranges.

Comp.	Chr. Coeff.	Range b - D	Range C - D	Range e - D	Range F - D	Range G'- D
a, f	- 991.83	5.1278	3.2523	-2.7176	-7.7859	-14.1039
b, e	1087.38	-5.8610	-3.7188	3.1643	9.0796	16.6260
c, d	- 270.09	1.0966	0.6914	-0.5699	-1.6286	- 2.9413
lch'		0.3634	0.2258	-0.1232	-0.3349	-0.4192

TABLE VIII.

Contributions to Tch' at 22° for Different Spectral Ranges.

Comp.	Chr. Coeff.	Range b - D	Range C - D	Range e - D	Range F - D	Range G'- D
a, f	-1.7013	.00880	.00558	-.00466	-.01336	-.02419
b, e	0.1610	-.00087	-.00055	.00047	.00134	.00246
c, d	2.5810	-.01048	-.00661	.00545	.01556	.02811
Tch'		-0.00255	-0.00158	0.00126	0.00354	0.00638

Using equation (2.67) for the longitudinal chromatic aberration we enter in Tables VI and VII the contributions of the pairs of components to the chromatic aberrations of the system at the zonal and paraxial regions for various spectral ranges. The sum of the contributions in each column gives the chromatic aberration of the system for the given spectral range. Similarly, using the relation of equation (2.69) we enter in Table VIII the contributions to the transverse chromatic aberration at an obliquity of 22°.

We may proceed now to see how the colour correction may be improved by the selection of slightly different glasses. A study of Table VI shows at once that an increase in the V-number of the either of the crown glasses will decrease the amounts of the contributions of these components and thus will improve the chromatic correction. Alternatively, of course, a decrease in the V-number of the light flint glass of components b and e will produce a similar result. The same changes will reduce the chromatic aberration in the paraxial region, but when we consider Table VIII it is seen that an increase in the V-number of the DBC glass of components a and f will increase the transverse aberration, but only by a small amount, while the increase in the V-number of the HC of components d and c will reduce it. Considering these two pencils only, then, we might well make a change to a DBC of slightly higher V-number, and also substitute a BSC for the HC of components c and d. By way of example we consider changing to the types DBC 613568 and BSC 517641. The changes in the dispersions for the different spectral ranges are listed in Table IX, and then, in Table X are set out the corresponding changes in the contributions of the components concerned for the zonal chromatic aberration. Adding these changes to the residuals in Table VI we obtain the new zonal chromatic aberration. The corresp-

TABLE IX.

The Changes in the Dispersions.

	DBC	BSC
$N_b - N_D$	+ .00015	+ .00023
$N_C - N_D$	+ .00010	+ .00015
$N_e - N_D$	- .00008	- .00009
$N_F - N_D$	- .00023	- .00037
$N_{G'} - N_D$	- .00042	- .00072

TABLE X.

The Changes in the Contributions to the Zonal Chromatic Aberration.

Comp.	Chr. Coeff.	Range b - D	Range C - D	Range e - D	Range F - D	Range G'-D
a, f	-1087.14	-0.1631	-0.1087	0.0870	0.2500	0.4567
c, d	- 277.27	-0.0638	-0.0416	0.0250	0.1026	0.1996
	New Lch'_z	0.1627	0.0920	-0.0199	-0.0059	+0.2097

TABLE XI.

The Changes in the Contributions to the Transverse Chromatic Aberration.

Comp.	Chr. Coeff.	Range b - D	Range C - D	Range e - D	Range F - D	Range G'-D
a, f	-1.7013	-.00026	-.00017	.00013	.00039	.00071
c, d	2.5810	.00060	.00039	-.00023	-.00095	-.00186
	New Tch'	-0.00221	-0.00136	0.00116	0.00298	0.00523

onding changes for the transverse chromatic aberration are given in Table XI. In this way the chromatic coefficients afford a most powerful means of analysing the colour correction of the system, and provide

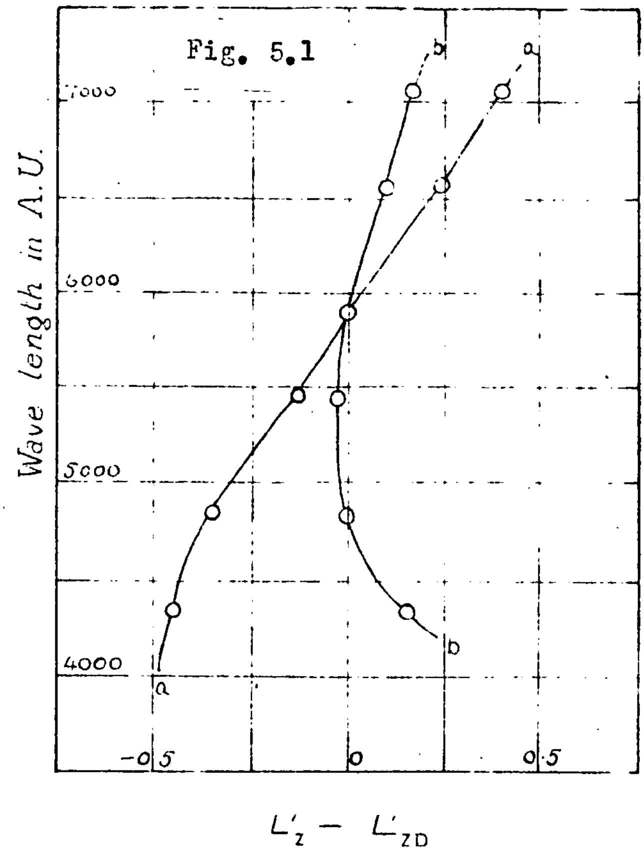


Fig. 5.1
Wave length in A.U.
7000
6000
5000
4000
a
b
-0.5
0
0.5
$L'_Z - L'_{ZD}$

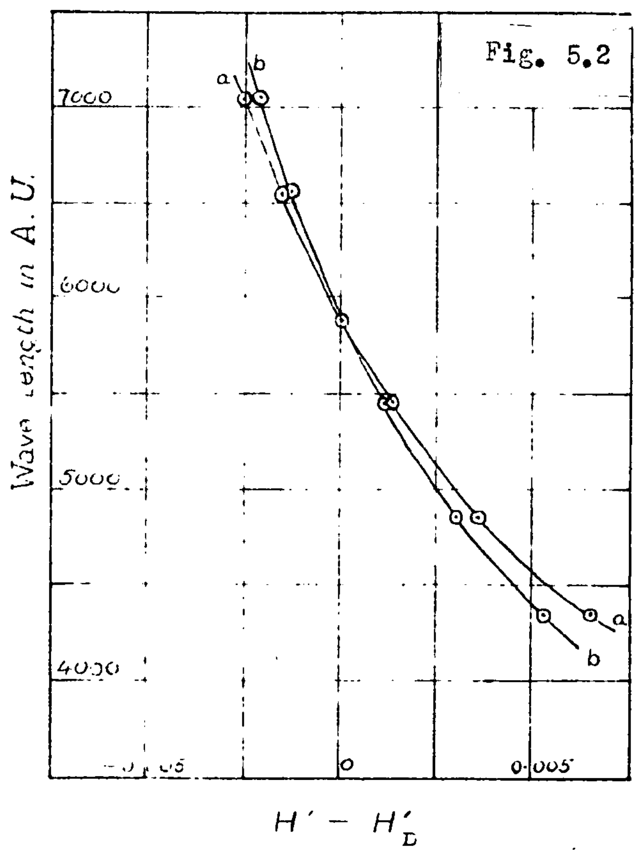


Fig. 5.2
Wave length in A.U.
7000
6000
5000
4000
a
b
0
0.005
$H' - H'_D$

a rapid means of adjusting the achromatism of the system to a desired type by varying the glasses selected. Graphs summarising the colour correction of the system for the 0.707 axial zone and the transverse chromatic aberration at 22° are shown in Figures 5.1 and 5.2. In each figure the curve <u>aa</u> represents the original adjustment and the curves <u>bb</u> the new colour correction state consequent upon the glass changes considered in the example. These graphs thus give a complete summary of the information available as to the secondary spectrum of the system.

3. A Brief Example of the Correction Method.

As an example of the way in which the coefficients may be used we will consider some changes made with this particular photographic objective. Only fragments of the correction process will be given as it is really unnecessary to multiply examples. At one stage some of the residuals were given by the trace as

$$LA'_z = -0.005 \qquad \overset{22^{\circ}}{X'_T} = -1.332$$

$$\mathrm{coma}'_T = 0.0014$$

$$\mathrm{dist}' = 0.0085$$

To be able to use available glass melts it became necessary to make the following glass changes

Glass	Component	δN_D
DBC	a, f	+.0016
LF	b, e	+.0023
HC	c, d	+.0003

The resulting changes in the residuals are calculated by means of the coefficients in Table III, a sample of the calculation being set out in Table XII. These changes gave the new values of the residuals for the

TABLE XII.

Computation of the Effects of the Glass Changes.

Comp.	δN_D	LA'_z	X'_T	$Coma'_T$	Dist'
a, f	.0016	.1525	-.2173	.00104	.0026
b, e	.0023	-.2115	.3743	-.00111	-.0035
c, d	.0003	.0021	-.0096	-.00021	.0000
		-.057	.147	-.0003	-.0009

system with the new glasses as

		22°	32°
$LA'_z = -0.060$		$X'_T = -1.185$	$X'_T = -3.520$
		$coma'_T = -0.0617$	$coma'_T = -0.128$
		$dist' = 0.0076$	$dist' = 0.015$

At this stage it was considered necessary to adjust the system so as to remove the strong negative curvature, bringing the zonal spherical aberration to a positive value of about 0.15 and holding the coma at 22° to a small value and the distortion likewise. If it was possible to reduce the coma at 32° at the same time this was also desired. With these general objects in view the coefficients in Tables II and I were studied closely and after some preliminary calculations the following set of changes were selected

$$\delta c_1 = +0.000165 \qquad \delta d_5 = -0.613 \qquad \delta d_7 = -0.817$$
$$\delta c_4 = -0.000199 \qquad \delta d_6 = +1.263$$

Multiplying each of these changes by their coefficients in Table II the following values of the principal residuals were expected from the changes

	22°	32°
$LA'_z = +0.15$	$X'_T = +0.10$	$X'_T = 0.00$
	$coma'_T = 0.00$	
	$dist' = 0.00$	

A new trace of the system gave the new residuals as

$$LA'_z = +0.21 \qquad X'_T = 0.40 \qquad X'_T = +0.01$$

$$\text{coma}'_T = 0.005 \qquad \text{coma}'_T = -0.053$$

$$\text{dist}' = -0.013 \qquad \text{dist}' = -0.049$$

showing that the selected changes have achieved quite closely the result desired. The order of agreement is quite good considering that in all eight changes, the three glass changes and the five changes listed above, had been made. In fact, there are really eleven changes for the glass changes were made at each of the six components.

The degree of accuracy of the predictions given by the transfer coefficients for finite changes depends of course on the relation between the parameter and the aberration which depends upon it. Looking at the matter geometrically, the transfer coefficient represents the slope of the tangent to the curve, which represents the relation between the aberration and the parameter, at the point corresponding to the value of the parameter in the design. If the slope of this curve is not changing rapidly the predicted values will agree quite closely with the true values of the aberrations obtained from a new trace. If, however, the present value of the parameter is close to a turning point of the curve, as sometimes happens, then the predicted values from a finite change may be seriously in error. This is not a fault of the coefficient itself, but a limitation upon its usefulness, which must be kept in mind. The writer's experience is that such unlucky situations are relatively infrequent.

CHAPTER SIX.

THE CONTRIBUTIONS MADE BY THE INDIVIDUAL SURFACES OF A LENS SYSTEM TO THE ABERRATIONS OF THE FINAL IMAGE.

Contributions to the Aberrations.

In the course of the design of an optical system it is frequently of considerable value to know the contributions which each surface makes to the various aberrations of the final image. Methods of computing the surface contributions to the primary aberrations are well known, but for aberrations other than these there is no general analysis into surface contributions available. Using the properties of the differential transfer coefficients developed in the preceding chapters it becomes possible to solve this problem, and it is proposed to outline in this chapter the development of a trigonometrical method of calculating the surface contributions characteristic of a lens system for a given aperture and position in the image field.

1. The Contributions to the Longitudinal and Transverse Chromatic Aberrations.

In equation (2.67) we have derived an expression for the longitudinal chromatic aberration of the system at a given zone in the form of a summation over all the components of the system, a component being a single glass lens. This relation is based on the trace of a single ray incident at the selected zone, the refractive indices used being those for some colour 'd' which is intermediate between the two colours 'r' and 'v' for which the system is to be achromatised. Writing this equation down again, we have

$$Lch' = \sum \frac{\partial L_d'}{\partial N_h} (N_r - N_v)_h \tag{6.1}$$

where

$$\frac{\partial L_d'}{\partial N_h} = \frac{\partial L_d'}{\partial n_2} \frac{1}{N_{h+1}} - \frac{\partial L_d'}{\partial n_1} \frac{N_{h-1}}{N_h^{\,2}} \tag{6.2}$$

Each term in the summation of (6.1) represents the contribution made by a component to the chromatic aberration of the final image. Thus for the general component h the trigonometrically determined contribution to the longitudinal chromatic aberration, which we denote by $TLchC_h'$, we have

$$\mathrm{TLchC}'_h \quad = \quad \frac{\partial L'_d}{\partial N_h} (N_r - N_v)_h \tag{6.3}$$

The contributions made by the components calculated in the foregoing manner are of great value in practical designing, being much more useful than the contributions made by the individual surfaces of the system. Their value lies in the fact that they are such simple functions of the dispersions, which renders the choice of glass types to produce achromatism a relatively simple matter. Their power will be appreciated more fully after the discussion of examples which will be considered later.

If the surface contributions are required rather than the component contributions it is obvious that these may be easily obtained from equation (6.3) ; each term of the summation is derived from a pair of terms as is seen from equation (6.2), the two members of each pair referring to single surfaces. Hence the longitudinal chromatic aberration may be expressed as a summation over the individual surfaces of the system. For each air-glass surface there is a single term while for a cemented surface there are two terms each involving the partial dispersion of one of the glasses forming the contact surface. The term, or sum of the two terms, relating to each surface gives the contribution of that surface to the final value of Lch'.

For the paraxial longitudinal chromatic aberration which is often required we have similarly,

$$\mathrm{lch}' \quad = \sum \frac{\partial l'_d}{\partial N_h} (N_r - N_v)_h \tag{6.4}$$

whence

$$\mathrm{TlchC}'_h \quad = \quad \frac{\partial l'_d}{\partial N_h} (N_r - N_v)_h \tag{6.5}$$

In a similar manner equation (2.69) gives a summation expression for the transverse chromatic aberration. This is based on the trace of a single principal ray of the desired obliquity using the refractive indices of the intermediate colour 'd' as before. Writing down this expression

Calculation by Conrady's Method.

LONGITUDINAL AND TRANSVERSE PRIMARY CHROMATIC ABERRATION.

BOOTH TELEPHOTO LENS.

Surface	1	2	3	4	5	6	7
N'	1·61517	1·000	1·66068	1·000	1·57338	1·57846	1·000
N	1·000	1·61517	1·000	1·66068	1·000	1·57338	1·57846
i'	·00382698	·00727649	·00444477	·00047640	·00447740	·000041532	·00440891
$N'i'$	·00618122	·00727649	·00733689	·00047640	·00704465	·00006556	·00440891
$l'u'$	1·00	·9299134	·928991	·912507	·491844	·497513	·505938
$l'u'N'i'$	·00618122	·00676661	·00681590	·000434718	·00346487	·0000326152	·00223064
$\delta N'$	·01108	0·0	·01933	0.	·01104	·01406	0.
$\delta N'/N'$	·00685996		·0117103		·00701674	·00890742	
$\delta N/N$		·00685996		·0117103		·00701674	·00890742
$(\delta N'/N'-\delta N/N)$	·00685996	·00685996	·0117103	·0117103	·00701674	·00189068	·00890742
u'							·00109435
$1/N'u'^2$							835002
$l'u'N'i'(\delta N'/N'-\delta N/N)$	$\cdot 0^4 424029$	$\cdot 0^4 46418$	$\cdot 0^4 798162$	$\cdot 0^5 509068$	$\cdot 0^4 243121$	$\cdot 0^7 616649$	$\cdot 0^4 198692$
$lch\,C'$	35.4065	38·7591	66·6467	4·2507	20·3006	0·05149	16·5908
i'_{pr}/i'	4·09388	34·4111	34·2411	291·654	8·48734	4126·13	3·57447
$u'_k\, i'_{pr}/i'$	·00448014	·0376578	·0374717	·319172	·00928812	4·51434	·00391172
[7·5°] $Tch\,C'$	·158626	1·45958	2·49737	1·35671	0·188554	·232445	·064899
[4°] $Tch\,C'$	·083389	·76729	1·31285	·713213	·099122	·122195	·034117
[10°] $Tch\,C'$	·209884	1·93123	3·30437	1·79512	·249483	·307567	·08587

$u'_{k4^\circ}/u'_{k7\cdot5^\circ} = 0\cdot525693$

$u'_{k10^\circ}/u'_{k7\cdot5^\circ} = 1\cdot82314$

again we have

$$Tch' = \sum \left[\frac{\partial H'_{prd}}{\partial N_h}\right]_{L'} (N_r - N_v)_h \qquad (6.6)$$

As noted previously in Chapter 2 the subscript L' indicates that the plane in which the ordinates, H' , are measured is a fixed plane. This plane might be either the paraxial plane or the plane of the Ab intersection point of the pencil under consideration. We shall always use the paraxial plane for this purpose. In a manner precisely similar to the longitudinal case it follows that the trigonometrically determined contribution to the transverse chromatic aberration for the general component, h , is

$$TTchC'_h = \left[\frac{\partial H'_{prd}}{\partial N_h}\right]_{L'} (N_r - N_v)_h \qquad (6.7)$$

If the surface contributions are desired then the equation (6.6) must be expressed as a summation over the surfaces using equation (2.71) for the purpose. These have not the practical interest or power of the contributions made by the components.

It is appropriate to consider a numerical example at this stage, and we shall use for the purpose the Booth Telephoto Lens which is analysed in detail in subsequent chapters. Conrady (1929) gives a method of calculating the contributions to the primary chromatic aberrations of a lens system, and we proceed to compare the contributions calculated by Conrady's method with those calculated by the present trigonometrical method. In the first place we consider the longitudinal chromatic aberration and calculate the primary contributions by Conrady's method. The computation is shown on the accomapnying sheet. Next, for the trigonometrical method we extract the chromatic coefficients for the individual surfaces calculated for the paraxial ray from the computation on page 226 and enter them in Table 6.1 together with the values of $(N_C - N_F)$ appropriate to each surface. The product of each chromatic coefficient with the corresponding dispersion

SURFACE CONTRIBUTIONS TO THE TRANSVERSE CHROMATIC ABERRATION.

Booth Telephoto Lens.

Surface	1.	2	3	4	5	6	7
$N_C - N_F$	$\overline{.01108}$	$\overline{.01108}$	$\overline{.01933}$	$\overline{.01933}$	$\overline{.01104}$	$\overline{.01104}$ / .01406	$\overline{.01406}$
				4° Pencil			
Chromatic Coefficient	7.7181	$\overline{70.223}$	68.873	$\overline{37.384}$	$\overline{9.170}$	$\overline{41.740}$ / 41.605	$\overline{2.397}$
T Tch C'	$\overline{.08552}$	.77807	$\overline{1.33130}$	.72264	.10124	$\overline{.12417}$	.03370
				7.5° Pencil			
Chromatic Coefficient	14.925	$\overline{139.408}$	136.740	$\overline{73.557}$	$\overline{17.989}$	$\overline{82.807}$ / 82.541	$\overline{4.6783}$
T Tch C'	$\overline{.16537}$	1.5446	$\overline{2.6432}$	1.4219	.19860	$\overline{.24633}$	.06578
				10° Pencil			
Chromatic Coefficient	20.084	$\overline{192.95}$	189.27	$\overline{100.84}$	$\overline{24.588}$	$\overline{114.319}$ / 114.151	$\overline{6.360}$
T Tch C'	$\overline{.22254}$	2.1379	$\overline{3.6587}$	1.9493	.27139	$\overline{.34067}$	.08943

value gives the trigonometrically determined contribution to the longitudinal chromatic aberration made by that surface. These are entered in the next column of the Table. In the final column the values of the primary contributions as calculated by the Conrady method are entered for comparison.

TABLE 6.1

Surface	Chromatic Coefficient	$N_C - N_F$	TLchC'	Conrady lchC'
1	$\overline{3195.50}$	$.\overline{01108}$	35.4061	35.4065
2	$\overline{3498.07}$	$.\overline{01108}$	38.7586	35.7591
3	3447.82	$.\overline{01933}$	$\overline{66.6464}$	$\overline{66.6467}$
4	219.90	$.\overline{01933}$	$\overline{4.2507}$	$\overline{4.2507}$
5	1838.77	$.\overline{01104}$	$\overline{20.3000}$	$\overline{20.3006}$
6	17.3106 $\overline{17.2549}$	$.\overline{01104}$ $.\overline{01406}$	0.0515	0.0515
7	$\overline{1179.99}$	$.\overline{01406}$	16.5907	16.5908

It will be seen that the contributions determined by the two methods are in exact agreement. Thus when applied to the paraxial region the trigonometrical method gives an analysis of the surface contributions, and consequently of the component contributions also, which is in exact agreement with the primary theory of Conrady.

We proceed now to compare the contributions to the primary transverse chromatic aberration as calculated by the Conrady method with the trigonometrically determined contributions at obliquities of 4°, 7.5°, and 10°, for the same lens. The calculation of the primary contbibutions appears on the same computing sheet as the longitudinal aberration, and the contributions according to the trigonometrical method are set out on the next computing sheet. The chromatic coefficients used are taken from the general computation on page 124, being those for the principal rays at the three obliquities. For comparison the values for the contributions given by the two methods are arranged in Table 6.2

TABLE 6.2

Surface	TTchC'	Conrady TchC'	TTchC'	Conrady TchC'	TTchC'	Conrady TchC'
	4°		7.5°		10°	
1	$\overline{0.0855}$	$\overline{0.0834}$	$\overline{0.1654}$	$\overline{0.1586}$	$\overline{0.2225}$	$\overline{0.2099}$
2	0.7781	0.7673	1.5446	1.4596	2.1379	1.9312
3	$\overline{1.3313}$	$\overline{1.3129}$	$\overline{2.6432}$	$\overline{2.4974}$	$\overline{3.6587}$	$\overline{3.3044}$
4	0.7226	0.7132	1.4219	1.3567	1.9493	1.7951
5	0.1012	0.0991	0.1986	0.1886	0.2714	0.2495
6	$\overline{0.1242}$	$\overline{0.1222}$	$\overline{0.2463}$	$\overset{6}{\overline{0.2324}}$	$\overline{0.3407}$	$\overline{0.3076}$
7	0.0337	0.0341	0.0658	0.0649	0.0894	0.0859

The Table shows that at an obliquity of 4° the values are in fairly close agreement, the small difference being due to abberations of higher order already present. As the obliquity increases the two sets of values diverge further, as is to be expected. At very small obliquity the two sets of values would be in exact agreement.

It will thus be seen that a perfectly general trigonometrical method of calculating the surface and component distributions for the chromatic aberrations of a system has been established. It is highly accurate, as the numerical examples given in Chapter 2, Section 8, have shown. When applied to the paraxial region and to pencils of small obliquity it is in exact agreement with existing primary theory.

2. The Surface Contributions to the Spherical Aberration.

In the ray-trace of the axial pencil we have traced rays incident at certain zones of the lens system. In the image formed after refraction at surface i of the system there is, for any such zone, a spherical aberration given by

$$LA'_i = l'_i - L'_i \tag{6.8}$$

which may be expressed in angular measure as

$$AA'_i = LA'_i \sin U'_i \, / \, S'_i$$

$$= LA'_i \sin U'_i \cos U'_i \, / \, (L'_i - X'_i) \tag{6.9}$$

The angle AA'_i measures the departure of the ray from the ideal direction after refraction at the surface $\underline{i}$. If the ray could be turned through an angle $dU'_i = - AA'_i$, the spherical aberration behind the surface $\underline{i}$ would be reduced to zero, and the effect at the final image would be the introduction of an amount of spherical aberration opposite to that introduced by the actual refractions over the first $\underline{i}$ surfaces of the system. In other words, the shift of the intersection point of the axial ray with the principal axis in the final image space due to this imagined rotation of the ray through the angle $dU'_i = - AA'_i$, at surface $\underline{i}$, will provide a measure of the sum of the spherical aberration contributions of the first $\underline{i}$ surfaces of the system. We shall denote this quantity by the symbol $\overset{i}{\Sigma}\, TSC'$, the letters standing for the trigonometrically determined spherical contribution. Hence

$$\overset{i}{\Sigma}\, TSC' = \frac{\partial L'_k}{\partial U'_i} \, dU'_i = - \frac{\partial L'_k}{\partial U'_i} \, AA'_i \tag{6.10}$$

and the individual contributions from the surfaces are given by

$$TSC' = \overset{i}{\Sigma}\, TSC' - \overset{i-1}{\Sigma}\, TSC' \tag{6.11}$$

This gives an unrestricted trigonometrical method of analysing the surface contributions to the final spherical aberration. Little additional computation is involved when the general methods of this paper are employed, for the necessary transfer coefficients are alredy obtained in the main computation. As to the accuracy of the method the only limitation is that we are using differential transfer coefficients to calculate the effects in the final image space of the rotations, dU'_i , of the ray. If the latter are large the transfer coefficients will not predict their effects accurately. This difficulty is met to a very considerable extent by using the appropriate second order correction term, as developed in Chapter IV. If dU'_k is the change in the direction of the final emergent ray due to the rotation, dU'_i , of the ray behind surface i , then the shift of the intersection point of the emergent ray along the principal axis is, by analogy with the first form of equation (4.1)

$$\delta L' = C(U'_i) \operatorname{cosec} U'_k \, dU'_i \, (1 - \cot U'_k \, dU'_k) \tag{6.12}$$

and with due attention to the sign of the longitudinal aberration this leads to

$$\sum^{i} TSC' = C(U'_i) \operatorname{cosec} U'_k \, dU'_i \, (1 - \cot U'_k \, dU'_k) \tag{6.13}$$

This equation (6.13) takes precedence over the earlier equation (6.10) and should be used for the normal calculation of $\sum^{i} TSC'$. The separate surface contributions then follow from equation (6.11).

3. The Surface Contributions to the Distortion.

From the results of the trace of an oblique pencil through the lens system we can obtain the value of the distortion for this obliquity in the images formed after refraction at the successive surfaces of the system. In the usual linear measure this is

$$\text{dist}'_i = (H'_{id} - H'_{prf})_i$$

and in angular measure

$$AD'_i = \text{dist}'_i \cos U'_{pri} \,/\, S'_{prfi} \tag{6.14}$$

$$= \text{dist}'_i \cos^2 U'_{pri} \,/\, (l'_i - X'_{pri}) \tag{6.15}$$

This angle, AD'_i , measures the departure of the principal ray from the ideal direction, as regards distortion, after refraction at surface i . If we could swing the principle ray through an angle, $dU'_{pri} = -AD'_i$, we would restore the ray to its ideal direction and thereby remove from the final image the amount of distortion introduced by the refractions at the first $\underline{i}$ surfaces of the system. The effect of such an imagined rotation, then, is to introduce into the final image an amount of distortion opposite to that contributed by the first $\underline{i}$ surfaces of the system, an amount which we denote by $-\sum^{i} TDC'$. Setting this down in symbols, we have

$$-\sum^{i} TDC' = d(H'_{id} - H'_{prf}) = -dH'_{prf}$$

$$= -\frac{\partial H'_{prf}}{\partial U'_i} dU'_{pri} = \frac{\partial H'_{prf}}{\partial U'_i} AD'_i$$

that is,

$$\sum^{i} TDC' = -\frac{\partial H'_{prf}}{\partial U'_i} AD'_i \qquad (6.16)$$

and the contribution from the surface $\underline{i}$ is given by

$$TDC'_i = \sum^{i} TDC' - \sum^{i-1} TDC' \qquad (6.17)$$

Equation (6.16) is adequate provided the angles, dU'_{pri}, are small. Improved accuracy is obtained by using the second order correction terms; the difference in the amount of computation involved is so little that equation (6.18), developed below, should be used as a matter of course. If dU'_k is the change in the direction of the principal ray as it leaves the last surface due to the rotation of the ray through dU'_i behind surface $\underline{i}$, then the change in the H' co-ordinate of the intersection point of the ray with the paraxial image plane may be written down by analogy with the equations preceding (4.3). Omitting the subscript 'pr', as all the symbols refer to the principal ray, we have

$$\delta H' = C(U'_i)_{prf} \sec U'_k \, dU'_i \, (1 + \tan U'_k \, dU'_k)$$

and hence,

$$\sum^{i} TDC' = C(U'_i)_{prf} \sec U'_k \, dU'_i \, (1 + \tan U'_k \, dU'_k) \qquad (6.18)$$

4. The Surface Contributions to the Tangential Coma.

We begin by computing the value of the tangential coma for a pencil of given obliquity in the images formed after refraction at the successive surfaces of the system. If we choose to regard the 'a' and 'b' rays of the pencil as aberrant, then, it is the displacement of their intersection point, Ab, from the intersection point Pr which afflicts the system with coma. If after refraction at surface $\underline{i}$ we could swing the 'a' and 'b' rays until both pass through the point Pr the tangential coma originally present in the image behind the surface $\underline{i}$ would be removed. Such a swing of the rays would also leave the curvature of the tangential field unchanged. The effect of such an imagined rotation of the rays on the final image would be the introduction of an amount of tangential coma opposite to that contributed by the first $\underline{i}$ surfaces of the system, that is, an amount which we denote by $- \;{}^{i}\,TCC'$. Thus we have the basis for the development of equations for computing the contribution of each surface of the system to the tangential coma of the final image.

The angle through which the 'a' ray must be roteted is

$$dU'_{ai} = -\, coma'_{Ti} \cos U'_{ai} \,/\, S'_{ai}$$

$$= -\, coma'_{Ti} \cos^2 U'_{ai} \,/\, (L'_{ab} - X'_{a})_i \tag{6.19}$$

The corresponding angle for the rotation of the 'b' ray is

$$dU'_{bi} = -\, coma'_{Ti} \cos^2 U'_{bi} \,/\, (L'_{ab} - X'_{b})_i \tag{6.20}$$

In order to calculate the contributions at the final image we must follow the shifts of the intersection points Ab and Pr in the final image space due to the rotations contemplated. These are

$$dL'_{ab} = \frac{\partial L'_{ab}}{\partial U'_{ai}}\, dU'_{ai} + \frac{\partial L'_{ab}}{\partial U'_{bi}}\, dU'_{bi} \tag{6.21}$$

$$dH'_{ab} = \frac{\partial H'_{ab}}{\partial U'_{ai}} dU'_{ai} + \frac{\partial H'_{ab}}{\partial U'_{bi}} dU'_{bi} \qquad (6.22)$$

$$dH'_{pr} = - dL'_{ab} \tan U'_{prk} \qquad (6.23)$$

It then follows that

$$\sum^{i} TCC' = dH'_{ab} - dH'_{pr} \qquad (6.24)$$

Finally the contribution made by the surface $\underline{i}$ is given by

$$TCC'_i = \sum^{i} TCC' - \sum^{i-1} TCC' \qquad (6.25)$$

As in the previous cases it is generally advisable to use the second order correction terms, depending of course on the values of the angles dU'_{ai} and dU'_{bi}. The appropriate forms are easily written down: equation (6.21) becomes

$$dL'_{ab} = C(U'_i)_a \operatorname{cosec} (U'_a - U'_b)_k \cos U'_{bk} dU'_{ai} (1 - \cot(U'_a - U'_b)_k dU'_{ak})$$
$$- C(U'_i)_b \operatorname{cosec}(U'_a - U'_b)_k \cos U'_{ak} dU'_{bi} (1 - \cot(U'_a - U'_b)_k dU'_{bk}) \qquad (6.2$$

and (6.22) becomes

$$dH'_{ab} = - C(U'_i)_a \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{bk} dU'_{ai} (1 - \cot(U'_a - U'_b)_k dU'_{ak})$$
$$+ C(U'_i)_b \operatorname{cosec}(U'_a - U'_b)_k \sin U'_{ak} dU'_{bi} (1 - \cot(U'_a - U'_b)_k dU'_{bk}) \qquad (6.2$$

These rather formidable-looking expressions compute very easily as large parts of them are available from the normal transfer coefficients. They only require three more operations than the apparently simple expressions in (6.21) and (6.22).

The essence of the method just described is to compute the effect at the final image of a change made in the refracted rays behind surface $\underline{i}$, the change being such that the coma present behind this surface is removed. If such a change is made the typical rays considered become incident on the next surface of the system as coma-free. Another change which would remove the coma in the image behind the surface $\underline{i}$ is a swing of the principal ray until it passes through the point Ab_i. It may seem more logical to make the changes in the directions of the rays 'a' and 'b' because we normally regard coma as due to the aberrant behaviour of the outer rays of the pencil , but it is simpler to make the change of the principal ray for the purpose of calculating the coma contributions as it involves less than half the amount of computation. The fact that the contributions computed either way are in close agreement confirms the validity of the general method developed in this chapter.

The angle through which the principal ray must be rotated to remove the coma behind surface <u>i</u> is

$$\begin{aligned} dU'_{pri} &= \text{coma}'_{Ti} \cos U'_{pri} \;/\; S'_{pri} \\ &= \text{coma}'_{Ti} \cos^2 U'_{pri} \;/\; (L'_{ab} - X'_{pr})_i \end{aligned} \tag{6.28}$$

Following out the effect of such a rotation we deduce that

$$\sum^{i} TCC' = -\frac{\partial H'_{pr}}{\partial U'_{pri}} \, dU'_{pri} \tag{6.29}$$

An additional advantage of the latter method is the ease with which the second order correction term may be applied. In a manner exactly similar to that used in the case of distortion, we have

$$\sum^{i} TCC' = - C(U'_i)_{pr} \sec U'_k \; dU'_{pri} (1 + \tan U'_k \, dU'_{prk}) \tag{6.30}$$

Finally, then, we adopt equations (6.28), (6.30), and (6.25) as the basic relations for the computation of the coma contributions.

5. <u>The Surface Contributions to the Curvature of the Tangential Field</u>.

We measure the curvature of the tangential field usually by the distance of the intersection point Ab from a plane at right angles to the principal axis through the focus of some chosen axial ray. The particular ray chosen may depend somewhat on the problem in hand, but generally it is the extreme ray of an axial pencil having the same relative aperture as the oblique pencil. Thus

$$X'_T = L'_{ab} - L'_m$$

'm' standing for the particular ray selected. As long as the point Ab lies in the plane through the intersection point of the ray 'm' with the axis the curvature of the tangential field would be zero, but the coma would be profoundly affected by the H' co-ordinate of the point. Considering the curvature as quite distinct from coma, we could say that in a comatic system the ideal location of the point Ab, as far as curvature is concerned, is in the plane of the 'm' focus at a point distant coma'_T from the intersection point, Prm, of the principal ray with this plane. This analysis of curvature and coma may be arbitrary, but so are most measures and analyses of the aberrations of a system. We shall use the above ideas as the basis for determining the surface contributions to the tangential curvature.

From the results of the ray trace we calculate the tangential curvature of the image formed after refraction at successive surfaces of the system. Considering the 'a' and 'b' rays after refraction at surface $\underline{i}$, a rotation of each ray so that their intersection point Ab_i now lies in the 'm' plane at a point distant coma'_{Ti} from the point Prm_i would remove the curvature originally present in the image formed behind the surface $\underline{i}$. The effect of such imagined rotations would be to introduce at the final image an amount of curvature opposite to that introduced by the first $\underline{i}$ surfaces of the system, that is, an amount $-\sum^{i} TXC'$. This leads, as before, to a measure of the surface contributions.

The angle through which the ray 'a' must be rotated is fairly obviously

$$dU'_{ai} = X'_{Ti}(\tan U'_{ai} - \tan U'_{pri}) \cos U'_{ai} \,/\, S'_{ai}$$

$$= X'_{Ti}(\tan U'_{ai} - \tan U'_{pri}) \cos^2 U'_{ai} \,/(L'_m - X'_a)_i \qquad (6.31)$$

with a corresponding expression for the angle dU'_{bi}, for the 'b' ray

$$dU'_{bi} = X'_{Ti}(\tan U'_{pri} - \tan U'_{bi})\cos^2 U'_{bi}/(L'_m - X'_b)_i \tag{6.32}$$

The effect of such changes in the directions of the rays on the curvature at the final image is given by

$$dX'_T = dL'_{ab} = \frac{\partial L'_{ab}}{\partial U'_{ai}} dU'_{ai} + \frac{\partial L'_{ab}}{\partial U'_{bi}} dU'_{bi} \tag{6.33}$$

and then,

$$\overset{i}{\Sigma}\, TXC' = -dL'_{ab} \tag{6.34}$$

and

$$TXC'_i = \overset{i}{\Sigma}\, TXC' - \overset{i-1}{\Sigma}\, TXC' \tag{6.35}$$

6. Numerical Example.

On the accompanying computing sheets a typical computation of the contributions to the aberrations for an approximate design of a Tessar type photographic objective is given. The spherical contributions are for the marginal zone at an aperture of f/3.5, and the oblique aberration contributions are for a pencil of obliquity about 12°.

SURFACE CONTRIBUTIONS TO THE ABERRATIONS.

Spherical Aberration - Marginal Ray.

Surface	1	2	3	4	5	6	7
LA'	3.268	3.433	$\overline{2.562}$	$\overline{83.36}$	$\overline{92.62}$	$\overline{580.5}$	0.4957
sin U' cos U'	0.1731	0.2890	0.1119	$\overline{0.05559}$	$\overline{0.04197}$	$\overline{0.01740}$	
L' - X	79.72	42.68	112.7	$\overline{216.8}$	$\overline{290.2}$	$\overline{716.9}$	
LA' sin U' cos U'	0.5658	0.9920	$\overline{0.2868}$	4.634	3.888	10.10	
dU'_1	$\overline{0.007097}$	$\overline{0.02324}$	0.002545	0.02138	0.01340	0.01409	
(U') cosec $U'_k . dU'_1$	8.279	15.39	$\overline{2.728}$	$\overline{12.74}$	$\overline{12.25}$	$\overline{14.36}$	0.4957
dU'_k	0.01105	0.02104	$\overline{0.003853}$	$\overline{0.01879}$	$\overline{0.01856}$	$\overline{0.02289}$	
cot U'_k .dU'_k	0.07596	0.1447	$\overline{0.02649}$	$\overline{0.1292}$	$\overline{0.1276}$	$\overline{0.1574}$	
1 - cot U'_k .dU'_k	0.9240	0.8553	1.026	1.129	1.128	1.157	
ΣTSC'	7.651	13.17	$\overline{2.800}$	$\overline{14.39}$	$\overline{13.81}$	$\overline{16.62}$	0.4957
TSC'	7.651	5.516	$\overline{15.97}$	$\overline{11.59}$	0.5775	$\overline{2.812}$	17.12

Distortion.

	1	2	3	4	5	6	7
ℓ' - Xpr	86.07	45.90	108.9	$\overline{297.8}$	$\overline{382.9}$	$\overline{1295.3}$	85.27
Dist'	$\overline{0.0564}$	0.2697	$\overline{0.0795}$	$\overline{0.9247}$	0.3974	1.733	$\overline{0.0104}$
$\cos^2$ U'pr	0.9719	0.9253	0.9731	0.9380	0.9727	0.9767	
Dist'.cos U^2pr	$\overline{0.05482}$	0.2496	$\overline{0.07737}$	$\overline{0.8674}$	0.3866	1.693	
dU'pr	0.0006368	$\overline{0.005437}$	0.0007103	$\overline{0.002912}$	0.001009	0.001307	
(U')sec U'pr.dU'pr	$\overline{0.1071}$	0.5075	$\overline{0.09994}$	0.2456	$\overline{0.1335}$	$\overline{0.1876}$	
dU'k	0.0008895	$\overline{0.004444}$	0.0009224	$\overline{0.002320}$	0.001349	0.001967	
tan U'k.dU'k	0.0001941	$\overline{0.000970}$	0.0002013	$\overline{0.0005064}$	0.0002943	0.0004293	
1 + tan U'k.dU'k	1.00019	0.9990	1.0002	0.9995	1.0003	1.0004	
ΣTDC'	$\overline{0.1071}$	0.5070	$\overline{0.09997}$	0.2454	$\overline{0.1336}$	$\overline{0.1877}$	$\overline{0.0104}$
TDC'	$\overline{0.1071}$	0.6141	$\overline{0.6070}$	0.3454	$\overline{0.3790}$	$\overline{0.05411}$	0.1773

Coma

	1	2	3	4	5	6	7
L'ab - Xpr	81.77	38.64	112.7	$\overline{172.0}$	$\overline{248.8}$	$\overline{553.0}$	
$\cos^2$ U'pr	0.9719	0.9253	0.9731	0.9380	0.9727	0.9767	
Coma'	0.3418	0.8124	1.931	2.262	2.091	5.535	
Coma' $\cos^2$ U'pr	$\overline{0.3322}$	0.7517	$\overline{1.879}$	2.122	2.034	5.406	
dU'pr	$\overline{0.004063}$	0.01946	$\overline{0.01668}$	$\overline{0.01234}$	$\overline{0.008175}$	$\overline{0.009775}$	
U')pr secU'_1pr.dU'pr	$\overline{0.6817}$	1.812	$\overline{2.342}$	$\overline{1.038}$	$\overline{1.078}$	$\overline{1.400}$	
dU'k	$\overline{0.005674}$	0.01590	$\overline{0.02166}$	$\overline{0.009832}$	$\overline{0.01092}$	$\overline{0.01472}$	
tan U'k .dU'_k	$\overline{0.001238}$	0.003471	$\overline{0.004727}$	$\overline{0.002146}$	$\overline{0.002384}$	$\overline{0.003212}$	
1 + tan U'k .dU'_k	0.9988	1.0035	0.9953	0.9979	0.9976	0.9968	
ΣTCC'	$\overline{0.6808}$	1.819	$\overline{2.330}$	$\overline{1.036}$	$\overline{1.076}$	$\overline{1.395}$	0.2133
TCC'	$\overline{0.6808}$	2.499	$\overline{4.149}$	1.295	$\overline{0.04001}$	$\overline{0.3192}$	1.182

Note: A bar over a figure indicates a negative sign.

CHAPTER SEVEN.

THE ESTIMATION OF THE TOLERANCES PERMISSIBLE IN THE PRODUCTION OF AN OPTICAL SYSTEM.

THE ESTIMATION OF TOLERANCES FOR PRODUCTION.

In the production of an optical system serious imperfections may be introduced by failure to achieve the curvatures and other dimensions specified in the design. On the other hand, the cost of production may be increased considerably by striving to attain the specified dimensions within unnecessarily fine limits. It is thus a very important part of the designer's work to specify the limits within which each quantity must be controlled. It is proposed now to examine this problem in the light of the analysis developed in the preceding chapters. It is hoped to show that a method of estimating tolerances of this kind can be obtained which links on logically and conveniently with the general method of final design which has been developed. It is to be emphasised that the tolerances under consideration are not those which are generally termed 'optical tolerances' which relate to the amounts of residual aberrations which can be permitted in a system which aims at a certain standard of definition in the image. The present considerations relate to the degree of control which is to be exercised in the optical shop in the production of a given system if a uniform product is to be produced.

The final design of a system always represents more or less of a compromise. It is characterised by certain residual aberrations which have been calculated during the final stages of the design, and which experience, or the performance of a carefully built prototype, shows to be compatible with satisfactory performance. The whole success of known methods of design depends on the fact that the mathematical analysis of the aberrations gives a fairly reliable guide to the actual physical aberrations of the system when built. In the course of production any variation from the specified dimensions will result in a variation in the actual residual aberrations, but production will be satisfactory provided these variations are small compared with the residual aberrations themselves. Hence it is reasonable to base of system of estimating tolerances on the calculation of the effect on the résiduals of a departure from the specified value of each quantity in the system. The reliability of these tolerances will be of the same order as the reliability of the design methods as a whole. The system of transfer coefficients developed in Chapter II provides the machinery for the method, little additional computation being involved.

1. The Tolerances for the Curvature of each Surface of the System.

We have seen in the chapter referred to how it is possible to calculate a transfer coefficient which measures the rate of change of each aberration with the curvature of any surface of the system. These coefficients are calculated normally for the purpose of the final differential correction of the system, as outlined in Chapter V. They are now available for the further purpose of estimating the tolerances permissible in production.

If we use the symbol, A', to denote any of the aberrations of the system then the calculated values of $\partial A'/\partial c$ reveal the effect of small curvature changes. After a close scrutiny of these quantities tolerances can be set by assigning a permissible curvature variation at each surface such that the sum over all the surfaces of the effects on the residual aberrations due to the use of such variations shall not exceed a specified fraction of the values of the residual aberrations. If a curvature tolerance, $\pm\, dc_i$, is selected at each surface in this way the corresponding tolerance range in the radius of curvature is $\pm\, r_i^2\, dc_i$. It is often convenient to express the tolerance in terms of the number of fringes across a surface of a certain diameter. It is easily shown that the difference in curvature, dc, between two spherical surfaces of curvatures $\underline{c}$ and $\underline{c} + \underline{dc}$, which show $\underline{x}$ fringes across a surface of diameter $\underline{2a}$, is given very closely by

$$dc \quad = \quad \frac{x\,\lambda}{a^2}\,(1 - 0.5\,a^2c^2) \qquad (7.1)$$

Writing $\partial A'/\partial c_f$ for the change in the aberration A' per fringe change in curvature we have for any surface

$$\frac{\partial A'}{\partial c_f} = \frac{\partial A'}{\partial c} \cdot \frac{\lambda}{a^2}(1 - 0.5\, a^2 c^2) \qquad (7.2)$$

In the production of precision optics close attention must be paid to the figure of the surface, and hence it becomes inadvisable to allow more than three or four fringes between a lens surface and its test plate as the figure of the surface cannot be carefully controlled if there is a large difference of curvature between them. The freedom provided by curvature tolerances in high grade work is therefore mainly used during the preparation of the test plates for an instrument.

2. <u>The Tolerance in the Axial Thickness of a Component or Airspace</u>.

The thickness of a lens is a quantity which is much more difficult to control than the curvatures of its surfaces and the question of tolerances becomes very important. For any component the transfer coefficient, $\partial A'/\partial d$, for its <u>second</u> surface specifies the change of the aberration per unit change of axial thickness. In accordance with the signs we have used a negative d-change increases the thickness of the component. Formally, then, we may introduce, if we desire, an axial thickness coefficient, $\partial A'/\partial t$, defined by

$$\frac{\partial A'}{\partial t_h} = -\left(\frac{\partial A'}{\partial d_2}\right)_h$$

the subscript $\underline{h}$ referring to the general component and the subscript 2 denoting the second surface of that component. Thus without further computation a set of coefficients is available for the estimation of the tolerances in thickness which may be allowed. We make use of them by assigning a permissible thickness variation to each component such that the sum over all the components of the effects on the residual aberrations due to such variations shall not exceed some small fraction of the values of the residual aberrations. In like manner tolerances are assigned for the thicknesses of the airspaces between components of which use is made during the assembly of the system. The transfer coefficients for the second surface bounding the airspace specifies the rate of change of the aberrations with the thickness of the airspace.

3. The Tolerances for the Refractive Index and Dispersion of the Glasses.

Equation (2.66) shows that for any monochromatic aberration, A', we can calculate a differential coefficient, $\partial A' / \partial N_h$, which measures the change of this aberration in the image formed by the system per unit change of refractive index of any component, $\underline{h}$, of the system. These coefficients provide full information as to the limits within which the refractive index of each component must be controlled for satisfactory production of the system, and permit a series of tolerances to be established. As before, we assign a permissible variation of refractive index to each component such that the sum over all the components of the effects on the residual aberrations due to the permitted variations is less than a prescribed small fraction of these residuals. As regards the variation of the dispersions of the glasses the coefficients, $\partial Lch' / \partial P_h$ and $\partial Tch' / \partial P_h$, defined in equations (2.68) and (2.70) furnish the basis for tolerances, but generally speaking the variation of the dispersion of a glass type between successive melts is negligible, so that these tolerances are seldom required.

4. The Tolerances for the Control of the Focal Length of the System.

Frequently it is important to ensure that the focal length of a particular system is controlled within fine limits during production. For this we require to know the effect of the variation of curvature, thickness of components, and the glass constants on the focal length. From the paraxial ray trace, we have

$$f' = y / u'_k$$

and hence

$$\frac{\partial f'}{\partial c_i} = -(y / u'^2_k) \frac{\partial u'_k}{\partial c_i}$$

$$= -(f'/u') \frac{\partial u'_k}{\partial c_i} \qquad (7.4)$$

Corresponding expressions hold for $\partial f'/\partial n_i$ and $\partial f'/\partial d_i$. For the effect on the focal length of a small change of refractive index of a component we have

$$\frac{\partial f'}{\partial N_h} = -\frac{\partial f'}{\partial n_1}\frac{N_{h-1}}{N_h^{\,2}} + \frac{\partial f'}{\partial n_2}\frac{1}{N_{h+1}} \qquad (7.5)$$

The transfer coefficients in equations (7.4) and (7.5) are quickly computed as the values of $\partial u'_k / \partial c$ etc., have been calculated in the general computation for the paraxial ray. Thus a series of tolerances for the control of the focal length may be set quite simply and with certainty.

5. The Use of Components Outside the Specified Tolerances.

When a large number of optical systems of a certain type are to be produced a certain percentage of rejected components with dimensions outside the specified tolerances is inevitable. The intelligent use of the information provided by the transfer coefficients which have been developed enables some at least of such rejects to be sorted into sets of optics which will give a perfectly satisfactory performance. Components rejected on account of non-spherical figure are excluded from our considerations.

The rejected components are classified according to the type and amount of their departure from specified dimensions. In connection with each aberration a limit is set for the additional amount of this aberration which can be admitted on account of departure from specifications. This will be some small fraction of the calculated residual aberration. A table is then prepared setting out the aberration changes for departure from specified dimensions in terms of the units used in the workroom. The trained worker then derives from this table the amount of each aberration introduced into the system if he employs a certain reject component, A. A study of the transfer coefficients in the table will now reveal whether some departure from dimensions of another component, B, of the system will introduce amounts of the various aberrations which will compensate those introduced by A, and thus bring their totals within the limits prescribed. If this is possible a reject of type A is paired with B etc. and in this way use may frequently be made of rejected components. This kind of attempt to salvage components is probably made in every optical shop by trial and error methods or bench tests. The use of a table of coefficients as suggested organises such attempts intelligently, giving a reliable guide as to what is possible in this direction.

6. <u>The Tolerances for the Centration of a System.</u>

The importance of accurate centration in the production of a lens system is too well known to need comment. It becomes of interest, however, to enquire what information the analysis we have developed can provide as to the effect on the aberrations of small departures from centration in a system. It is necessary to remember, of course, that any information which may be provided is based only the behaviour of the tangential rays. The effect of decentering each component will not be the same, and hence any information we may derive will be useful in indicating, at least, which components need most care in this respect. We consider the effect of the simple type of decentering which results in a single component being displaced a small distance dy at right angles to the principal axis, the axis of the component remaining parallel to the principal axis of the system. We use the subscript $\underline{h}$ to denote the component as usual, and for simplicity we will call the first surface of this component the $\underline{i}^{th}$

surface of the system. To produce the type of decentration considered we displace the component $\underline{h}$ and all components behind it in a direction at right angles to the axis of the system through a distance dy. A traced ray falling on the $\underline{i^{th}}$ surface now meets the surface at a new point given relative to the original incidence point by

$$dp_i = - dy \cos U_i$$

Next, we must restore the components behind the component $\underline{h}$ to their original positions on the specified axis of the system by giving to each a displacement - dy. The effect of this is an incidence point displacement for all rays meeting the first surface of the component h + 1. Let us call this the $(i + 2)^{th}$ surface. The change of incidence point is given by

$$dp_{i+2} = dy \cos U_{i+2}$$

The total effect of the two changes considered is to leave the component $\underline{h}$ decentred by an amount dy, and it now remains to deduce the aberration changes from the two dp changes in terms of which the decentration is described. The effect at the last surface is

$$dU'_k = \frac{\partial U'_k}{\partial p_{i+2}} \cdot dp_{i+2} + \frac{\partial U'_k}{\partial p_i} \cdot dp_i$$

$$= \left(\frac{\partial U'_k}{\partial p_{i+2}} \cos U_{i+2} - \frac{\partial U'_k}{\partial p_i} \cos U_i\right) dy$$

Hence,
$$\frac{\partial U'_k}{\partial y_h} = \frac{\partial U'_k}{\partial p_{i+2}} \cos U_{i+2} - \frac{\partial U'_k}{\partial p_i} \cos U_i \qquad (7.6)$$

The corresponding expression giving $\partial p'_k / \partial y_h$ can be written down immediately

$$\frac{\partial p'_k}{\partial y_h} = \frac{\partial p'_k}{\partial p_{i+2}} \cos U_{i+2} - \frac{\partial p'_k}{\partial p_i} \cos U_i \qquad (7.7)$$

The computation now follows the standard route through the C quantities to the transfer coefficients of the intersection points, and thus to the aberration transfer coefficients, but only involves the first surface of each component. Thus a set of transfer coefficients for the aberrations with respect to centration changes may be obtained, and these afford a measure of the seriousness of decentration in terms of the residual aberrations. The writer has lacked opportunity so far to follow up this investigation into decentration and tilt of a component, so that what is presented in this section remains for further work.

CHAPTER EIGHT.

TRANSFER COEFFICIENTS FOR THE ASTIGMATISM OF A LENS SYSTEM AT SMALL APERTURE.

The Astigmatism of a Lens System at Small Aperture.

In the new designing methods which have been outlined in the preceding chapters we have considered only the tangential aberrations. The problem of astigmatism has been entirely omitted. A full solution of this problem requires the extension of the present type of analysis to the general skew trace, a task which has only been commenced as yet. In the meantime the writer has contented himself for routine designing with a simple analysis of the astigmatism at small aperture and any obliquity, checking the final design by the usual skew trace at the full aperture. This does represent a considerable advance, however, inasmuch as the changes in the astigmatism at small aperture due to changes within the system can be estimated for the differential correction process. In order to do this we make use of the formulae for the s-trace and the t-trace, which locate the sagittal and tangential foci of narrow fans of rays close to the principal ray. The general theory of the transfer coefficients for these will now be developed.

1. The Transfer Coefficients for the Shift of the Tangential Focus.

In addition to the principal ray of the oblique pencil we trace another ray of the same obliquity close to the principal ray and calculate the single surface coefficients and the transfer coefficients for it in the normal way. From the trace of the principal ray we have, as in equation (2.73),

$$t'_k = \frac{\partial p'_k}{\partial p_1} \Big/ \frac{\partial U'_k}{\partial p_1} \tag{8.1}$$

Denoting the 'close principal' ray, i.e. the additional ray traced for the present purpose, by the subscript 'cp', we can write down an expression for the separation of the emergence points of the rays 'pr' and 'cp' at the last surface of the system as

$$\delta p'_{prk} = t'_k \, (U'_{cp} - U'_{pr})_k = t'_k \, \delta U'_k \, . \tag{8.2}$$

Now let us consider a change of curvature at surface $\underline{i}$ within the system. Differentiating equation (8.2) we obtain

$$\frac{\partial}{\partial c_i}(\delta p'_{prk}) = t'_k \frac{\partial}{\partial c_i}(U'_{cp} - U'_{pr})_k + \frac{\partial t'_k}{\partial c_i}(U'_{cp} - U'_{pr})_k$$

For the term on the left hand side we have

$$\frac{\partial}{\partial c_i}(\delta p'_{prk}) = \frac{\partial p'_{cpk}}{\partial c_i} - \frac{\partial p'_{prk}}{\partial c_i}$$

and combining these two relations we obtain

$$\frac{\partial t'_k}{\partial c_i} = \frac{1}{\delta U'_k}\left[\frac{\partial p'_{cpk}}{\partial c_i} - \frac{\partial p'_{prk}}{\partial c_i} - t'_k\left(\frac{\partial U'_{cpk}}{\partial c_i} - \frac{\partial U'_{prk}}{\partial c_i}\right)\right] \qquad (8.3)$$

The corresponding relations for thickness changes and refractive index changes may be written down, giving

$$\frac{\partial t'_k}{\partial d_i} = \frac{1}{\delta U'_k}\left[\frac{\partial p'_{cpk}}{\partial d_i} - \frac{\partial p'_{prk}}{\partial d_i} - t'_k\left(\frac{\partial U'_{cpk}}{\partial d_i} - \frac{\partial U'_{prk}}{\partial d_i}\right)\right] \qquad (8.4)$$

$$\frac{\partial t'_k}{\partial n_i} = \frac{1}{\delta U'_k}\left[\frac{\partial p'_{cpk}}{\partial n_i} - \frac{\partial p'_{prk}}{\partial n_i} - t'_k\left(\frac{\partial U'_{cpk}}{\partial n_i} - \frac{\partial U'_{prk}}{\partial n_i}\right)\right] \qquad (8.5)$$

If changes are made during the process of differential correction which are too large to be treated as differentials an improvement in accuracy is obtained by calculating the new value of $\delta p'_k$ and the new value of $\delta U'_k$ and hence the new value of t'_k directly from them. Thus, for a curvature change, δc_i,

$$\text{new }(\delta p'_k) = \delta p'_k + \left(\frac{\partial p'_{cpk}}{\partial c_i} - \frac{\partial p'_{prk}}{\partial c_i}\right)\delta c_i$$

$$\text{new }(\delta U'_k) = \delta U'_k + \left(\frac{\partial U'_{cpk}}{\partial c_i} - \frac{\partial U'_{prk}}{\partial c_i}\right)\delta c_i$$

and the change in the t-focus is given by

$$\delta t'_k = \frac{(\delta p'_k)_{new}}{(\delta U'_k)_{new}} - t'_k$$

The transfer coefficients for the t-focus thus require little calculation beyond the additional work involved in the trace and coefficients of the ray 'cp'.

2. Transfer Coefficients for the Shift of the Sagittal Focus.

The system of transfer coefficients for the shift of the sagittal focus of a narrow fan of rays close to the principal ray of an oblique pencil is based on an s-trace along this principal ray, in addition to the data already available from the ordinary trace of the principal ray and its general transfer coefficients. The relation giving the position of the s-focus behind any surface $\underline{i}$ of the system is

$$\frac{N'}{s'} = \frac{N' \cos I' - N \cos I}{r} + \frac{N}{s} \tag{8.6}$$

We now write $\quad S' = 1/s' , \qquad S = 1/s ,$ (8.7)

and
$$G = \frac{N' \cos I' - N \cos I}{N'} \tag{8.8}$$

whereby equation (8.6) now becomes

$$S' = Gc + nS \tag{8.9}$$

This procedure clears our equation of reciprocals and makes it easier to handle. The use of the symbol S' as the reciprocal of s' in this section should not cause any confusion with the extensive use we have made of it previously for the slant distance measured along a ray to some intersection point. The use of it in the present connection is limited to the section.

Changes at a Single Surface. Suppose that a small change of curvature is made at surface $\underline{i}$ of the system. The effect on the position of the s-focus behind the surface will be determined by the derivative

$$\frac{\partial S_i'}{\partial c_i} = G_i + c_i \frac{\partial G_i}{\partial c_i} + n_i \frac{\partial S_i}{\partial c_i} \tag{8.10}$$

The change in S_i due to a small curvature change is of second order only and hence we may write

$$\frac{\partial S_i'}{\partial c_i} = G_i + c_i \frac{\partial G_i}{\partial c_i} \tag{8.11}$$

To compute the derivative $\partial S_i'/\partial c_i$ we require an expression for $\partial G_i/\partial c_i$. This is obtained by differentiation of (8.8) and the refraction law as follows: From (8.8)

$$\frac{\partial G}{\partial I} = -\sin I' \frac{\partial I'}{\partial I} + n \sin I$$

$$= -\sin I' \; n \frac{\cos I}{\cos I'} + n \sin I, \quad \text{using the refraction la}$$

$$= \sin I'\left(1 - n \frac{\cos I}{\cos I'}\right)$$

$$= \sin I'\left(1 - \frac{\partial U'}{\partial U}\right) \tag{8.12}$$

the last step following from equation (2.7). From the computing equation

$$(L - r) \sin U = r \sin I$$

we obtain, as before,

$$\frac{\partial I}{\partial c} = L \sin U / \cos I$$

Hence,

$$\frac{\partial G}{\partial c} = \frac{\partial G}{\partial I} \frac{\partial I}{\partial c}$$

$$= \sin I'\left(1 - \frac{\partial U'}{\partial U}\right) \frac{L \sin U}{\cos I}$$

$$= \sin I' \frac{\partial U'}{\partial c} \tag{8.13}$$

as follows from equation (2.10). The computation of $\partial G / \partial c$ at any surface thus involves only a single operation beyond the ordinary single surface coefficients already computed for the principal ray. Two further operations complete the calculation of $\partial S' / \partial c$.

If a small change is made in the refractive index, n, at any surface <u>i</u> of the system, we require the derivative

$$\frac{\partial S_i'}{\partial n_i} = S_i + c_i \frac{\partial G_i}{\partial n_i} \tag{8.14}$$

Differentiating equation (8.8) with respect to <u>n</u>, we obtain

$$\frac{\partial G}{\partial n} = -\sin I' \frac{\partial I'}{\partial n} - \cos I$$

$$= -\sin I' \frac{\sin I}{\cos I'} - \cos I$$

and by equation (2.9) this becomes

$$\frac{\partial G}{\partial n} = \sin I' \frac{\partial U'}{\partial n} - \cos I \qquad (8.15)$$

Thus two operations beyond the single surface coefficients suffice to calculate $\partial G/\partial n$, and a further two bring us to the value of $\partial S'/\partial n$ for the surface.

If a small change is made in the axial separation between two consecutive surfaces, we have from equation (8.9)

$$\frac{\partial S'}{\partial d} = c\frac{\partial G}{\partial d} + n\frac{\partial S}{\partial d} \qquad (8.16)$$

Expressions for the derivatives on the right hand side of the last equation may be developed as follows: By differentiation of (8.8)

$$\frac{\partial G}{\partial d} = -\sin I' \frac{\partial I'}{\partial d} + n \sin I \frac{\partial I}{\partial d}$$

$$= \sin I' \left(\frac{\partial I}{\partial d} - \frac{\partial I'}{\partial d} \right)$$

$$= \sin I' \left(\frac{\partial I}{\partial d} - n \frac{\cos I}{\cos I'} \frac{\partial I}{\partial d} \right)$$

$$= \sin I' \left(1 - \frac{\partial U'}{\partial U} \right) \frac{\partial I}{\partial p} \frac{\partial p}{\partial d}$$

From equation (2.8) and the one preceding it this becomes

$$\frac{\partial G}{\partial d} = \sin I' \frac{\partial U'}{\partial p} \frac{\partial p}{\partial d}$$

$$= \sin I' \sin U \frac{\partial U'}{\partial p} \qquad (8.17)$$

The transfer formula in the s-trace is

$$s = s'_- - D'_-$$

whence $$\frac{\partial s}{\partial d} = \frac{\partial}{\partial d}\left(\frac{1}{S}\right) = -\frac{\partial D'_-}{\partial d}$$

giving $$\frac{\partial S}{\partial d} = S^2 \frac{\partial D'_-}{\partial d}$$

Now $$D'_- = (-d + X - X_-) / \cos U$$

and $$X = r(1 - \cos(U + I))$$

whence we obtain by differentiation

$$\begin{aligned} \frac{\partial D'_-}{\partial d} &= \sec U \left(-1 + \frac{\partial X}{\partial d} \right) \\ &= \sec U \left(-1 + \frac{\partial X}{\partial I} \frac{\partial I}{\partial p} \frac{\partial p}{\partial d} \right) \\ &= -\sec U \left[1 - \frac{\sin U \ \sin(U + I)}{\cos I} \right] \end{aligned}$$

Finally then we have

$$\frac{\partial S}{\partial d} = -S^2 \sec U \left[1 - \frac{\sin U \ \sin(U + I)}{\cos I} \right] \tag{8.18}$$

Often the bracketed expression may be replaced by unity with a good degree of approximation. It is accurately unity for a plane surface. Equations (8.16), (8.17), and (8.18) permit the calculation of $\partial S'/\partial d$ at any surface of the system.

3. The Transfer Coefficients for the Sagittal Focus of a Narrow Pencil.

It now remains to develop a method of computing the transfer coefficients for S'-changes so that the effect on S'_k of a change within the system may be determined. Since G is a function of the angles I and I' its value at any surface will change with change of the point of incidence. In general, however, it is found that the change of G along the path of the pencil due to small alterations within the system has a negligible effect on the transfer coefficients, and, on account of the resulting simplification, it is proposed to develop a set of transfer coefficients which neglect this factor.

Suppose that a change dS'_i is produced in the pencil refracted at surface i. To determine the effect of this change at the next surface of the system we require an expression for $\partial S_{i+1} / \partial S'_i$. We may write the transfer formula of the s-trace in the form

$$S_+ = S'(1 - D' S')^{-1}$$

and $$\frac{\partial S_+}{\partial S'} = (1 - D'S')^{-1} + D'S'(1 - D'S')^{-2}$$

$$= (1 - D'S')^{-2} \tag{8.20}$$

which provides a simple relation for the computation of this derivative. The change which will result behind surface $(i + 1)$ must now be determined, and by differentiation of equation (8.9) we obtain

$$\frac{\partial S'}{\partial S} = n \tag{8.21}$$

whence

$$\frac{\partial S'_{i+1}}{\partial S'_i} = n_{i+1} \frac{\partial S_+}{\partial S'_i}$$

and the process may be repeated surface by surface through the system. We may therefore write down an expression for the transfer coefficient for S'-changes as follows:

$$\frac{\partial S'_k}{\partial S'_i} = \frac{\partial S_+}{\partial S'_i} n_{i+1} \frac{\partial S_+}{\partial S'_{i+1}} n_{i+2} \cdots\cdots \frac{\partial S_+}{\partial S'_{k-1}} n_k \tag{8.22}$$

and the relation between these transfer coefficients at successive surfaces is thus

$$\frac{\partial S'_k}{\partial S'_i} = \frac{\partial S_+}{\partial S'_i} n_{i+1} \cdot \frac{\partial S'_k}{\partial S'_{i+1}} \tag{8.23}$$

This provides the main framework of the computation of these basic transfer coefficients. Equation (8.20) is used to calculate $\partial S_+/\partial S'$ at each surface and, commencing at the last surface where we have $\partial S'_k / \partial S'_k = 1$, equation (8.23) is used for the calculation of $\partial S'_k / \partial S'$ at each surface in turn.

Finally we have the transfer coefficients for changes of curvature, refractive index and axial separation in the form

$$\frac{\partial S'_k}{\partial c_i} = \frac{\partial S'_k}{\partial S'_i} \frac{\partial S'_i}{\partial c_i} \tag{8,24}$$

and returning to the quantity s' instead of its reciprocal, we have

$$\frac{\partial s_k'}{\partial c_i} = - s_k'^2 \frac{\partial S_k'}{\partial c_i}$$

Assembling the final expressions for convenience, then,

$$\frac{\partial s_k'}{\partial c_i} = - s_k'^2 \frac{\partial S_k'}{\partial S_i'} \frac{\partial S_i'}{\partial c_i} \tag{8.25}$$

$$\frac{\partial s_k'}{\partial d_i} = - s_k'^2 \frac{\partial S_k'}{\partial S_i'} \frac{\partial S_i'}{\partial d_i} \tag{8.26}$$

$$\frac{\partial s_k'}{\partial n_i} = - s_k'^2 \frac{\partial S_k'}{\partial S_i'} \frac{\partial S_i'}{\partial n_i} \tag{8.27}$$

3. Numerical Example of the Shift of the s-Focus and the t-Focus.

In an f/1.5 lens which the writer was computing there was made a change of curvature of 0.001 mm.$^{-1}$ at the seventh surface of the system at one stage of the design, there being ten surfaces in the complete system. As a result of the change the radius of curvature of the surface changed from 1091 mm. to 521.78 mm. On the following page the computation of the necessary coefficients for this particular change are set out.for the pencil of obliquity of approximately 17°. For this surface we have thus

$$\frac{\partial s_k'}{\partial c_7} = \overline{2112.5} \qquad\qquad \frac{\partial t_k'}{\partial c_7} = \overline{2416.0}$$

and hence $ds_k' = \overline{2.1125}$ $\qquad$ $dt_k' = \overline{2.416}$

giving new $s_k' = 47.273$ and new $t_k' = 47.661$

Using the alternative relations (8.5a) we obtain

$$dt_k' = \overline{2.335} \qquad \text{and} \qquad \text{new } t_k' = 47.742$$

A full trace of the altered system gives the final values for the foci as:

$$s_k' = 47.338 \qquad\qquad t_k' = 47.801$$

giving an accuracy of about three percent which is decidedly useful.

Single Surface Coefficients - Sagittal Focus.

Surface.	7.
c	0.000916506
$\sin I'$	$\overline{0.345888}$
$\partial U'/\partial c$	1.07656
$\partial G/\partial c$	$\overline{0.372369}$
$c\ \partial G/\partial c$	$\overline{0.000341}$
G	0.378746
$\partial S'/\partial c$	0.378405

Transfer Coefficients - Sagittal Focus.

s'	7.	8.	9.	10.
D'/s'	0.014439	0.117242	0.063284	
$1 - D'/s'$	0.985561	0.882758	0.936716	
$(1 - D'/s')^2$	0.971332	0.779262	0.877437	
$\partial S_+/\partial S'$	1.029514	1.28327	1.13968	
n_+	0.920497	1.03477	1.5960	
$n_+\ \partial S_+/\partial S'$	0.94766	1.32789	1.81893	
$\partial S'_k/\partial S'$	2.2889	2.4153	1.8189	1.0000
$\partial S'_k/\partial c$	0.86614			
$\partial s'_k/\partial c$	$\overline{2112.5}$			
s'				49.3856

Transfer Coefficients - Tangential Focus.

	7.	10.
$\partial p'_{cpk}/\partial c - \partial p'_{prk}/\partial c$	$\overline{8.8492}$	
$\partial U'_{cpk}/\partial c - \partial U'_{prk}/\partial c$	0.44240	
$-\ t'_k\ ()$	$\overline{22.1540}$	
$\partial t'/\partial c$	$\overline{2416.0}$	
t'		50.0772
$\delta U'_k$		0.012833

CHAPTER NINE.

A NEW TRIGONOMETRICAL ANALYSIS OF THE ABERRATIONS OF A LENS SYSTEM.

A New Trigonometrical Analysis.

In the methods considered so far there are various deficiencies which become more and more apparent as systems of wider aperture and field are designed. In the first place the vignetting which frequently occurs in oblique pencils introduces a difficulty. The iris diaphragm of a phpto-graphic objective, for example, is not the aperture stop of the system for all incident rays, so that in the incident fan of rays of an oblique pencil the extreme rays which pass through the system are not symmetrically situated with respect to the principal ray of the pencil. In our usual notation the pencil is vignetted on the side of the 'a' ray. In the orthodox trigonomatrical analysis of the aberrations of the oblique pencil we use a pair of rays, 'a' and 'b', symmetrically situated about the principal ray in the incident fan, to determine the value of the coma and the curvature of field. When the aperture and obliquity are such that vignetting appears in the oblique pencils this analysis of the aberrations neglects all that part of the fan of rays which lies outside the 'b' ray, and which in large aperture lenses may so modify the distribution of light in the neighbourhood of the image disc that the values of the aberrations so measured cease to be a reliable guide to the state of correction of the pencils.

Secondly, it is essential that the final correction of the system should be made in such a way that as the iris diaphragm is closed progressively the definition in the image should improve as rapidly as possible. In other words, the compromise made in the final adjustments of the residuals should be such that the most aberrant rays are the first to be eliminated on closing the diaphragm. It is not suggested that this can always be done, but it is highly desirable to have in the designing methods the means of effecting such a correction when it is possible. The conventional analysis of the aberrations makes no provision for estimating the variation of definition with aperture unless we are prepared to trace a pair of rays at selected intervals of aperture, and even this process fails as soon as vignetting appears in the pencil.

Another feature limiting the usefulness of the usual analysis of the aberrations is the difficulty of clearly analysing the colour correction of the system. The full meaning of the the colour residuals is really only to be found by considering the chromatic variation of the ordinary aberrations and the magnitude of the secondary spectrum. A paper showing how this analysis may be made is given in Chapter 10 , but there is still the question of exhibiting the dependence of the chromatic aberration on the aperture of the system.

Finally, the desire for a simple graphical method of representing the correction state throughout the field of the system, a very great consideration in systems such as photographic objectives with wide fields and large apertures, and the attempt to reduce the work involved in the computation of the transfer coefficients, each played a part in urging the search after some other analysis of the aberrations which would be more flexible for design purposes than the conventional one. The system of the type of the photographic objective was in mind principally in this work.

1. The Analysis of the Aberrations in Monochromatic Light.

We are concerned principally with tangential rays in our designing methods, and the most usual correction state sought is that in which the tangential image field is flat. In what follows we will take as our ideal correction state that which gives a flat tangential field coincident with the paraxial image plane. We may depart subsequently from the paraxial image plane, if we so desire, but for the time being we retain this as our ideal state of correction. Any state of correction must be specified by certain numbers which measure its departure from the ideal correction state, and further, it is desirable that the measures employed should be of the same kind for each pencil regardless of its position in the field. By using transverse measures only we cease to differentiate between the axial and the oblique pencils, treating the former as the pencil of zero obliquity. To determine the correction state of a system we trace a number of pencils through it, an axial pencil and several oblique pencils at selected intervals across the field. For the axial pencil a marginal, zonal, and paraxial ray will suffice in general, though additional rays may be desirable in large aperture systems. For each oblique pencil the usual rays 'a' , 'pr', and 'h' should be traced, and for some pencils at least two additional rays, 'c' and 'd', should be traced, the former being a 'zonal' ray in the upper half pencil and the latter a 'zonal' ray in the lower half pencil. We regard each pencil as sampling the state of correction at its own region in the image field. The point at which each rays intersects the paraxial image plane is then calculated, its H' co-ordinat being given by

$$H' = (L_k' - l_k') \tan U_k' \tag{9.1}$$

Departure from the ideal state of correction may be measured for each traced ray by the quantity

$$\Delta' = H' - H'_{pr} \tag{9.2}$$

where H'_{pr} is the intersection height of the principal ray of the pencil to which the ray belongs, which in the special case of the axial pencil is zero. A positive value of Δ' means that the ray concerned intersects the paraxial image plane above the corresponding principal ray. A zero value of Δ' for all rays would give ideal definition of the image, but the image would in general still suffer from distortion. Hence we retain the usual measure of the distortion defined by

$$\text{dist}' = H'_{pr} - H'_{id} \tag{9.3}$$

which, conveniently, is a measure of the same type as Δ'. The aberrations dist' and Δ' will then suffice for a description of the performance of the lens system in monochromatic light. The chromatic aberration is easily fitted into the scheme, but we leave this for separate discussion in a subsequent section. It may be felt at this stage that the proposed aberration measures overlook completely the possibility of a perfect state of correction in a plane parallel to, but slightly displaced from, the paraxial image plane. This is not so, as the manner in which the measures are used in practical designing will show, and which we leave for discussion in connection with an example shortly. The interpretation of the aberration quantities is very simple. Dist' measures, as usual, an actual defect present in the image formed by the system even at very small apertures, while Δ' measures the blur contributed by a very narrow bundle of rays in the vicinity of the traced ray in question as the diaphragm is opened to admit them. In using these measures the designer must pay careful attention to his interpretation of the importance of the dimensions of the blur from the point of view of the relative illumination contributed by the zones which are sampled by the rays of the trace.

2. An Analysis of the Aberrations of a Telephoto Photographic Objective.

It is proposed to discuss the details of the present methods in connection with the analysis of an actual lens, believing that the information conveyed in this way is more definite for our present purpose than the discussion of expressions only. The lens chosen as an example is a telephoto lens of 36 in. focal length designed originally by Booth. It is not an ideal type to consider as an example on account of its restricted field and moderate aperture, but as the traces were easily available at the time of writing it was thought that it would serve the purpose. The analysis is that of a

THE BOOTH TELEPHOTO LENS. F/6.3.

Focal length 36 in.

Surface	Radius	d'	N_d	N'_d	ν
1	+ 161.78	29.77	1.000	1.61517	55.5
2	- 432.35	0.18	1.61517	1.0000	
3	- 420.12	7.38	1.0000	1.65068	33.7
4	+ 361.80	205.63	1.65068	1.0000	
5	- 98.39	10.87	1.0000	1.57338	52.0
6	- 1036.8	16.16	1.57338	1.57846	41.1
7	- 152.64		1.57846	1.0000	

Diaphragm 73.15 mm. behind surface 4.

Glass

Lens.	Type	N_b	N_c	N_d	N_e	N_F	V
I	DBC	1.60993	1.61181	1.61517	1.61781	1.62289	55.5
II	EDF	1.64192	1.64503	1.65068	1.65526	1.66436	33.7
III	BLF	1.56820	1.57007	1.57338	1.57601	1.58111	52.0
IV	LF	1.57197	1.57429	1.57846	1.58179	1.58835	41.1

Specification supplied by Ross Ltd. London.

specification of the lens as manufactured by Ross Ltd. The specification is interleaved.

In the first place an analysis of the state of correction of the lens as specified will be made. A trace of the system was made comprising an axial pencil, and oblique pencils at obliquities of approximately 4°, 7.5°, and 10°. This was a very liberal sampling, but we will only consider the oblique pencils at the first two stated obliquites as consideration of the additional pencil at 10° will contribute nothing essentially new to the discussion. The detailed computation is collected in a later chapter purely to afford opportunity for inspection of the details if this is desired. From the detailed computation we extract certain relevant information for the analysis of the correction state of the lens, and for convenience this is collected in Table 9.1 . The first three columns of the Table need no comment. In the fourth column are tabulated the heights, Y_o , of the intersection points of the various rays in the plane of the diaphragm. These are given in this particular case by the relation

$$Y_o = (L_4' - 73.15) \tan U_4' \tag{9.4}$$

the diaphragm being 73.15 mm. behind the fourth surface of the lens. We might also have chosen the intersection heights in the plane of the entance pupil, but the diaphragm is probably better. This quantity, Y_o, enables us to correlate the definition of the image with the aperture of the system in a very simple manner. As a further help in this direction we add to the Table in column five the f-numbers of the aperture which just admits the various rays. The remaining columns of the Table will not concern us for a little while so that we may postpone their discussion. To represent the analysis of the correction which is contained in this Table in an easily comprehensible way we plot the values of Δ' against Y_o.

TABLE 9.1

Ray	H' mm.	Δ' mm.	Y_o mm.	Apert. f/	0.5 tanU' mm.	δΔ' mm.	Δ'_+ mm.	Δ'_- mm.
				Axial Pencil.				
M	0.3718	0.3718	56.77	6.25	$\overline{0.0395}$	$\overline{0.0395}$	0.3323	0.4113
QM	0.1304	0.1304	52.21	6.79	$\overline{0.0366}$	$\overline{0.0366}$	0.0938	0.1670
Q	$\overline{0.0369}$	$\overline{0.0369}$	45.73	7.75	$\overline{0.0324}$	$\overline{0.0324}$	$\overline{0.0693}$	$\overline{0.0046}$
Z	$\overline{0.0852}$	$\overline{0.0852}$	39.58	8.96	$\overline{0.0282}$	$\overline{0.0282}$	$\overline{0.1134}$	$\overline{0.0570}$
Qz	$\overline{0.0705}$	$\overline{0.0705}$	30.72	11.54	$\overline{0.0220}$	$\overline{0.0220}$	$\overline{0.0925}$	$\overline{0.0485}$
				4^o Pencil.				
a	$\overline{63.9934}$	0.1075	50.49	7.02	$\overline{0.0886}$	$\overline{0.0325}$	0.0750	0.1400
c	$\overline{64.2038}$	$\overline{0.1029}$	35.17	10.1	$\overline{0.0783}$	$\overline{0.0251}$	$\overline{0.1280}$	$\overline{0.0778}$
pr	$\overline{64.1009}$	0	0	0	$\overline{0.0531}$	0	0	0
d	$\overline{64.0447}$	0.0562	—		$\overline{0.0284}$	0.0247	0.0809	0.0315
b	$\overline{64.2185}$	$\overline{0.1176}$	$\overline{49.08}$	7.22	$\overline{0.0184}$	0.0347	$\overline{0.0829}$	$\overline{0.1523}$
				7.5^o Pencil.				
a	$\overline{123.0771}$	0.1744	45.24	7.84	$\overline{0.1841}$	$\overline{0.0320}$	0.1424	0.2064
c	$\overline{123.3051}$	$\overline{0.0536}$	31.50	11.25	$\overline{0.1247}$	$\overline{0.0226}$	$\overline{0.0762}$	$\overline{0.0310}$
pr	$\overline{123.2515}$				$\overline{0.1021}$			
d	$\overline{123.1962}$	0.0553	$\overline{32.08}$	11.05	$\overline{0.0790}$	0.0232	0.0785	0.0321
b	$\overline{123.3408}$	$\overline{0.0893}$	$\overline{45.42}$	7.81	$\overline{0.0697}$	0.0325	$\overline{0.0568}$	$\overline{0.1218}$

In order to bring out clearly the correspondence between the axial and the oblique pencils we plot the portion of the curve associated with the axial rays on the lower side of the principal axis of the system. The abcissa interval between the extreme values of Δ' in the plot of each pencil represents the 'diameter' of the blur patch on a plate placed

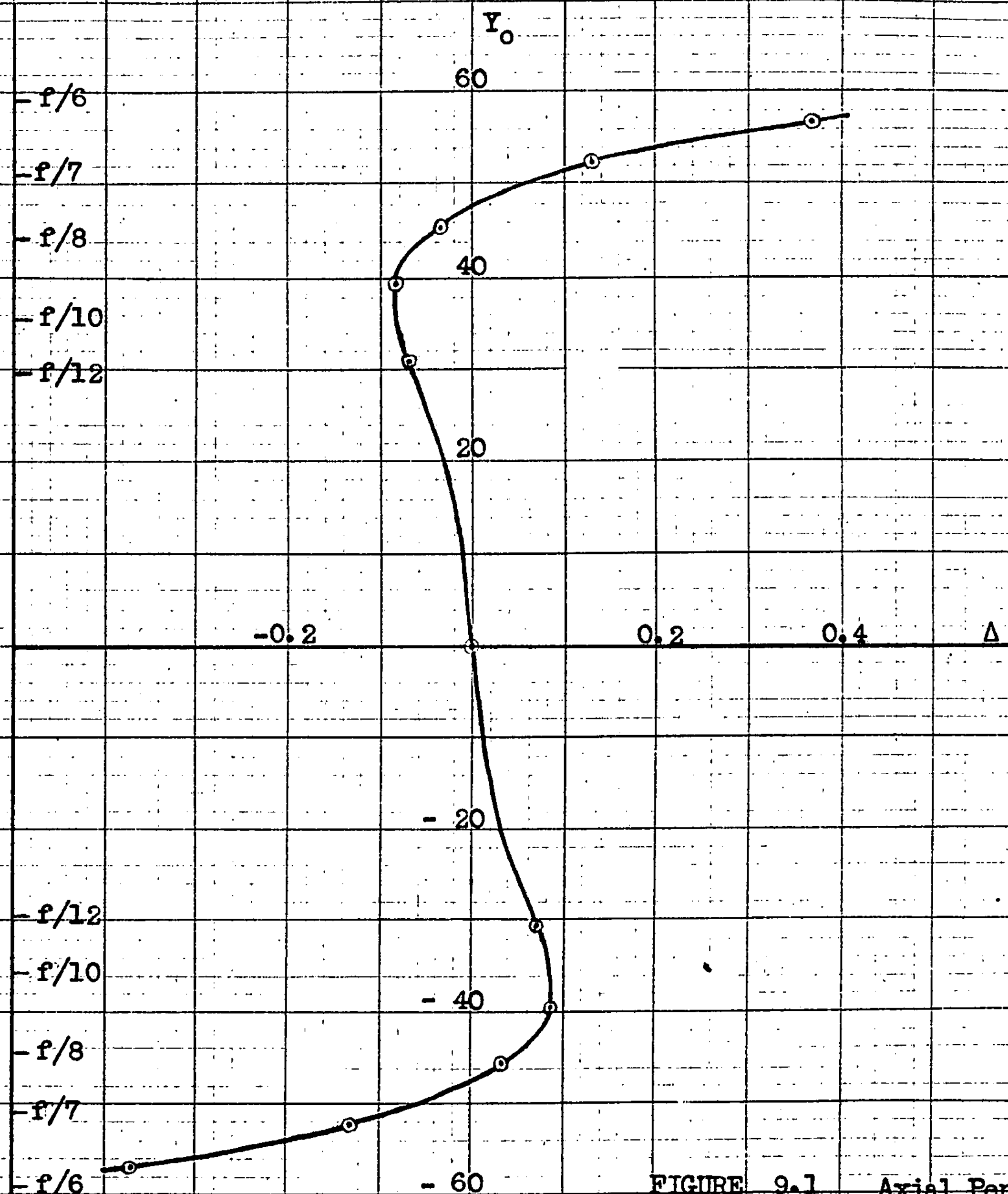


FIGURE 9.1 Axial Pencil.

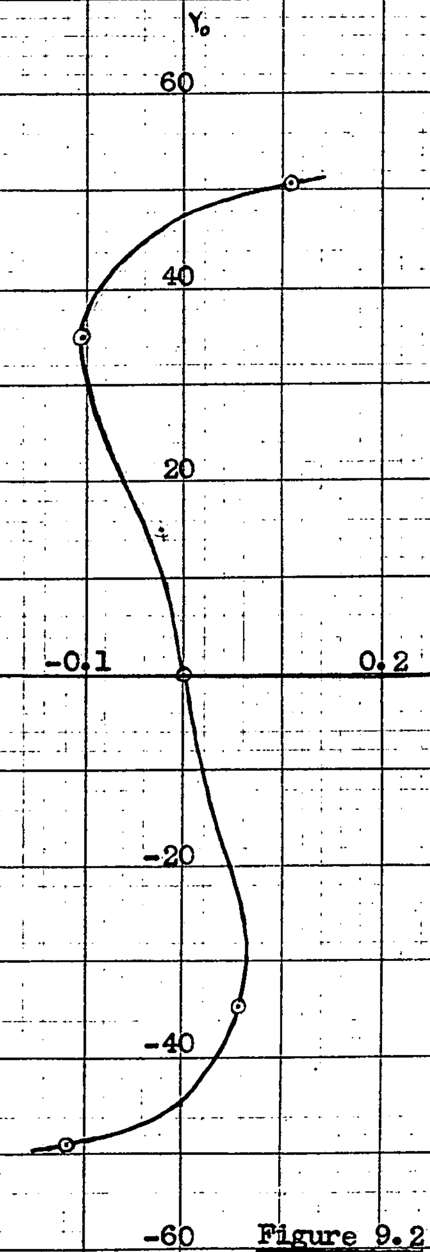


Figure 9.2

4° Oblique Pencil.

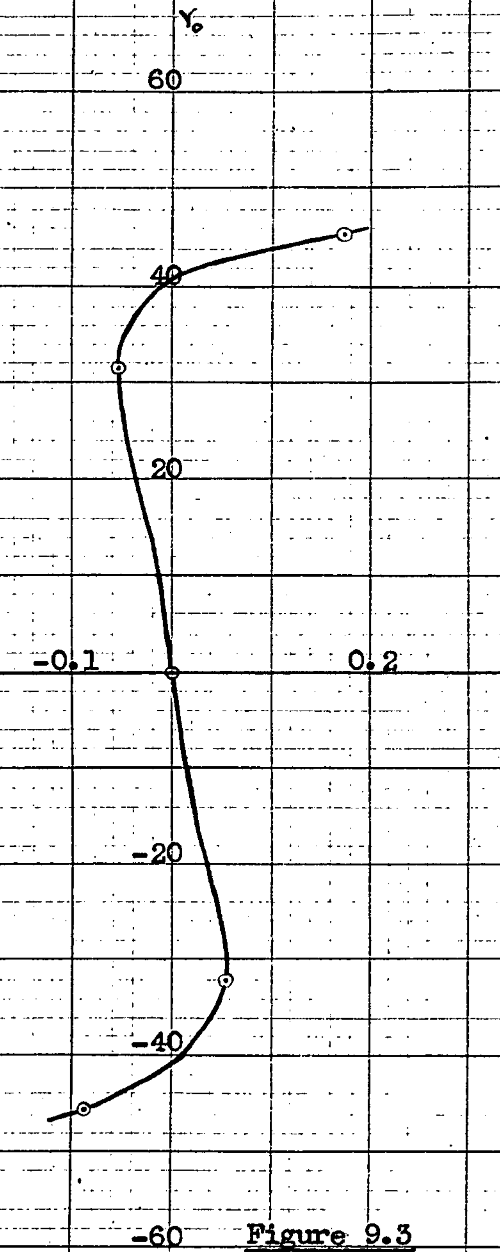


Figure 9.3

7.5° Oblique Pencil.

in the paraxial image plane when the lens aperture is f/6.3 . It is to be remembered that this calculation of the blur patch takes into consideration the effect of tangential rays only. The plots are shown in Figures 9.1 , 9.2 , and 9.3 . One feature of the lens is immediately apparent from these plots and calls for comment, and that is that the useful aperture is over-rated. It can be seen from a drawing of the system on which the path of the traced rays is shown that the 'a' ray of the 4^{o} pencil is the extreme ray of the pencil on one side, but the 'b' ray is not quite the limiting ray on its side of the pencil. Effectively, then, the aperture of the 4^{o} pencil is only about f/7. By the time an obliquity of 7.5^{o} has been reached the effective aperture is reduced to f/7.8 . A large area of the image then will be illuminated by pencils limited by an aperture not greater than f/7, and in any case these areas are also subject to the decrease in illumination proportional to $\cos^4 U$ quite independently of the vignetting produced by the lens mount. Hence there is nothing to be gained by illuminating the centre of the image by pencils of aperture greater than f/7 particularly when the blur patch increases in extent as rapidly as Figure 9.1 shows. It would seem reasonable then to limit the aperture of the lens to f/7.

3. <u>Correction States in Planes Adjacent to the Paraxial Plane</u>.

We should consider now the question of the possibility that a better state of correction exists in a plane parallel to but slightly displaced from the paraxial image plane. In other words, the plane of best focus does not necessarily coincide with the paraxial image plane, and may vary with aperture. We therefore require a process in our technique of designing which corresponds with the operation of focussing when the lens is projecting its image on a ground glass screen. A simple graphical means is readily available for this. If the plane in which the Δ' quantities are measured is displaced through a distance dl'_k the intersection height of any ray in the new plane differs from its value in the paraxial image plane by an amount $-dl'_k \tan U'_k$. The addition of this amount to the H' values of the rays in the paraxial plane gives the corresponding values in the new plane from which the Δ' values in the new plane are obtained. If we denote the Δ' values in two planes half a millimetre on either side of the paraxial plane by the sysmols Δ'_+ and Δ'_-, then a comparison of the plots for Δ'_+, Δ', and Δ'_- will enable us to deduce the effect of 'focussing' on the definition of the image within this range.

Returning now to Table 9.1 we consider the information contained in the last four columns. In column six the values of $-0.5 \tan U'_k$ are tabulated for each ray. This gives the change in H' for a shift of half a millimetre in the position of the image plane in the positive direction, i.e. away from the lens. Column seven gives the change in the values of Δ' which result from the shift, and columns eight and nine the values of Δ'_+ and Δ'_-. These quantities are plotted for the three pencils in Figures 9.4 , 9.5 , and 9.6 . The dotted curve in each of these is the original Δ' curve for the paraxial plane which has been added to the Figures to show the effect of the displacement more clearly. It will thus be seen that the effect of a shift of the image plane away from the lens is to swing the Δ' curve in a counter-clockwise sense about its origin, while a displacement of the image plane towards the lens (dl'_k - ve) swings the curve about its origin in a clockwise sense. We immediately infer from these Figures that for an aperture of f/7 the plane of best focus will be slightly on the lens side of the paraxial plane, and that as the aperture is decreased the plane of best focus will move further and further towards the lens, for the Δ' values are decreased by this movement. The change in the shape of the curves with small displacements of the plane in which the Δ' values are measured is much more marked with lenses

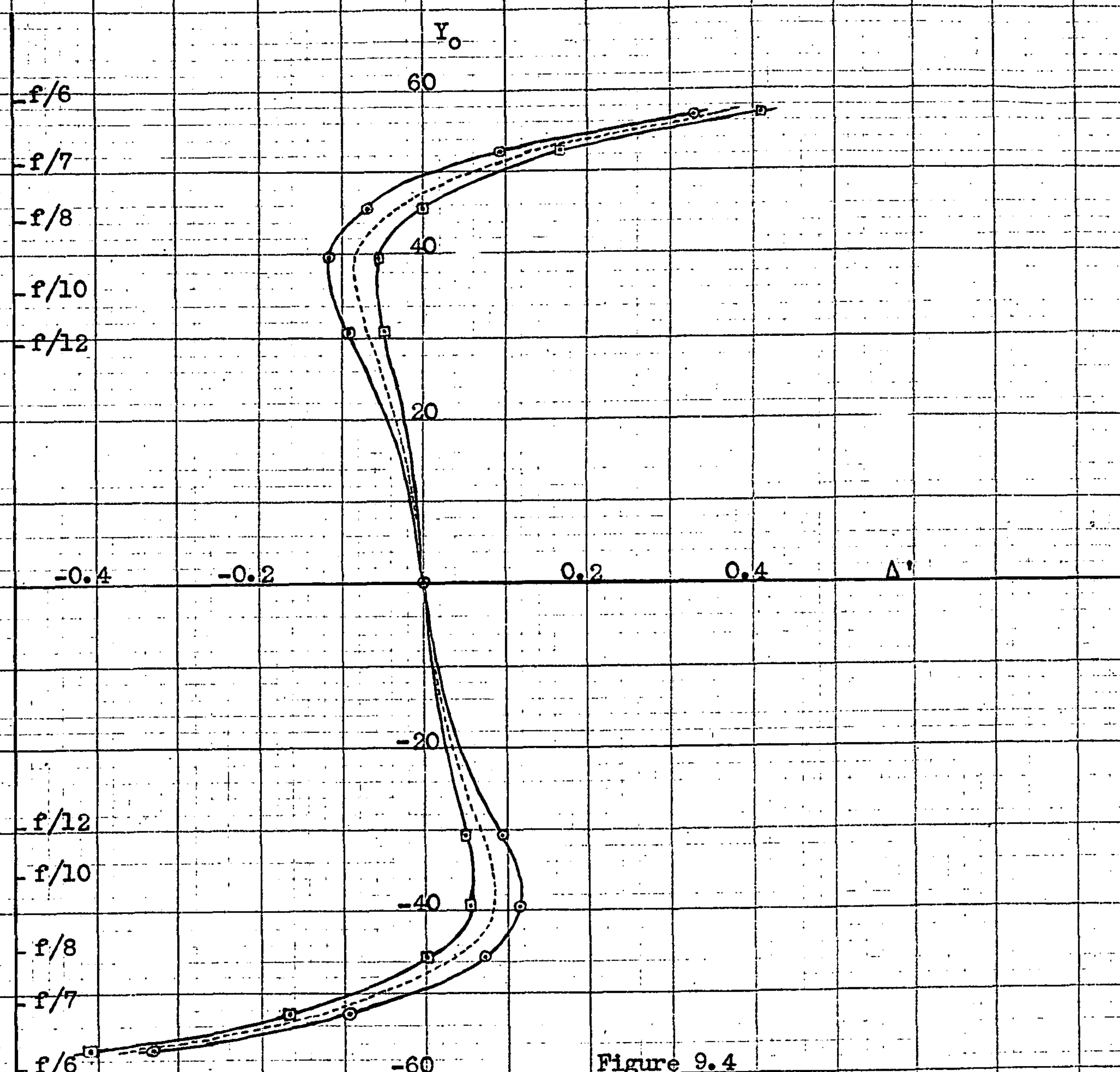


Figure 9.4

Axial Pencil

Δ'_+ points plotted ⊙

Δ'_- points plotted ⊡

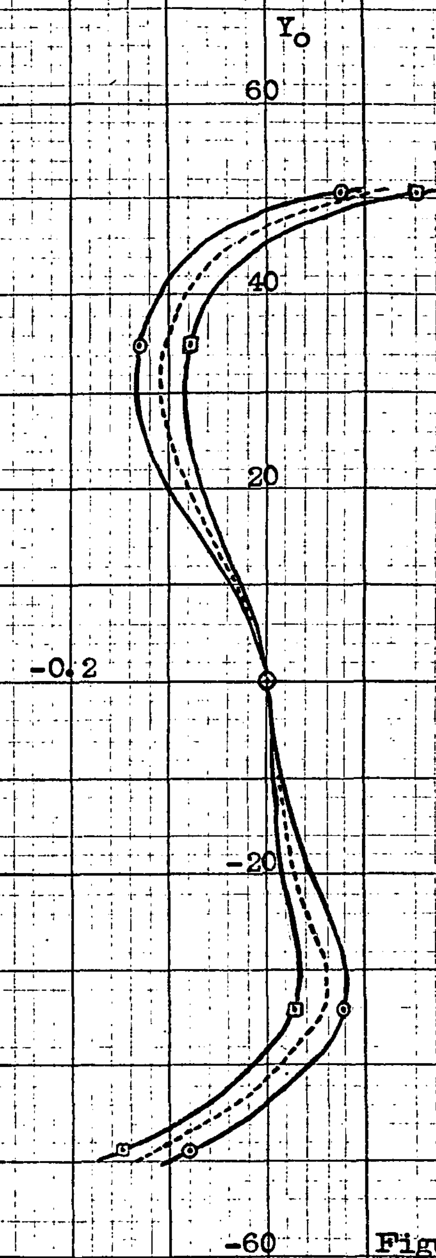


Figure 9.5

4° Oblique Pencil.

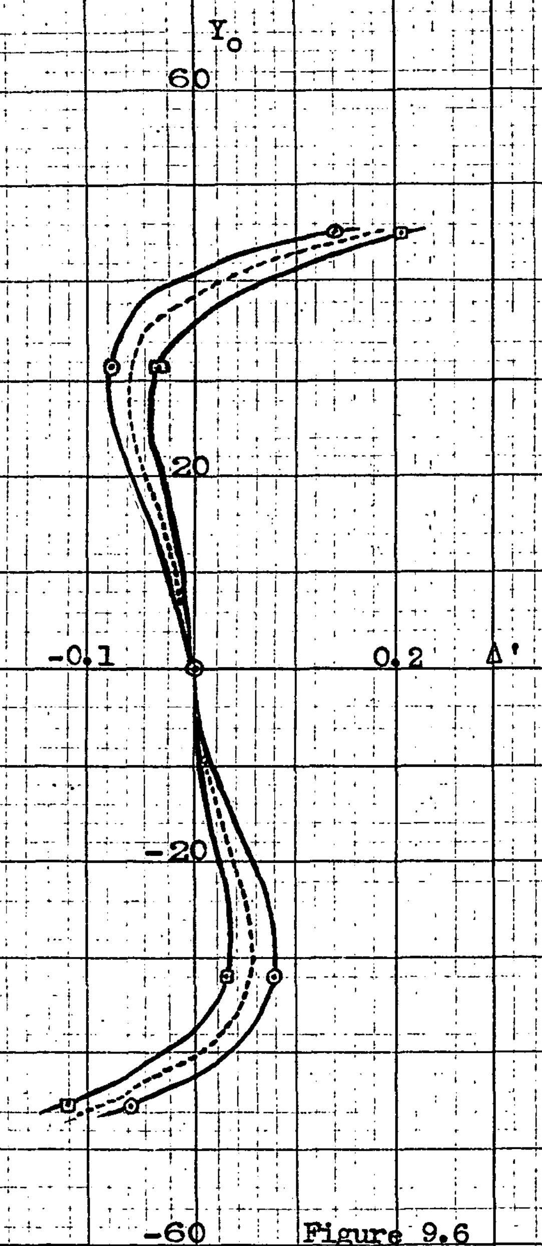


Figure 9.6

7.5° Oblique Pencil.

Δ'_+ points plotted ⊙

Δ'_- points plotted ⊡

of wider fields.

4. The Analysis of the Chromatic Aberration of a Lens System.

It is necessary to consider now the way in which the chromatic aberration of a lens system is to be treated in this new analysis of the general aberrations of the system. In the orthodox method of analysing the chromatic aberration it is usual to determine the longitudinal chromatic aberration for the axial pencil, generally at the paraxial and 0.707 zones. Conrady states as a rule 'that a prescribed state of chromatic correction will be most perfectly realised for the whole aperture if it is established trigonometrically for the 0.707 zone.' The same longitudinal aberration is expected in the oblique pencils and its correction at the axial pencil is intended to control it for the whole field of the system. In addition, the chromatic difference of magnification, or transverse chromatic aberration, is calculated from the data from the trace of the principal ray of an oblique pencil. There is no provision for determining the variation of the chromatic co correction with aperture or the effects of vignetting.

In the first place we decide to use only transverse measures for the specification of the chromatic aberration. For any specified incident ray we define the chromatic blur, $\Delta ch'$, for the colours $\underline{r}$ and $\underline{v}$ as

$$\Delta ch' \quad = \quad H'_r - H'_v \qquad (9.5)$$

where H'_r and H'_v denote the intersection heights in the paraxial image plane of rays of light of the colours $\underline{r}$ and $\underline{v}$ which enter the system along the specified incident path. The quantity $\Delta ch'$ may be reagrded as the measure of the chromatic aberration of a very narrow pencil surrounding the given ray. When applied to a principal ray this definition is identical with the transverse aberration, Tch' , but we propose to extend its use to any ray of any pencil.

Provided that the path of a ray incident in the prescribed way has been traced through the system using the refractive indices appropriate to some intermediate colour, $\underline{d}$, then, by analogy with the equation immediately preceding equation (2.69)

$$H'_r = H'_d + \Sigma\left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_r - N_d)_h \tag{9.6}$$

$$H'_v = H'_d + \Sigma\left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_v - N_d)_h \tag{9.7}$$

and

$$\Delta ch' = \Sigma\left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_r - N_v)_h \tag{9.8}$$

The last equation shows that the chromatic blur is given by the sum over all the components of the system of the products of the dispersion, $(N_r - N_v)_h$, with the chromatic coefficients $(\partial H'_d / \partial N_h)_{L'}$. For each ray traced through the system for the purpose of its analysis and correction by the present methods a value of $\Delta ch'$ may be calculated quite simply, and thus we obtain a measure of the chromatic aberration of a narrow pencil in the neighbourhood of each of these traced rays. For an oblique pencil for which five rays are traced we can thus sample the chromatic aberration in the vicinity of each of the traced rays, and thus form a clear picture of the change of the chromatic correction as we move across the pencil. It should be carefully noted that this sampling of the chromatic correction at different positions across the pencil will allow us to deduce how the over-all chromatic correction will be affected by the change of aperture of the system. Again, since we are dealing with chromatic coefficients, it should be noticed that we may calculate the chromatic blur for any pair of colours provided we know the corresponding dispersion values for the glasses, and hence a complete analysis of the secondary spectrum is possible in the vicinity of each traced ray. We thus have the basis of a very direct and powerful analysis of the chromatic correction of a lens system.

5. Numerical Example. The Analysis of the Chromatic Correction of the Telephoto Lens.

Continuing with the example begun in the earlier sections, we will proceed to assemble the data for the calculation of the chromatic correction of the system. A full analysis would require an examination over the spectral range from 7000 A. U. to about 5000 A.U. , but for the purpose of our example we will consider only the range C to F . In Table 9.2 the partial dispersions of the glasses of the components are set out.

TABLE 9.2

The Partial Dispersions of the Glasses.

Component	$N_C - N_d$	$N_e - N_d$	$N_F - N_d$
I	$.\overline{00336}$	.00264	.00772
II	$.\overline{00565}$	.00458	.01368
III	$.\overline{00331}$	.00263	.00773
IV	$.\overline{00417}$	.00333	.00989

TABLE 9.3

	Axial Pencil						4° Pencil					7.5° Pencil.				
	M	QM	Q	Z	QZ	Pxl	a	c	pr	d	b	a	c	pr	d	b
	$\overline{639.19}$	$\overline{571.71}$	$\overline{491.06}$	$\overline{408.13}$	$\overline{307.59}$	$\overline{7.3251}$	$\overline{624.35}$	$\overline{428.43}$	$\overline{62.504}$	289.04	467.91	$\overline{639.46}$	$\overline{464.29}$	$\overline{124.48}$	211.05	372.38
	363.31	322.73	271.10	227.14	170.05	4.0138	358.08	239.23	31.489	$\overline{160.78}$	$\overline{260.26}$	369.99	260.61	63.183	$\overline{119.95}$	$\overline{207.05}$
	164.35	149.17	128.49	109.69	83.799	2.0312	90.554	44.959	$\overline{50.910}$	$\overline{147.78}$	$\overline{194.20}$	24.978	$\overline{14.695}$	$\overline{100.80}$	$\overline{193.34}$	$\overline{236.80}$
	$\overline{103.84}$	$\overline{94.536}$	$\overline{81.756}$	$\overline{70.033}$	$\overline{53.717}$	$\overline{1.3102}$	$\overline{51.185}$	$\overline{22.423}$	39.208	100.93	129.63	$\overline{3.186}$	22.229	77.863	136.72	163.51

TABLE 9.4

	2.1477	1.9210	1.6500	1.3713	1.0335	0.0246	2.0978	1.4395	0.2100	$\overline{0.9712}$	$\overline{1.5722}$	2.1486	1.5600	0.4183	$\overline{0.7091}$	$\overline{1.2512}$
	$\overline{2.0527}$	$\overline{1.8234}$	$\overline{1.5317}$	$\overline{1.2833}$	$\overline{0.9608}$	$\overline{0.0227}$	$\overline{2.0232}$	$\overline{1.3516}$	$\overline{0.1779}$	0.9084	1.4705	$\overline{2.0904}$	$\overline{1.4724}$	$\overline{0.3570}$	0.6777	1.1698
	$\overline{0.5440}$	$\overline{0.4938}$	$\overline{0.4253}$	$\overline{0.3631}$	$\overline{0.2774}$	$\overline{0.0067}$	$\overline{0.2997}$	$\overline{0.1488}$	$\overline{0.1685}$	0.4892	0.6428	$\overline{0.0827}$	0.0486	0.3336	0.6400	0.7838
	0.4330	0.3942	0.3409	0.2920	0.2240	0.0055	0.2134	0.0935	0.1635	$\overline{0.4209}$	$\overline{0.5406}$	0.0133	$\overline{0.0927}$	$\overline{0.3247}$	$\overline{0.5701}$	$\overline{0.6818}$
$H'_c - H'_d$	$\overline{0.0160}$	$\overline{0.0020}$	0.0339	0.0169	0.0193	0.0007	$\overline{0.0126}$	0.0326	$\overline{0.0371}$	0.0055	0.0005	$\overline{0.0112}$	0.0435	0.0702	0.0385	0.0206

TABLE 9.5

	$\overline{1.6875}$	$\overline{1.5093}$	$\overline{1.2964}$	$\overline{1.0775}$	$\overline{0.8120}$	$\overline{0.0193}$	$\overline{1.6483}$	$\overline{1.1311}$	$\overline{0.1650}$	0.7631	1.2353	$\overline{1.6882}$	$\overline{1.2257}$	$\overline{0.3286}$	0.5572	0.9831
	1.6640	1.4781	1.2416	1.0403	0.7788	0.0184	1.6400	1.0957	0.1442	$\overline{0.7410}$	$\overline{1.1920}$	1.6946	1.1936	0.2894	$\overline{0.5494}$	$\overline{0.9483}$
	0.4322	0.3922	0.3379	0.2885	0.2204	0.0053	0.2382	0.1182	$\overline{0.1339}$	$\overline{0.3887}$	$\overline{0.5107}$	0.0657	$\overline{0.0386}$	$\overline{0.2651}$	$\overline{0.5085}$	$\overline{0.6228}$
	$\overline{0.3458}$	$\overline{0.3148}$	$\overline{0.2722}$	$\overline{0.2332}$	$\overline{0.1789}$	$\overline{0.0043}$	$\overline{0.1704}$	$\overline{0.0747}$	$\overline{0.1306}$	0.3361	0.4317	$\overline{0.0106}$	0.0740	0.2593	0.4553	0.5445
$H'_e - H'_d$	0.0629	0.0463	0.0109	0.0181	0.0083	0.0001	0.0595	0.0081	$\overline{0.0241}$	$\overline{0.0305}$	$\overline{0.0357}$	0.0615	0.0033	$\overline{0.0450}$	$\overline{0.0454}$	$\overline{0.0435}$

TABLE 9.6

	$\overline{4.9345}$	$\overline{4.4136}$	$\overline{3.7910}$	$\overline{3.1508}$	$\overline{2.3746}$	$\overline{0.0568}$	$\overline{4.8200}$	$\overline{3.3075}$	$\overline{0.4825}$	2.2314	3.6123	$\overline{4.9366}$	$\overline{3.5843}$	$\overline{0.9610}$	1.6293	2.8748
	4.9701	4.4149	3.7086	3.1073	2.3263	0.0549	4.8985	3.2727	0.4825	$\overline{2.1995}$	$\overline{3.5604}$	5.0615	3.5651	0.8643	$\overline{1.6409}$	$\overline{2.8324}$
	1.2704	1.1531	0.9932	0.8479	0.6478	0.0157	0.7000	0.3475	$\overline{0.3935}$	$\overline{1.1423}$	$\overline{1.5012}$	0.1931	$\overline{0.1136}$	$\overline{0.7792}$	$\overline{1.4945}$	$\overline{1.8305}$
	$\overline{1.0270}$	$\overline{0.9350}$	$\overline{0.8086}$	$\overline{0.6926}$	$\overline{0.5313}$	$\overline{0.0140}$	$\overline{0.5062}$	$\overline{0.2218}$	0.3878	0.9982	1.2820	$\overline{0.0315}$	0.2198	0.7701	1.3522	1.6171
$H'_F - H'_d$	0.2790	0.2194	0.1022	0.1118	0.0682	0.0008	0.2723	0.0909	$\overline{0.0574}$	$\overline{0.1122}$	$\overline{0.1673}$	0.2865	0.0870	$\overline{0.1058}$	$\overline{0.1539}$	$\overline{0.1710}$

Making use of equation (9.8) we have

$$H'_C - H'_d = \sum \left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_C - N_d)_h \tag{9.9}$$

$$H'_e - H'_d = \sum \left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_e - N_d)_h \tag{9.10}$$

$$H'_F - H'_d = \sum \left[\frac{\partial H'_d}{\partial N_h}\right]_{L'} (N_F - N_d)_h \tag{9.11}$$

The calculations based on these equations are set out in Tables 9.4 , 9.5 , and 9.6 . First of all we assemble in Table 9.3 the chromatic coefficients of the traced rays, collecting them from the detailed computation on page Taking Table 9.4 as typical, each entry in the first four rows of this Table is the product of the partial dispersion, $(N_C - N_d)$, of the glass of the component concerned with the corresponding chromatic coefficient. This product measures the contribution made by the component to the chromatic blur, for the C and d lines, of a thin pencil which enters the system along the incident path of the traced ray concerned. Each entry in the fifth row of this Table is the sum of the four contributions terms in the column above it: these entries thus give the values of the final chromatic blur, for the C and d lines, of thin pencils entering the system along the paths of incidence of all the rays traced through the system. In this way the state of chromatic correction for the C and d lines is sampled for sixteen narrow pencils distributed over the image field of the system. The process is repeated in Tables 9.5 and 9.6 for the e and d lines and for the F and d lines respectively.

To order the information obtained in these calculations so that it may be easily interpreted we plot a series of simple graphs in which the variation of $H' - H'_d$ with wave-length is shown for each ray. These are seen in Figures 9.7 to 9.22. These graphs present a fairly complete analysis of the chromatic correction of the system considered alone, and present it in a form which is readily comprehended, and in which its trends are clearly seen.

We proceed now to a detailed examination of the graphs. To estimate the chromatic correction at small aperture we examine the plots for the paraxial ray and principal rays of each oblique pencil. The paraxial correction leaves nothing to be desired, but the correction state deteriorates with increasing obliquity, the blur having already a value of 0.095 mm. at 4°, increasing to 0.176 mm. at 7.5°, and (details not shown in Tables) finally reaching 0.226 mm. at 10° from the axis. Next the state of correction of the axial pencil may be followed as the diaphragm is opened. By the time an aperture of f/11 is reached (sampled by ray 'QZ') the chromatic blur, according to the graphs, has reached 0.068 mm. We have to remember, however, that there is a ray corresponding to 'QZ' in the other half of the axial pencil on the opposite side of the principal axis, and its effect is to contribute a chromatic blur of opposite sign making the total blur twice the above amount, i.e. 0.138 mm. Increasing the aperture to f/9 (ray 'Z') the maximum blur has now reached the value of 0.224 mm. A further change to f/7.7 produces no deterioration from the f/9 value, but by the time f/6.8 has been reached the blur has attained a value of 0.44 mm. and continues to grow rapidly thereafter. It is the heavy swing away at the blue end of the spectrum which is responsible for these large values of the aberration.

Taking the 7.5° pencil next we see that the chromatic blur increases from the value 0.176 mm. at small aperture to a value of approximately 0.241 mm. by the time the aperture has reached f/11, at which stage the rays 'c' and 'd' are admitted. At an aperture of f/7.8, which represents the maximum aperture at this obliquity, rays of greater aperture being vignetted by the mount, the chromatic blur reaches a value of 0.457 mm.

Proceeding in this way it is possible to form a clear picture of the way in which the chromatic correction varies across the image field at different apertures. The setting out on a few sheets of graph paper a comprehensive

Variation of $\Delta ch'$ ($= H' - H'_d$) with Wave-length. Axial Pencil.

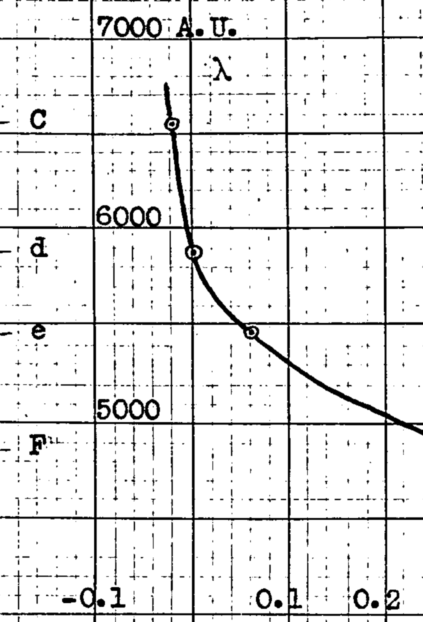


Figure 9.7 Ray 'M'.

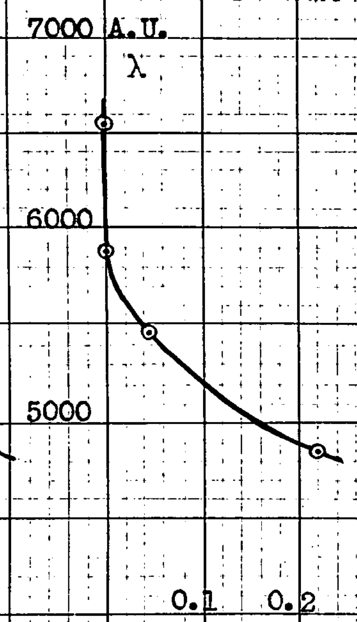


Figure 9.8 Ray 'QM'.

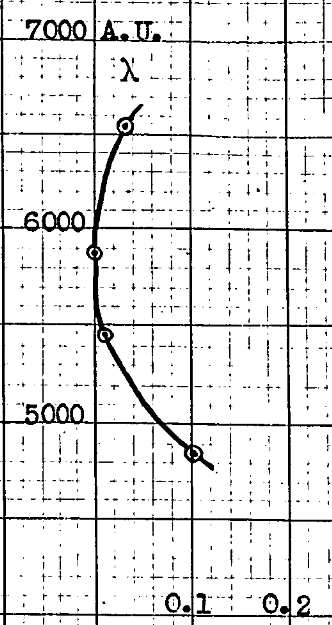


Figure 9.9 Ray 'Q'.

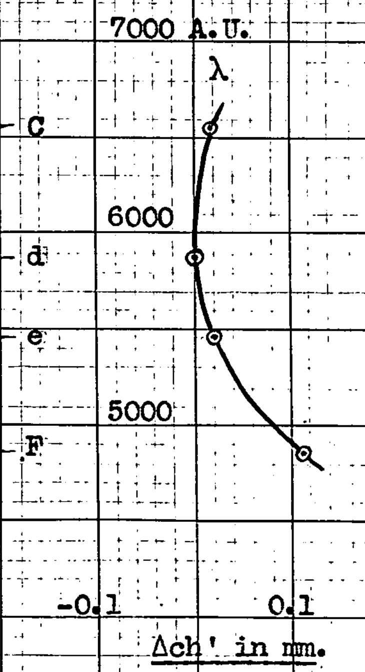


Figure 9.10 Ray 'Z'.

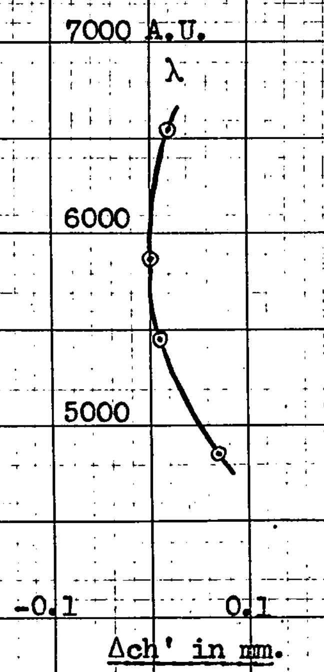


Figure 9.11 Ray 'QZ'.

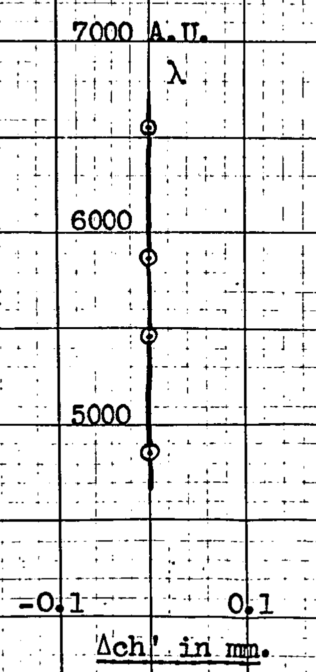


Figure 9.12 Ray 'pxI'

Variation of $\Delta ch'$ ($= H' - H'_d$) with Wave-length. 4° Oblique Pencil.

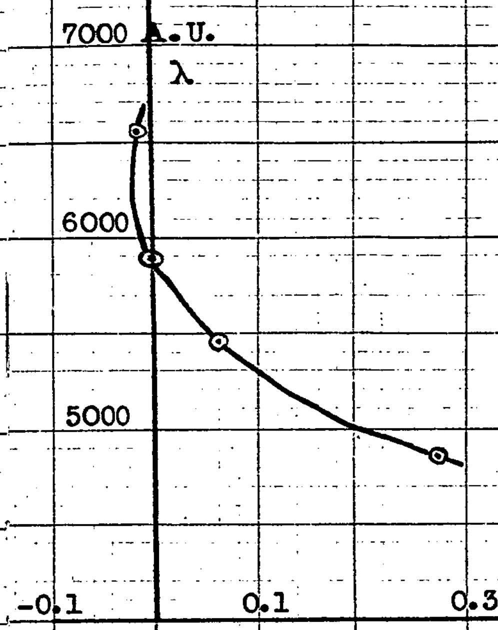


Figure 9.13 Ray 'a'.

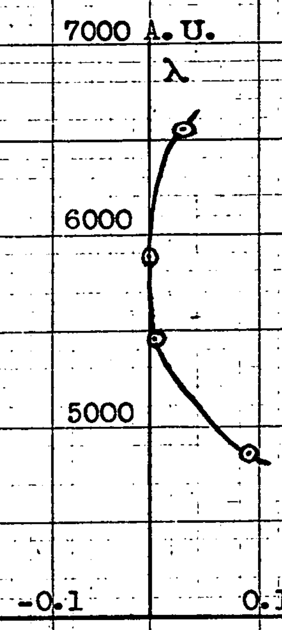


Figure 9.14 Ray 'c'.

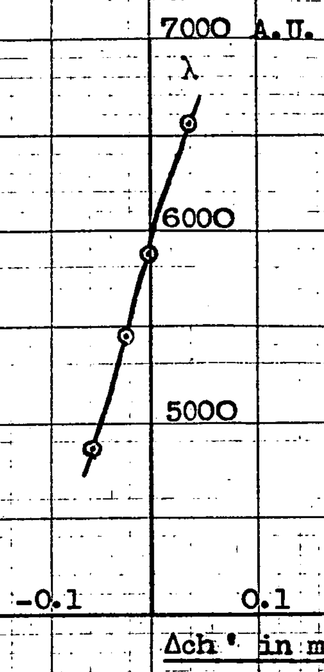


Figure 9.15 Ray 'pr'.

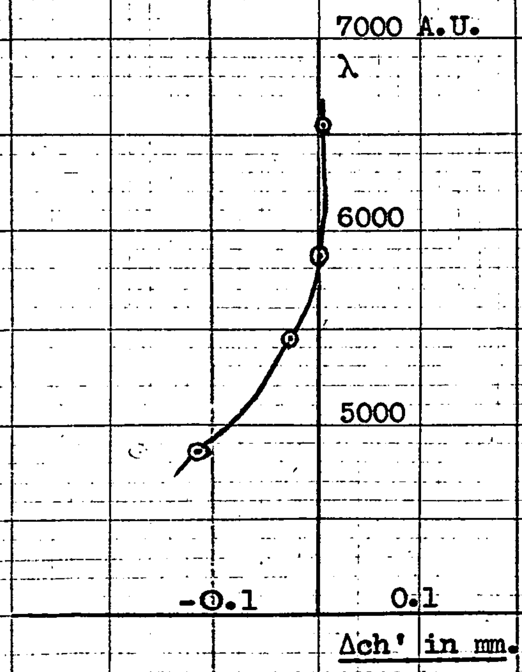


Figure 9.16 Ray 'd'.

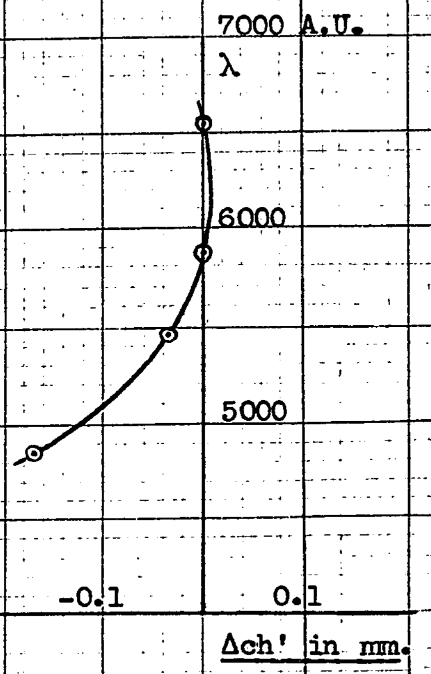


Figure 9.17 Ray 'b'.

Variation of $\Delta ch'$ ($= H' - H'_d$) with Wave-length. 7.5° Oblique Pencil.

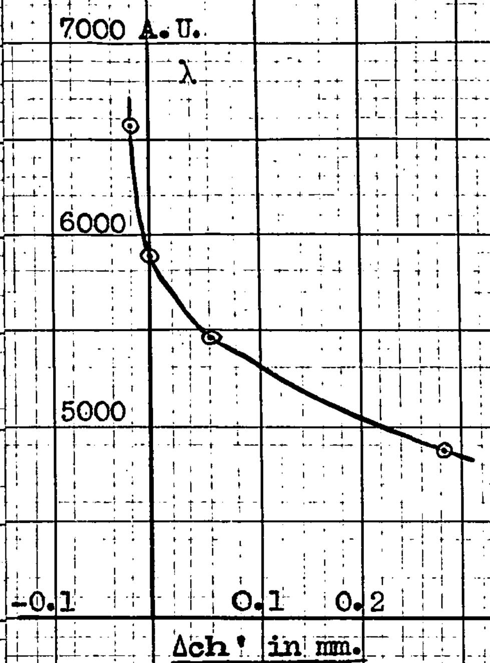


Figure 9.18 Ray 'a'

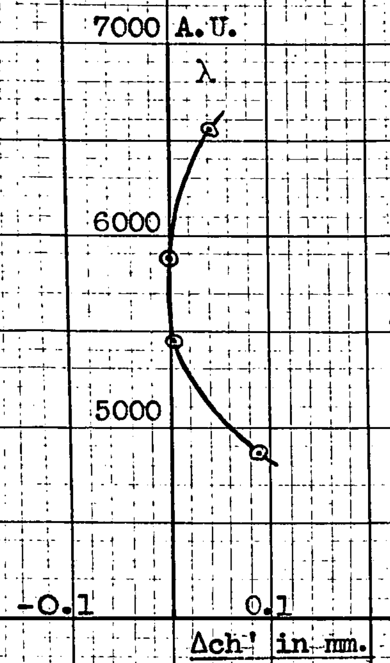


Figure 9.19 Ray 'c'.

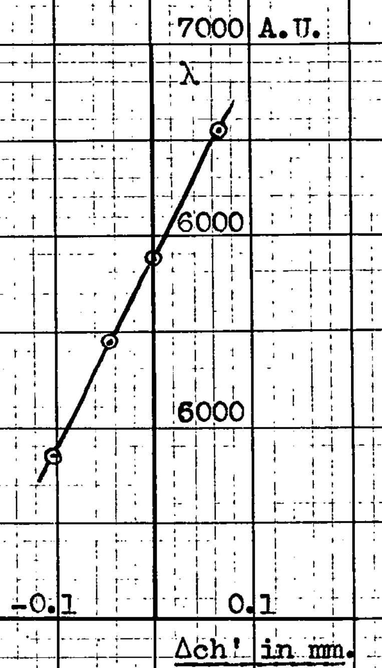


Figure 9.20 Ray 'pr'.

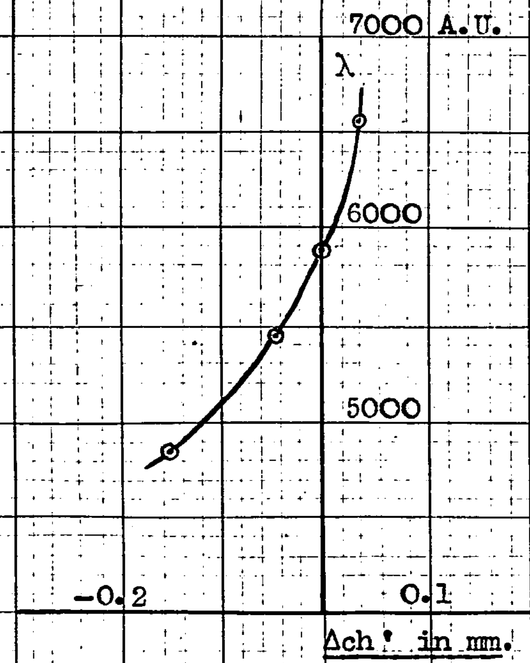


Figure 9.21 Ray 'd'.

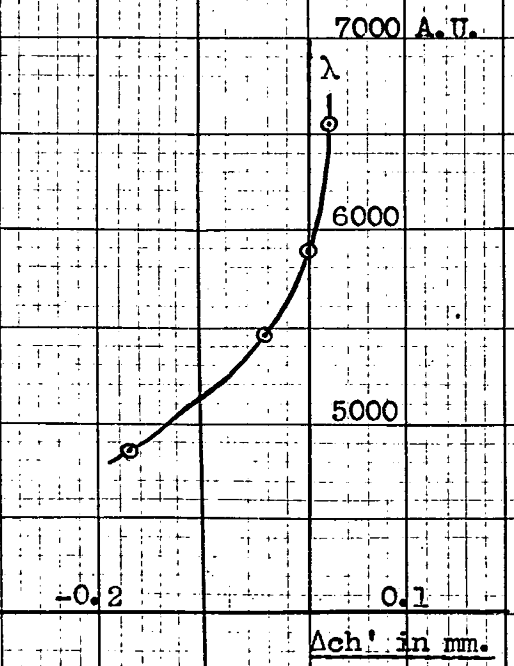


Figure 9.22 Ray 'b'.

picture of the variation of chromatic correction with wave-length and with aperture reveals the power of the designing method which has just been described. Before the method is really complete, however, there remains one further step. In the first place we have analysed the aberrations of a lens system in monochromatic light, and secondly, we have analysed the chromatic correction of the system completely separately from the other aberrations. To judge the over-all correction of the system we must take these two analyses into account together. Then, and only then, are we in a position to undertake the adjustment of the system with a view to improving the definition of the image. The combination of the two analyses is described in the next section.

6. The Chromatic Variation of the General Aberrations.

The remaining step of combining the two portions of the analysis already described is most easily made by plotting the $\Delta' - Y_o$ curves of the various pencils for a series of different wave-lengths. The graphs of Figures 9.1 to 9.3 presented the $\Delta' - Y_o$ curves for yellow light in the case of the Booth Telephoto Lens. By using the information in the second set of graphs in Figures 9.7 to 9.22 we may add to the first set the curves for the red and violet light corresponding to the C and F lines. Developing our numerical example in this way, we gather the relevant information in Table 9.7. In the third column of the table the Δ' quantities for each ray in yellow light are entered, these being denoted by Δ'_d, defined by

$$\Delta'_d = H'_d - H'_{prd} \tag{9.12}$$

These are identical with the values in column three of Table 9.1. In columns four and five the values of the chromatic blurs, $H'_C - H'_d$ and $H'_F - H'_d$, are entered, being taken from Tables 9.4 and 9.6 respectively. Finally, the Δ' values for each ray in C and F light are obtained by adding the corresponding entries in columns three and four, and three and five, respect-

TABLE 9.7

Ray	Y_o	Δ'_d	$H'_C - H'_d$	$H'_F - H'_d$	Δ'_C	Δ'_F
M	56.77	0.3718	$\overline{0.0160}$	0.2790	0.3558	0.6508
QM	52.21	0.1304	$\overline{0.0020}$	0.2194	0.1284	0.3498
Q	45.73	$\overline{0.0369}$	0.0339	0.1022	$\overline{0.0030}$	0.0653
Z	39.58	$\overline{0.0852}$	0.0169	0.1118	$\overline{0.0683}$	0.0266
QZ	30.72	$\overline{0.0705}$	0.0193	0.0682	$\overline{0.0512}$	$\overline{0.0023}$

4^o Oblique Pencil.

Ray	Y_o	Δ'_d	$H'_C - H'_d$	$H'_F - H'_d$	Δ'_C	Δ'_F
a	50.49	0.1075	0.0126	0.2723	0.1201	0.3798
c	35.17	$\overline{0.1029}$	0.0326	0.0055	$\overline{0.0703}$	$\overline{0.0974}$
pr			0.0371	$\overline{0.0574}$	0.0371	$\overline{0.0574}$
d	$\overline{34.50}$	0.0562	0.0081	$\overline{0.0305}$	0.0643	0.0257
b	$\overline{49.08}$	$\overline{0.1176}$	0.0005	$\overline{0.1673}$	$\overline{0.1171}$	$\overline{0.2849}$

7.5 Oblique Pencil.

Ray	Y_o	Δ'_d	$H'_C - H'_d$	$H'_F - H'_d$	Δ'_C	Δ'_F
a	45.24	0.1744	$\overline{0.0112}$	0.2865	0.1632	0.4609
c	31.50	$\overline{0.0536}$	0.0435	0.0870	$\overline{0.0101}$	0.0334
pr			0.0702	$\overline{0.1058}$	0.0702	$\overline{0.1058}$
d	$\overline{32.08}$	0.0553	0.0385	$\overline{0.1539}$	0.0938	$\overline{0.0986}$
b	$\overline{45.42}$	$\overline{0.0893}$	0.0206	$\overline{0.1710}$	$\overline{0.0687}$	$\overline{0.2603}$

these quantities being denoted by Δ'_C and Δ'_F and defined by

$$\Delta'_C = H'_C - H'_{prd} \tag{9.13}$$

$$\Delta'_F = H'_F - H'_{prd} \tag{9.14}$$

Each of these quantities is now plotted against Y_o , the plots for each pencil being made on the same sheet. These are shown in Figures 9.23, 9.24, and 9.25. and provide a complete summary of the state of correction of the three pencils considered in relation to the variation of the correction state with the colour of the incident light and with aperture. The writer usually summarises the correction state of a lens system on a large sheet of graph paper by plotting three rows of diagrams across the sheet. In the central row or band are plotted a set of curves similar to figures 9.23 to 9.25 showing the blur in the paraxial plane and its variation with colour plotted against the aperture. Across the upper band of the graph sheet are plotted the curves which give for one colour the variation with aperture of the blur quantities in a plane parallel to the paraxial plane but displaced towards the lens by a small interval of about 0.25 mm. to 0.50 mm., that is, the curves of Δ'_- vs. Y_o. Finally, across the lower band of the page are plotted the corresponding curves for Δ'_+ against Y_o. This incorporates all the information showing how the blur quantities are related to aperture, colour variation, and the process of focussing.

7. The Significance of the Form of the Curves.

We consider now the question of interpreting the particular form of the Δ' vs. Y_o curves in terms of the usual aberration types. The curves for the axial pencil call for little comment, for their forms are essentially the same as any graphical representation of the spherical aberration. In figure 9.23 there appears the familiar zonal undercorrection and marginal overcorrection. Considering the oblique pencils, however, there are a few points of interest. If the system is afflicted with coma only the 'a' and 'b' rays will fall both above or below the the principal ray at the paraxial image plane, and the Δ' curves will take forms similar to those in Figure 9.26. In the absence of coma, but

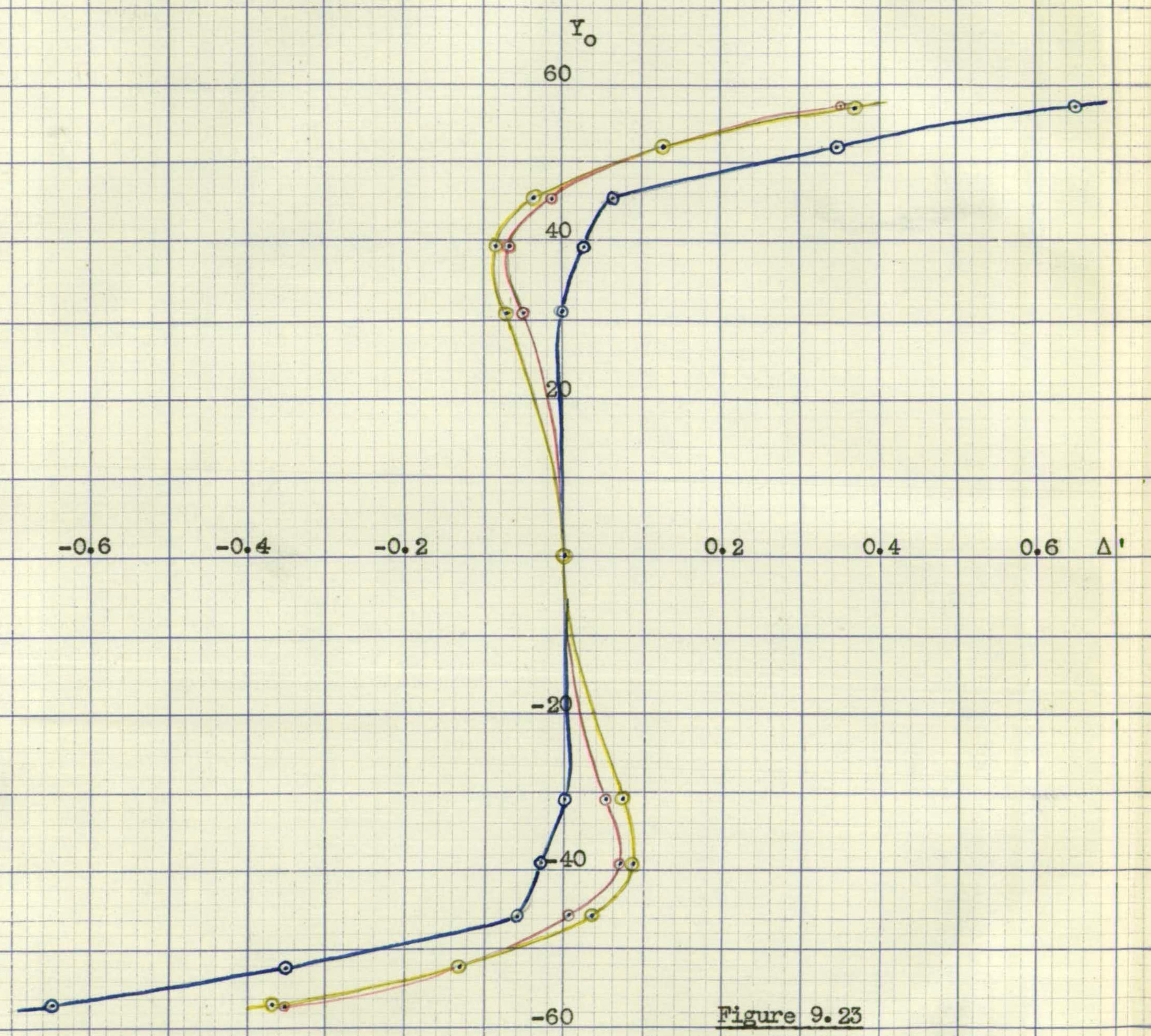


Figure 9.23

Axial Pencil.

Curve for Δ'_d drawn in yellow.

Curve for Δ'_C drawn in red.

Curve for Δ'_F drawn in blue.

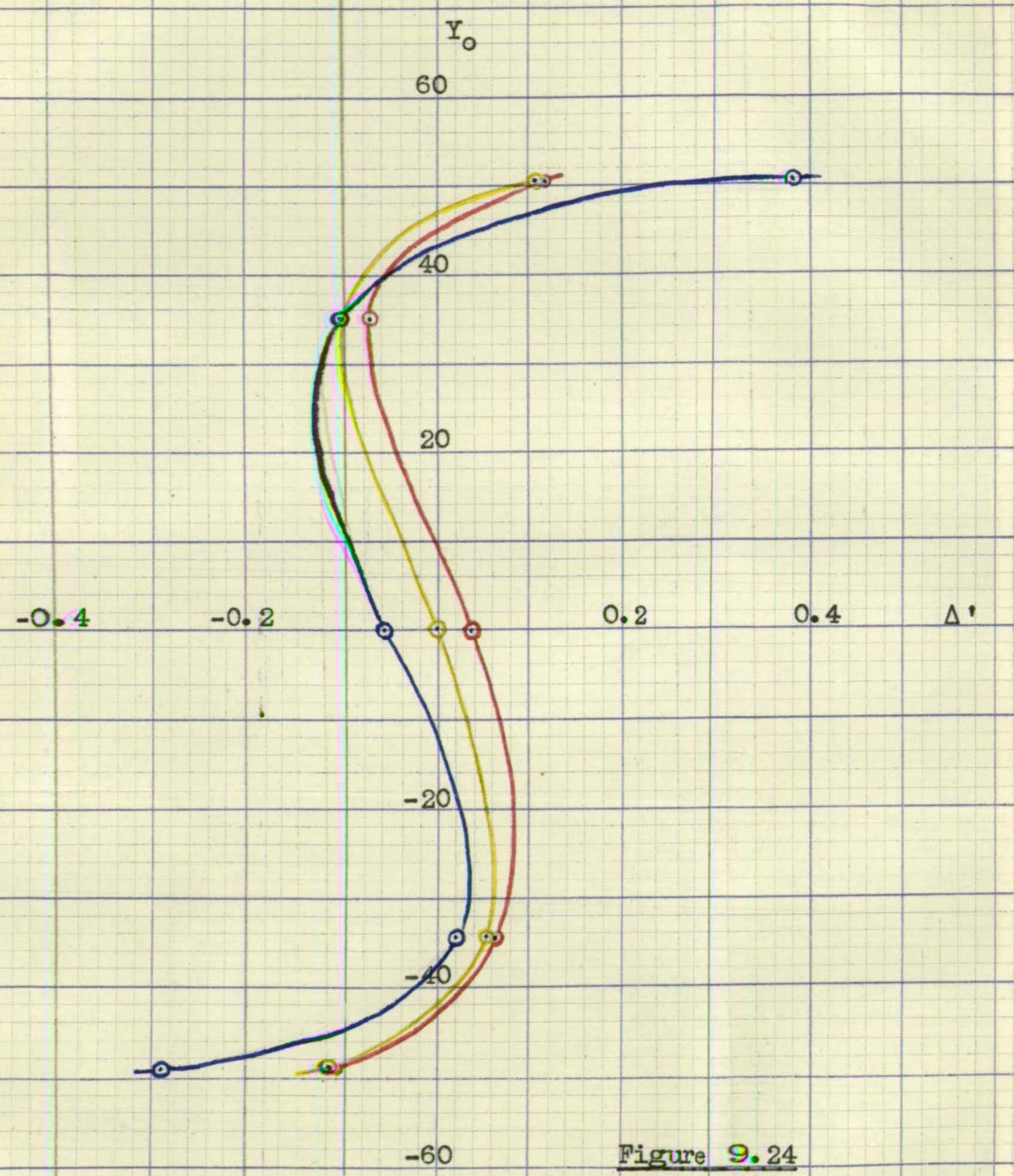


Figure 9.24

4^o Oblique Pencil.

Curve for Δ'_d drawn in yellow.

Curve for Δ'_C drawn in red.

Curve for Δ'_F drawn in blue.

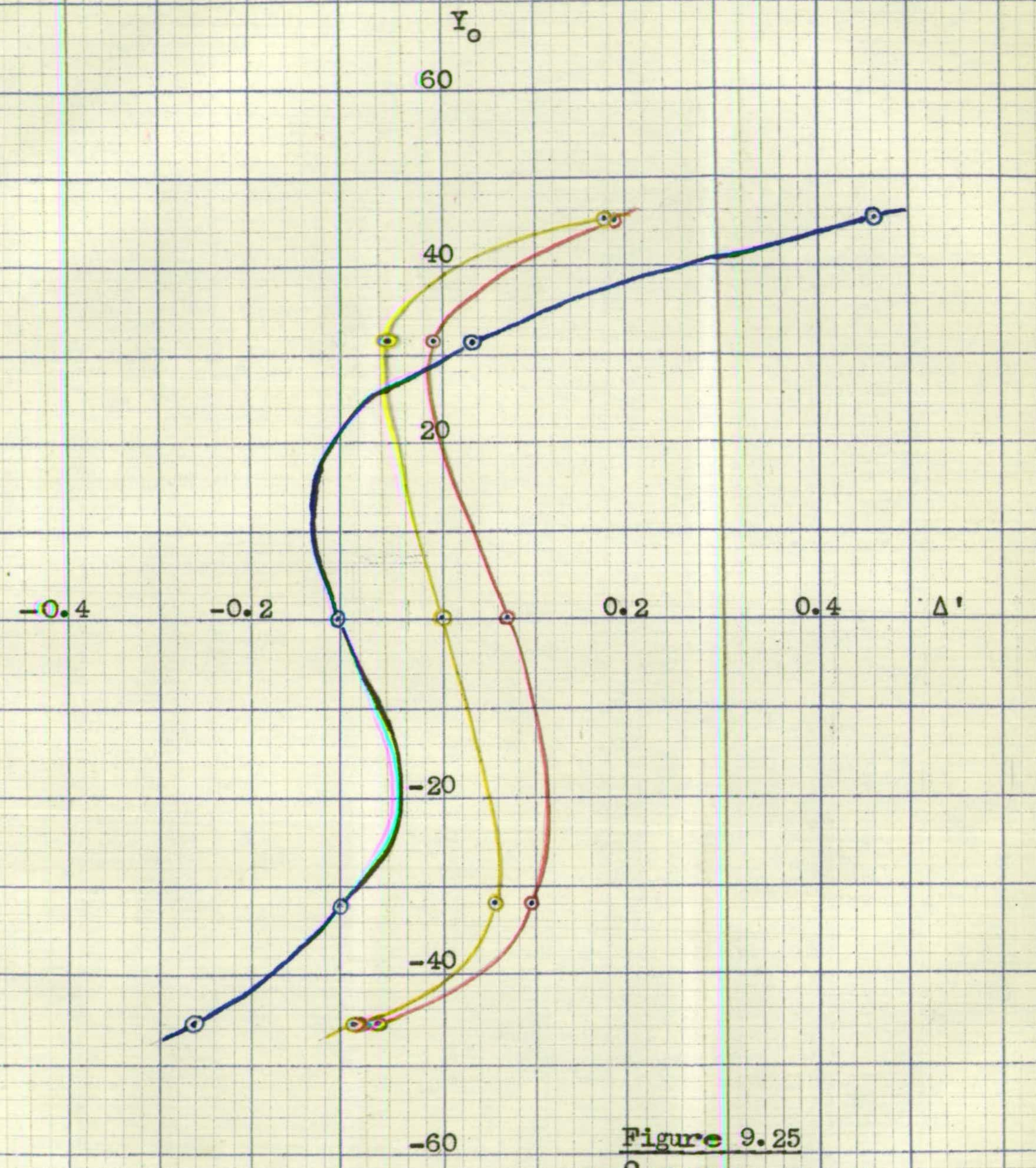


Figure 9.25

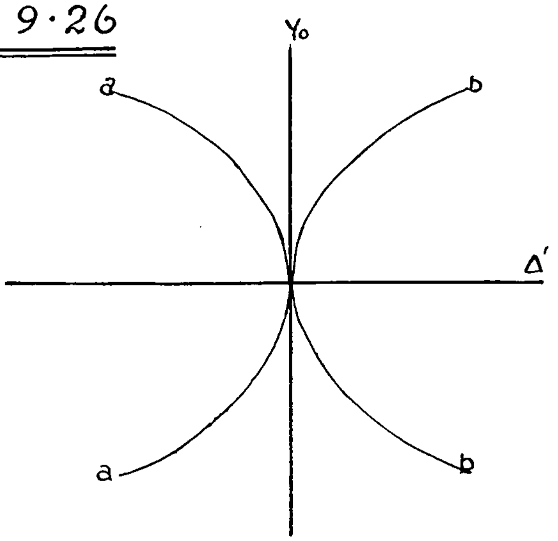
7.5^o Oblique Pencil.

Curve for Δ'_d drawn in yellow.

Curve for Δ'_C drawn in red.

Curve for Δ'_F drawn in blue.

Fig. 9·26

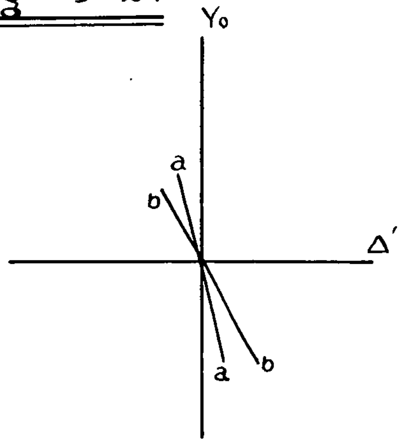


Form of curve for pencil afflicted with coma only.

aa. Positive coma.

bb. Negative coma.

Fig. 9·27



Slope of curve near origin. If t-focus falls on lens side of paraxial plane indicating negative curvature of field at small aperture the slope of curve passing through origin will be as shown.

aa. small negative curvature

bb. larger negative curvature.

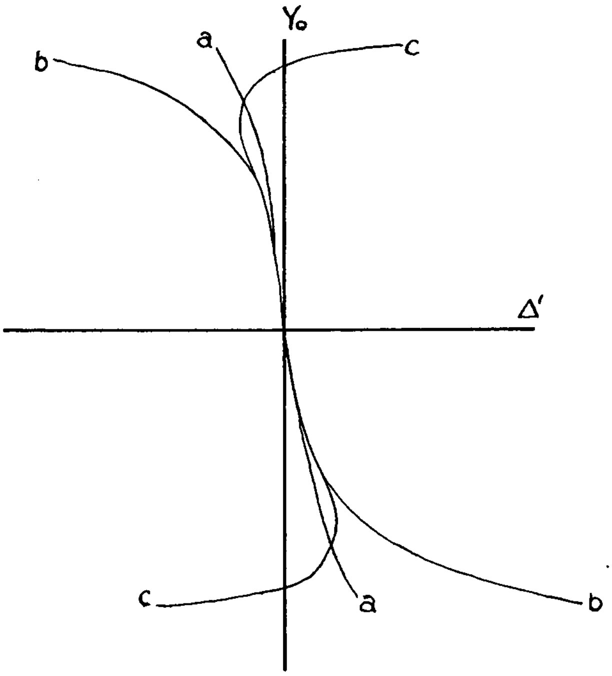


Fig. 9·28

Pencil not comatic. Negative curvature of field at small aperture.

aa. Negative field curvature only.

bb. Negative field curvature increasing rapidly with aperture, or alternatively, negative field curvature of small aperture with undercorrected oblique spherical aberration.

cc. Negative field curvature at small aperture with undercorrected zonal oblique spherical aberration and overcorrected marginal oblique spherical aberration.

with curvature of field present we may obtain curves of the forms shown in Figure 9.28 . In the first place consider the slope of the curve in the vicinity of the origin. If the t-focus falls on the lens side of the paraxial plane, indicating that there is negative curvature of field at small aperture, the slope of the curve will be as shown in Figure 9.27. The greater the curvature at small aperture the greater, the angle through which the curve will be turned from the Y_o axis. A glance at the slope of the curve near the origin, then, will enable an inference to be drawn as to the Petzval curvature. If, as the aperture is increased, the rays in the tangential fan intersect at points very close to the t-focus, so that a well defined image is formed by these rays, then the system suffers only from field curvature and for the negative case a plot of the form 'aa' in Figure 9.28 would be obtained. If, however, the curvature increases with increasing aperture we obtain a curve of the form 'bb' in Figure 9.28. In such a state successive 'a' and 'b' ray pairs intersect at points further and further away from the t-focus, with the result that the Δ' values increase rapidly. This is most easily thought of as an oblique spherical aberration of the tangential fan, and in the curve 'bb' we would call it undercorrected. A more common form, particularly for small obliquities, is that shown in curve 'cc' of Figure 9.28 . At small aperture the field has negative curvature, at larger aperture the presence of undercorrected oblique spherical aberration is seen, while at larger aperture still the oblique spherical aberration is overcorrected, the 'a' and 'b' ray pairs intersecting beyond the image plane. In all of these curves, coma being absent, there is an approximate symmetry, the corresponding portions of the curves lying in opposite quadrants. If now a system is adjusted to have a curve of the form of 'cc' in Figure 9.28 and then an alteration is made which admits a considerable amount of negative coma, the curve will be altered to one resulting from the combination of the form 'cc' , Figure 9.28 , with the form 'bb' , Figure 9.26 . Thus the upper portion will be pushed towards the right and the lower portion towards the right also, resulting in a strong departure from symmetry. In this general way it becomes very simple to interpret these curves in terms of the usual aberration quantities when desired. The oblique pencil whose correction state is summarised in Figure 9.25 , taking the curve drawn in yellow for example, shows negative curvature of field at small aperture, zonal oblique spherical aberration which is undercorrected, marginal oblique spherical aberration which is overcorrected, and a little negative coma the measure of which increases appreciably from the zonal to the outer parts of the tangential fan.

8. <u>The Slope of the Δ' Aberration Curves Near the Origin.</u>

In the way in which the present aberration analysis is used the rays which are traced through the system lie in the outer portions of the pencil, from the zonal region outwards. It therefore becomes very necessary to know the slope of the curve as it passes through the origin, in order that the general form of the aberration curve may be correctly inferred in the region between the principal ray and the first traced rays. A very simple means is available for determining this, and thus the possible guess-work is eliminated with only a couple of computing operations. We consider the case of the infinitely distant object first. Consider the ray which is incident on the first surface of the system parallel to the principal ray of the oblique pencil considered, its incidence point being displaced from that of the principal ray by dp_1 . The height at which this ray would intersect the plane of the entrance pupil is given by

$$SA = dp_1 \sec U_{pr}$$

The ray which intersects the entrance pupil plane at unit height will have an incidence point on the first surface displaced from that of the principal ray by $dp_1 = \cos U_{pr}$. After passing through the system this ray would emerge from the last surface with an incidence point and inclination angle given relative to those of the principal ray by

$$dp'_k = (\partial p'_k / \partial p_1) \cos U_{pr} \tag{9.15}$$

$$dU'_k = (\partial U'_k / \partial p_1) \cos U_{pr} \tag{9.16}$$

The distance of the t-focus from the last surface, measured along the principal ray as usual, is given by

$$t'_k = (\partial p'_k / \partial p_1) / (\partial U'_k / \partial p_1) \tag{9.17}$$

and the Δ' value for this ray under consideration is given by

$$\Delta' = (t'_k - S'_k) \sec U'_k \, dU'_k \tag{9.18}$$

as a glance at Figure 9.29 will confirm.

The remaining case of an object plane at finite distance from the lens is very little more complicated and is required for the design of process lenses etc. In Figure 9.30 a principal ray is shown aimed at the centre of the entrance pupil of the system. A near ray from the same object point intersects the pupil at a height SA, and the geometry of the figure obviously gives

$$SA = (l - L_{pr})_1 \sec^2 U_{pr} \, dU_1$$

The ray which intersects the entrance pupil at unit height above the axis is defined relative to the principal ray by

$$dU_1 = \cos^2 U_{pr} / (l - L_{pr})_1 \tag{9.19}$$

This ray strikes the first surface of the system with an inclination angle, relative to the principal ray, given by equation (9.19) and with an incidence point displacement

$$dp_1 = S_1 \, dU_1 \tag{9.20}$$

where S_1 is the distance of the object point from the first surface measured along the principal ray. After passing through the system the ray under consideration will emerge from the last surface with

$$\begin{aligned} dp'_k &= \frac{\partial p'_k}{\partial p_1} dp_1 + \frac{\partial p'_k}{\partial U_1} dU_1 \\ &= \left(S_1 \frac{\partial p'_k}{\partial p_1} + \frac{\partial p'_k}{\partial U'_1} \frac{\partial U'_1}{\partial U_1}\right) dU_1 \end{aligned} \tag{9.21}$$

and $$dU'_k = \left(S_1 \frac{\partial U'_k}{\partial p_1} + \frac{\partial U'_k}{\partial U'_1} \frac{\partial U'_1}{\partial U_1}\right) dU_1 \tag{9.22}$$

This gives for the t-focus

$$t'_k = \left(S_1 \frac{\partial p'_k}{\partial p_1} + \frac{\partial p'_k}{\partial U'_1} \frac{\partial U'_1}{\partial U_1}\right) \Big/ \left(S_1 \frac{\partial U'_k}{\partial p_1} + \frac{\partial U'_k}{\partial U'_1} \frac{\partial U'_1}{\partial U_1}\right) \tag{9.23}$$

and finally the Δ' value for the ray is given by

$$\Delta' = (t'_k - S'_k) \sec U'_k \; dU'_k \tag{9.24}$$

The Y_o value of these rays which intersect the entrance pupil at unit height above the axis is given by the value of the magnification between these two planes, i.e.

$$Y_o = \sin U_{pr} / \sin U'_{pri} \tag{9.25}$$

if the diaphragm is behind the surface $\underline{i}$ of the system. This allows the Δ' values for the two rays whose Y_o values are plus and minus the Y value of equation (9.25) to be plotted on the aberration curve, and so defines the slope of the curve near the origin.

9. The Transfer Coefficients for the Δ' Aberration Analysis.

The transfer coefficients needed for the differential correction of a lens system using the present analysis of the aberrations are all of a single type, each specifying the rate of change of the intersection height of a ray in the paraxial image plane with some variable of the system. This simplifies the computation very considerably, the more cumbersome expressions belonging to the Ab intersection in the previous method being eliminated. In writing down the expressions for the transfer coefficients we will dispense with any subscript denoting the plane in which the intersection point lies as in every case it is the paraxial image plane. By analogy with equations (2.35) and (2.36), we have for any ray

$$\frac{\partial H'}{\partial U_i'} = C(U_i') \sec U_k' \tag{9.26}$$

$$\frac{\partial H'}{\partial p_i} = C(p_i) \sec U_k' \tag{9.27}$$

and by analogy with equations (2.41), (2.48), and (2.55) we have

$$\frac{\partial H'}{\partial c_i} = C(c_i) \sec U_k' - \frac{\partial l'}{\partial c_i} \tan U_k' \tag{9.28}$$

$$\frac{\partial H'}{\partial d_i} = C(d_i) \sec U_k' - \frac{\partial l'}{\partial d_i} \tan U_k' \tag{9.29}$$

$$\frac{\partial H'}{\partial n_i} = C(n_i) \sec U_k' - \frac{\partial l'}{\partial n_i} \tan U_k' \tag{9.30}$$

where
$$\frac{\partial l'}{\partial c_i} = C(c_i) / u_k' \tag{9.31}$$

with corresponding expressions for $\partial l' / \partial d_i$, and $\partial l' / \partial n_i$. Considering the ideal image point, in the case of an infinitely distant object equation (2.43) gives

$$\frac{\partial H'_{id}}{\partial c_i} = \frac{f'}{u'_k} \tan U_{pr} \frac{\partial u'_k}{\partial u'_i} \frac{\partial u'_i}{\partial c_i} \tag{9.32}$$

$$= -\frac{H'_{id}}{u'_k} \frac{\partial u'_k}{\partial u'_i} \frac{\partial u'_i}{\partial c_i} \tag{9.33}$$

The transfer coefficient in the case of a near object is given by (9.33) also. Corresponding expressions follow for transfer coefficients with respect to changes of $\underline{d}$ and $\underline{n}$. From the foregoing equations it follows that

$$\frac{\partial \Delta'}{\partial c_i} = \frac{\partial H'}{\partial c_i} - \frac{\partial H'_{pr}}{\partial c_i} \tag{9.34}$$

$$\frac{\partial \mathrm{dist}'}{\partial c_i} = \frac{\partial H'_{pr}}{\partial c_i} - \frac{\partial H'_{id}}{\partial c_i} \tag{9.35}$$

with corresponding expressions for the transfer coefficients with respect to the other parameters $\underline{d}$ and $\underline{n}$. Finally, the effect of changing the glass of a complete component is specified by transfer coefficients of the type

$$\frac{\partial H'_d}{\partial N_h} = -\frac{N_{h-1}}{N_h^2} \frac{\partial H'_d}{\partial n_1} + \frac{1}{N_{h+1}} \frac{\partial H'_d}{\partial n_2} \tag{9.36}$$

The computation of these coefficients calls for no comment at this stage, but the interleaved computing sheets show a systematic arrangement of the work. On the paraxial transfer sheet the quantities listed below the line for $\partial l'/\partial d$ are connected with the second order correction terms, and are used only when large changes and higher accuracy are in view. Similarly, the the general transfer coefficient sheet the normal computation ends with the calculation of $\partial H'/\partial d$. Detailed examples of the computations will be found in the last chapter.

Computer: **TRANSFER COEFFICIENTS.** System

Date: Pencil Ray

Surface

$\partial U_+'/\partial U_+$
D_+
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$
$(\partial U_k'/\partial p_+)D_+$
$\partial U_k'/\partial U'$
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$
$(\partial p_k'/\partial p_+)D_+$
$\partial p_k'/\partial U'$
$\partial U'/\partial p$
$\partial p'/\partial p$
$(\partial U_k'/\partial U')(\partial U'/\partial p)$
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$
$\partial U_k'/\partial p$
$(\partial p_k'/\partial U')(\partial U'/\partial p)$
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$
$\partial p_k'/\partial p$

$-S_k'.\partial U_k/\partial U'$
$C(U')$
$C(U')\sec U_k'$
$-S_k'.\partial U_k'/\partial p$
$C(p)$
$C(p)\sec U_k'$

$\partial U'/\partial c$
$C(c)\sec U_k'$
$-\partial l'/\partial c \tan U_k'$
$\partial H'/\partial c$
$\partial U'/\partial n$
$C(n)\sec U_k'$
$-\partial l'/\partial n \tan U_k'$
$\partial H'/\partial n$
$\partial p/\partial d$
$C(d)\sec U_k'$
$-\partial l'/\partial d \tan U_k'$
$\partial H'/\partial d$

$\partial U_k'/\partial c$
$\partial U_k'/\partial n$
$\partial U_k'/\partial d$

$\nu(c)$
$\gamma(c)$
$\pi(c)$
$\rho(c)$

$\nu(d)$
$\gamma(d)$
$\pi(d)$
$\rho(d)$

Computer:

PARAXIAL TRANSFER COEFFICIENTS.

System..

Date:

Surface

$\partial u_+'/\partial u_+$

d_+

$(\partial u_k'/\partial u_+')\ (\partial u_+'/\partial u_+)$

$(\partial u_k'/\partial p_+)\, d_+$

$\partial u_k'/\partial u'$

$(\partial p_k'/\partial u_+')\ (\partial u_+'/\partial u_+)$

$(\partial p_k'/\partial p_+)\, d_+$

$\partial p_k'/\partial u'$

$\partial u'/\partial p$

[redacted]

$(\partial u_k'/\partial u')\, (\partial u'/\partial p)$

$(\partial u_k'/\partial p_+)$ [redacted]

$\partial u_k'/\partial p$

$(\partial p_k'/\partial u')\, (\partial u'/\partial p)$

$(\partial p_k'/\partial p_+)\, (\partial p'/\partial p)$

$\partial p_k'/\partial p$

$-l'.\partial u_k'/\partial u'$

$C(u')$

$\partial l'/\partial u'$

$-l'.\partial u_k'/\partial p$

$C(p)$

$\partial l'/\partial p$

$\partial u'/\partial c$

$\partial l'/\partial c$

$\partial u'/\partial n$

$\partial l'/\partial n$

$\partial p/\partial d$

$\partial l'/\partial d$

$\partial u_k'/\partial c$

$\partial u_k'/\partial n$

$\partial u_k'/\partial d$

$-\partial l'/\partial c.1/u_k'$

$\lambda'(c)$

$-\partial l'/\partial d.1/u_k'$

$\lambda'(d)$

$1/u_k'$

f/u_k'

ADDITIONAL PAPERS.

I. ON THE PRIMARY CHROMATIC COEFFICIENTS OF A LENS SYSTEM.

II. THE PARAXIAL DIFFERENTIAL TRANSFER COEFFICIENTS OF A LENS SYSTEM.

III. TRANSFER COEFFICIENTS FOR THE PRIMARY ABERRATIONS OF A LENS SYSTEM.

IV. THE CHROMATIC VARIATION OF THE TANGENTIAL ABERRATIONS.

V. THE TRIGONOMETRICAL CORRECTION OF MICROSCOPE OBJECTIVES.

ON THE PRIMARY CHROMATIC COEFFICIENTS OF A LENS SYSTEM.

by F. D. CRUICKSHANK B. Sc.

Department of Physics, University of Tasmania.

In a discussion of the theory of the primary chromatic aberration of a lens system Conrady[1] arrives at expressions for the contributions made by a surface of the system to the longitudinal and transverse chromatic aberrations of the final image having the forms

$$\mathrm{LchC'} = \frac{N' u' l' i'}{N'_k u'^2_k} \left(\frac{\delta N'}{N'} - \frac{\delta N}{N}\right) \tag{1}$$

$$\mathrm{TchC'} = \frac{N' u' l' i'_{pr}}{N'_k u'_k} \left(\frac{\delta N'}{N'} - \frac{\delta N}{N}\right) \tag{2}$$

$$= \mathrm{LchC'}\ u'_k \left(\frac{i'_{pr}}{i'}\right)$$

The total aberration of each kind is then given by the sum of the contributions from all the surfaces of the system. To compute these quantities the traces of an axial paraxial ray and a principal paraxial ray are required. It is proposed to show now that there are advantages to be gained by considering the contributions made by the separate components of the system rather than the contributions of the separate surfaces, a component being any singlet lens in the system. The desired expressions may be obtained by algebraic manipulation of equations (1) and (2), but another development is given on account of the insight which it affords into the significance of the final expressions and as illustrating a method which has fruitful applications in optical theory.

1. The Effect of Small Changes in Refractive Index on the Final Intersection Length of a Paraxial Ray.

Consider a ray from an axial object point the path of which is traced paraxially through the lens system. The ray emerges from the last surface of the system with an intersection length l_k' and an inclination angle u_k'. At the i^{th} surface of the system the ray is incident with an intersection length l_i and an inclination angle u_i, and its path after refraction at the surface is specified by the corresponding quantities l_i' and u_i'. Suppose now that a small change of refractive index of amount dn_i is made at this surface, where $n_i = N_i / N_i'$. As a result of this change the ray after refraction will follow a new path which may be specified by the quantities $l_i' + dl_i'$ and $u_i' + du_i'$. From Figure 1 it is easily seen that

$$dl_i' = - l_i' \, du_i' / u_i' \tag{3}$$

Again, since

$$\begin{aligned} u' &= u + i - i' \\ &= u + i(1 - n) \end{aligned}$$

we have

$$\frac{\partial u'}{\partial n} = - i \tag{4}$$

and equation (3) becomes

$$\begin{aligned} dl_i' &= - l_i' \frac{\partial u_i'}{\partial n_i} dn_i / u_i' \\ &= l_i' \, i_i \, dn_i / u_i' \end{aligned} \tag{5}$$

As a result of the change made at surface i the ray will emerge from the last surface of the system along a new path specified by $l_k' + dl_k'$ and $u_k' + du_k'$. The change, dl_k', in the intersection length may be expressed in terms of dl_i' by means of the longitudinal magnification law which gives

$$N_k' \, dl_k' \, u_k'^2 = N_i' \, dl_i' \, u_i'^2$$

and combining this with equation (5) we obtain

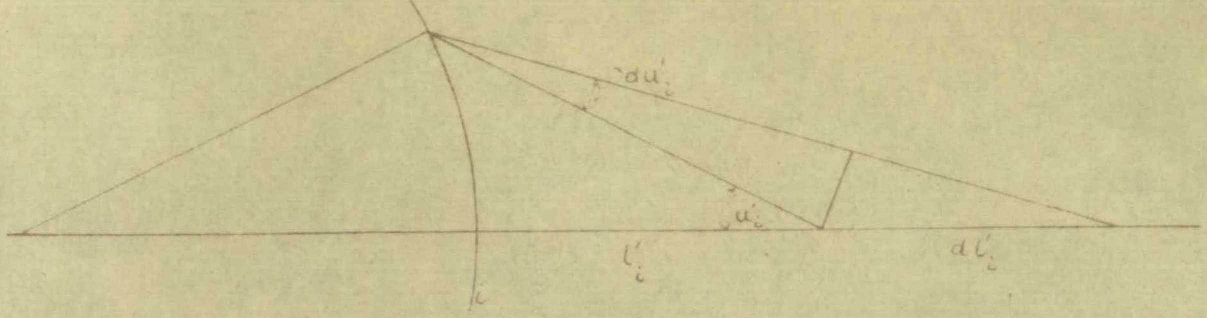
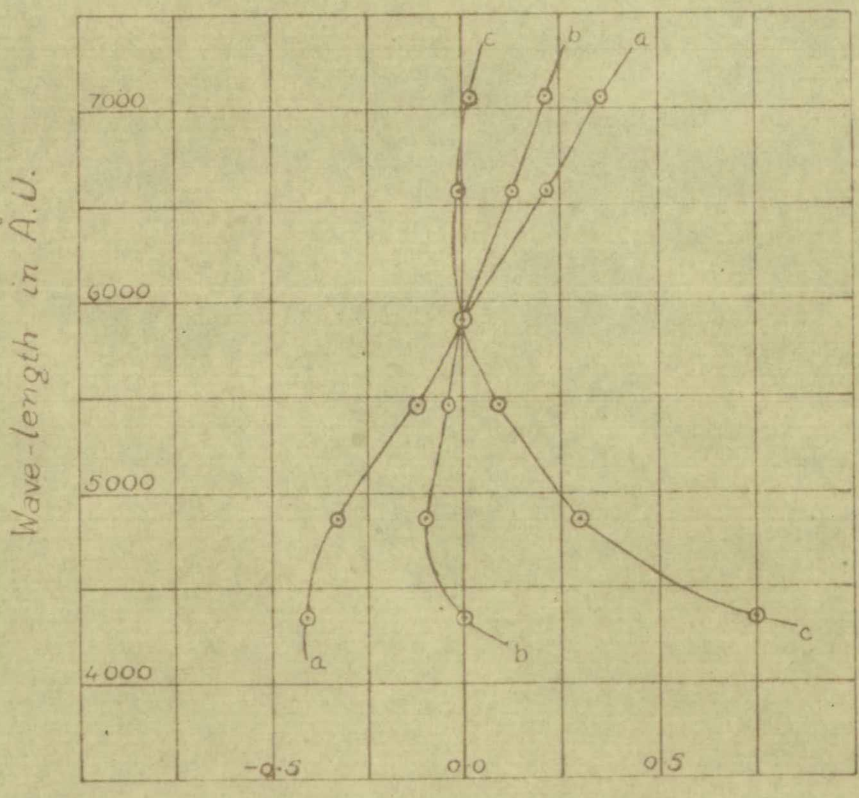


Figure 1.



Paraxial Chromatic Aberration $(l_k' - l_{DR}')$

Figure 2.

$$dl' = \frac{N_1' u_1' l_1' i_1}{N_k' u_k'^2} dn_1$$

whence

$$\frac{\partial l_k'}{\partial n_1} = \frac{N_1' u_1' l_1' i_1}{N_k' u_k'^2}$$

$$= \frac{N_1' y_1 i_1}{N_k' u_k'^2} \qquad (6)$$

Now consider the general component, h , of the system which is any singlet lens. We shall use the subscripts 1 and 2 to denote the first and second surfaces of this component. The refractive index of the component is N_h , while the media on either side of it have refractive indices N_{h-1} and N_{h+1} respectively. Suppose that the refractive index of the component is changed to the value $N_h + dN_h$. This change involves n-changes of different amounts at the two surfaces of the component, and their effect on the path of the ray entering the final image space can be determined with the aid of equation (6). Thus

$$\frac{\partial l_k'}{\partial N_h} = \frac{\partial l_k'}{\partial n_1}\frac{\partial n_1}{\partial N_h} + \frac{\partial l_k'}{\partial n_2}\frac{\partial n_2}{\partial N_h}$$

$$= \frac{\partial l_k'}{\partial n_1}\cdot\frac{\partial}{\partial N_h}\left(\frac{N_{h-1}}{N_h}\right) + \frac{\partial l_k'}{\partial n_2}\frac{\partial}{\partial N_h}\left(\frac{N_h}{N_{h+1}}\right)$$

$$= -\frac{N_1' y_1 i_1}{N_k' u_k'^2}\cdot\frac{N_{h-1}}{N_h^2} + \frac{N_2' y_2 i_2}{N_k' u_k'^2}\cdot\frac{1}{N_{h+1}}$$

$$= \frac{1}{N_k' u_k'^2}\left(-n_1 y_1 i_1 + y_2 i_2\right)$$

$$= \frac{1}{N_k' u_k'^2}\left(-y_1 i_1' + y_2 i_2\right) \qquad (7)$$

In the next section we shall use equation (7) in the derivation of an expression for the primary longitudinal chromatic aberration, but it is of considerable interest in itself as affording a simple expression for the rate of change of the back focal length of the system with the refractive index of the glass of any singlet lens of the system. Its form is easily remembered as the incidence angles are those within the glass of the lens concerned, while the negative sign goes with the dashed angle.

2. The Primary Longitudinal Chromatic Aberration.

Equation (7) may now be used to investigate the paraxial chromatic properties of the lens system if, instead of thinking of glass changes at the various components, we consider the passage through the system of rays of different colours. The traced paraxial ray gives the path through the system for a ray of some intermediate colour $\underline{d}$, which very commonly may be sodium light. To change from this ray to a ray of red light, $\underline{r}$, incident on the system along the same path in the initial medium, only involves a refractive index change from N_d to N_r at each component. Hence we may write

$$\begin{aligned} l'_{rk} &= l'_{dk} + \frac{\partial l'_k}{\partial N_b}(N_r - N_d)_a + \frac{\partial l'_k}{\partial N_b}(N_r - N_d)_b + \dots \\ &= l'_{dk} + \Sigma \frac{\partial l'_k}{\partial N_h}(N_r - N_d)_h \end{aligned} \tag{8}$$

Similarly, for a ray of colour $\underline{v}$ at the other end of the spectrum

$$l'_{vk} = l'_{dk} + \Sigma \frac{\partial l'_k}{\partial N_h}(N_v - N_d)_h \tag{9}$$

If the colours $\underline{r}$ and $\underline{v}$ are those for which the system is to be achromatised, then the longitudinal chromatic aberration is given by

$$\text{lch}' = l'_{rk} - l'_{vk} = \Sigma \frac{\partial l'_k}{\partial N_h}(N_r - N_v)_h \tag{10}$$

This equation is of considerable value as giving the chromatic aberration of a lens system as a sum of terms taken over all the components of the system, each term being the product of a certain differential coefficient with the dispersion of the glass of that component. We shall call these differential coefficients, $\partial l'_k / \partial N_h$, the primary longitudinal <u>chromatic coefficients</u> of the system. It is obvious that each term in the summation in equation (10) represents the contribution of one component to the final chromatic aberration. Hence if the dispersion of the glass of one component is altered the resultant effect on the chromatic aberration is easily determined. Writing $(N_v - N_r)_h = P_h$ we have

$$lch = - \Sigma \frac{\partial l'_k}{\partial N_h} P_h$$

and hence
$$\frac{\partial lch}{\partial P_h} = - \frac{\partial l'_k}{\partial N_h} \tag{11}$$

Thus the chromatic coefficients specify the rate of change of the back focal length of the system with the refractive index of each component, and also give the negative rate of change of the longitudinal chromatic aberration with the dispersion of each component. It should be noticed further that once the chromatic coefficients of the system are calculated for a paraxial ray of some intermediate colour it is possible to derive very quickly the final intersection length of a paraxial ray of any colour from the same object point. Hence we can obtain complete and accurate information of the variation of the chromatic correction throughout the whole spectrum, or, in other words, an accurate analysis of the secondary spectrum of the system.

3. A Numerical Example.

We will consider an example now in order to give an idea of the order of accuracy of the chromatic coefficients and also to show the type of question to which they provide a rapid answer. A paraxial trace was made through a rough design of a photographic objective of the wide angle Ross Xpres type, and the chromatic coefficients of the six components were calculated. In Table I the calculated values of the coefficients and the constants of the glasses are listed.

TABLE I.

Component.	$\partial l_k' / \partial N$	N_D	V
a	- 587.858	1.6151	55.3
b	625.028	1.5491	46.7
c	- 155.558	1.5185	60.4
d	- 114.533	1.5185	60.4
e	459.352	1.5491	46.7
f	- 403.975	1.6151	55.3

Since the same glass is used in components a and f we may combine the chromatic coefficients of these two components, for in the summation of equation (10) each coefficient will be multiplied by the same dispersion. The same holds for components b and e, and for components c and d. Hence we have

$$\partial l_k' / \partial N_{a,f} = -991.833$$

$$\partial l_k' / \partial N_{b,e} = 1087.38$$

$$\partial l_k' / \partial N_{c,d} = -270.091$$

In Table II the details of the computation of the contributions of the components to the chromatic aberration, and the total chromatic

TABLE II.

Detailed computation of the contributions to the primary longitudinal chromatic aberration made by the three pairs of components of a wide angle Ross Xpres type photographic objective for various spectral ranges.

Comp.	Chr. Coeff.	$N_b - N_D$	Contrbn.	$N_C - N_D$	Contrbn.	$N_e - N_D$	Contrbn.	$N_F - N_D$	Contrbn.	$N_{G'} - N_D$	Contrbn.
a,f	-991.833	-0.00517	5.1278	-0.00328	3.2532	0.00274	-2.7176	0.00785	-7.7859	0.01422	-14.1039
b,e	1087.38	-0.00539	-5.8610	-0.00342	-3.7188	0.00291	3.1643	0.00835	9.0796	0.01529	16.6260
c,d	-270.091	-0.00406	1.0966	-0.00256	0.6914	0.00211	-0.5699	0.00603	-1.6286	0.01089	- 2.9413
		lch'_{bD} =	0.3634	lch'_{CD} =	0.2258	lch'_{eD} =	-0.1232	lch'_{FD} =	-0.3349	$lch'_{G'D}$ =	-0.4192

aberration, of the system for various spectral ranges are set out.

TABLE II.

(See accompanying sheet.)

In the first place consider the general accuracy of the results obtained by the use of the coefficients. From the trace we have l'_k = 106.525 , and considering the spectral range C - F , for example, equations (8) and (9) together with some results from Table II give at once

$$l'_{Ck} = 106.525 + 0.226 = 106.751$$
$$l'_{Fk} = 106.525 - 0.335 = 106.190$$
$$lch'_{CF} = 0.561$$

A full trace of the system for paraxial rays of colours C and F gives

$$l'_{Ck} = 106.748$$
$$l'_{Fk} = 106.181$$
$$lch'_{CF} = 0.567$$

from which it will be seen that the chromatic coefficients give predictions of the final intersection lengths of rays of other colours, and hence of the chromatic aberration, to a very valuable order of accuracy.

In the results of Table II we have essentially an analysis of the secondary spectrum of the system. By plotting the values of $l'_k - l'_{Dk}$ against the wave-length of the spectral line concerned we summarise the chromatic correction state of the system by the line <u>aa</u> , which describes a very undesirable state of affairs. The chromatic coefficients may be used next to indicate the way in which improvement may be obtained. A study of Table II shows that the contributions from components a and f are too great, or alternatively, the contributions from components b and e are too small. Improvement may be obtained by replacing the glass of a and f by another DBC of higher V-number, or alternatively, by replacing the flint glass of components b and e by one of lower V-number. Calculating the effect of the first possibility by changing to a DBC of the type $N_D = 1.6132$, $V = 58.8$, we show in Table III the computation of the new secondary spectrum values.

TABLE III.

Computation of the change of the secondary spectrum, $l'_K - l'_{DK}$, due to change of glass in two components of the system.

Partial Dispersion, $P = N - N_D$	dP	$d(l'_K - l'_{DK})$	New $(l'_K - l'_{DK})$
− 0.00502	0.00015	− 0.1498	0.2146
− 0.00518	0.00010	− 0.0992	0.1266
0.00266	− 0.00008	0.0795	− 0.0459
0.00762	− 0.00025	0.2231	− 0.1068
0.01830	− 0.00042	0.4166	− 0.0025

The resulting chromatic correction state is exhibited in the curve bb in Figure 2, which shows that the foci for D and G' have been brought together giving achromatism of the usual photovisual type. A change to a DBC glass with a still higher V-number will produce another type of achromatism. Thus if we use the glass $N_D = 1.6134$ $V = 59.3$, we get a correction state summarised by the curve cc in Figure 2, a type of achromatism which would be suitable over the spectral range from 7000 A. U. to 5100 A.U., corresponding to the use of a panchromatic emulsion with a K 2 or blue minus filter. Smaller changes could be made by suitable choice of glass for components c and d, for which the chromatic coefficients are smaller, but the available range of glasses is somewhat more restricted. The example provides, however, sufficient evidence of the power of the chromatic coefficients in analysing the chromatic correction and directing the choice of glasses for achromatism of a desired type.

4. The Primary Transverse Chromatic Aberration.

Suppose that a principal paraxial ray for some given obliquity has been traced through the lens system. After refraction at surface i this ray intersects the paraxial image plane behind this surface (i.e. a plane at right angles to the principal axis distant l'_i from the pole of the surface) at a point of which the height above the axis is h'_i. If a change of refractive index of amount dn_i were made at this surface the principal paraxial ray would undergo a change of direction du'_{ipr} after refraction, and would intersect the paraxial image plane at a height $h'_i + dh'_i$ such that

$$\begin{aligned} dh'_i &= - l'_i \, du'_{ipr} \\ &= - l' \frac{\partial u'_{ipr}}{\partial n_i} dn_i \\ &= l'_i \, i_{ipr} \, dn_i \end{aligned} \tag{12}$$

If the traced ray intersects the final paraxial image plane at a height h'_k above the principal axis of the system, the change dn_i will produce a shift of this intersection point through a distance dh'_k which is given by the Lagrange magnification law as

$$dh'_k = N'_i u'_i dh'_i / N'_k u'_k$$

which on combination with equation (12) leads to the relation

$$\frac{\partial h'_k}{\partial n_i} = \frac{N'_i y_i i_{ipr}}{N'_k u'_k} \qquad (13)$$

If we wish to relate this derivative to the corresponding longitudinal one, we have

$$\frac{\partial h'_k}{\partial n_i} = u'_k \left(\frac{i_{ipr}}{i_i} \right) \frac{\partial l'_k}{\partial n_i} \qquad (14)$$

If the refractive index of the general component, h , is changed by an amount dN_h , then by analogy with the derivation of equation (7) we have

$$\frac{\partial h'_k}{\partial N_h} = \frac{1}{N'_k u'_k} (- y_1 i'_{pr1} + y_2 i_{pr2})_h \qquad (15)$$

the subscripts 1 and 2 referring to the first and second surfaces of component h . Equation (15) provides the most convenient form for the computation, requiring less operations than the alternative use of equation (14). In precise analogy to equation (10) we may write down an expression for the transverse chromatic aberration as

$$tch' = h'_{rk} - h'_{vk} = \Sigma \frac{\partial h'_k}{\partial N_h} (N_r - N_v)_h \qquad (16)$$

It should be noticed that the intersection height, h'_{rk} , for example, is not the intersection height of the principal paraxial ray for red light in the paraxial image plane for red light, but it is the height at which a red ray entering the system along the traced path of the principal ray for the intermediate colour d intersects the plane at right angles to the principal axis through the paraxial focus for the colour d . This does not affect the validity of our expression for tch' , for equation (17) still measures the separation, in a fixed plane through the mean paraxial focus, of the intersection points of two principal paraxial rays of the colours r and v . Each term in the summation of equation (16) gives the contribution to the transverse chromatic aberration of the final image made by one component of the system. The quantities $\partial h'_k / \partial N_h$ are called the primary transverse chromatic coefficients of the system. Their computation requires very little work, only four operations for each component, which is considerably less than the surface contribution analysis given by Conrady. Once these coefficients have been computed the variation of the correction state of the system in respect of primary transverse chromatic aberration may be examined quickly throughout the spectrum. A study of the values of the coefficients allows the designer to vary his selection of glasses to give the best best state of correction for this aberration in exactly the same way as was exemplified in the case of the longitudinal aberration. If the surface contributions are required in any particular case they are easily obtained as in both equations (7) and (15) the two terms entering into the expressions each have reference to a single surface. The extraction from the computation of the terms referring to each surface, one in number for a glass-air interface and two for a contact surface, permits the calculation of the surface contributions. The transverse chromatic coefficients are calculated for the particular obliquity at which the principal paraxial ray has been traced, while those for other obliquities vary in direct proportion to the obliquity.

References.

1. A.E.Conrady, Applied Optics and Optical Design, Oxford University Press, London. 1929. pp. 312 - 3.

THE PARAXIAL DIFFERENTIAL TRANSFER COEFFICIENTS OF A LENS SYSTEM.

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Introduction.

Various attempts have been made to solve the important problem of the transfer coefficients of an optical system which specify the effects in the final image space of changes made in the curvatures, the thicknesses, and the refractive indices of the components of the system. In America, Hertzberger[1] has discussed the use of Gaussian brackets for the evaluation of certain of these coefficients; Stephens[2] has a solution of the problem of the dependence of the focal length of a system on the constructional parameters, of which the writer has only seen an abstract; and Stempel[3] has published an account of differential changes at a single surface in connection with the design of the Huyghens eyepiece, but has not used a transfer theorem.
In Australia, an earlier account of the differential coefficients for small changes at a single surface was given by McAulay[4] for rays of any aperture and obliquity. Subsequently McAulay[5] also proposed certain approximate transfer methods for rapid use in the design of lens systems. From the basis of McAulay's single surface work the writer[6] developed a system of exact differential transfer coefficients

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1. M. Hertzberger, J. Opt. Soc. Am. 33, 651, (1943)
2. R. E. Stephens, J. Opt. Soc. Am. 33, 684 (A), (1943)
3. W. M. Stempel, J. Opt. Soc. Am. 33, 278, (1943)
4. A. L. McAulay, Secret Papers, Optical Munitions Panel, Aust. Nov. 1942.
5. A. L. McAulay, Secret Papers, Opt. Mun. Panel, Aust. July, 1943.
6. F. D. Cruickshank, Secret Papers, Opt. Mun. Panel, Nov. 1942.

for meridional rays of any aperture and obliquity through any centered system of spherical refracting surfaces. These coefficients predict accurately the effect on the ray paths in the final image space of differential changes in the parameters of the system. In a later paper[7] the power and usefulness of the general methods were extended considerably.

In the present paper it is proposed to give an account of the general differential transfer coefficients of a lens system for paraxial rays only. These coefficients provide a simple means of predicting the effect on the paths of paraxial rays in the final image space of any small change which may be made in the constructional parameters of the system, and hence on related quantities such as the focal length, back focus, and magnification. The values of the coefficients are obtained from an orderly computation based on the trace of a single paraxial ray through the system. In general, the notation employed follows that of Conrady[8].

The Fundamental Transfer Coefficients.

We consider any centered lens system comprising k spherical refracting surfaces. Suppose that a paraxial ray having an inclination angle u_1 enters the first surface of the system at a height y_1 above the axis, and, after passing through the system, emerges from the last surface at a height y_k above the axis in a direction making an angle u'_k with the axis. On account of the linear character of the paraxial ray-tracing equations we may write

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7. F. D. Cruickshank, Secret Papers, Opt. Mun. Panel, Aust. May 1943
8. A. E. Conrady, Applied Optics and Optical Design , Oxford Univ. Press, 1929
<u>Note</u>. The work described in references 4 - 7 will appear in the Proc. Phys. Soc. when security permits.

$$y_k = \frac{\partial y_k}{\partial y_1} y_1 + \frac{\partial y_k}{\partial u_1} u_1 \tag{1}$$

$$u'_k = \frac{\partial u'_k}{\partial y_1} y_1 + \frac{\partial u'_k}{\partial u_1} u_1 \tag{2}$$

and it may be easily shown that these derivatives are subject to the relation

$$\frac{\partial y_k}{\partial y_1} \frac{\partial u'_k}{\partial u_1} - \frac{\partial y_k}{\partial u_1} \frac{\partial u'_k}{\partial y_1} = N_1 / N'_k \tag{3}$$

Compare, for example, equations (1) and (2) of reference 1. If we know the values of the four constants of equations (1) and (2), then given the data y_1, u_1 for any ray incident on the first surface, we may calculate the values of y_k, u'_k of the emergent ray, and vice versa. The consideration of particular incident and emergent rays enables other quantities such as the focal length and the positions of the first and second focal points to be deduced, as Hertzberger[1] has shown. Thus the equivalent focal length of the system is

$$f' = 1 / \frac{\partial u'_k}{\partial y_1} \tag{4}$$

the back focal distance is given by

$$l'_F = \frac{\partial y_k}{\partial y_1} / \frac{\partial u'_k}{\partial y_1} = f' \frac{\partial y_k}{\partial y_1} \tag{5}$$

and the front focal distance by

$$l_F = - \frac{\partial u'_k}{\partial u_1} / \frac{\partial u'_k}{\partial y_1} = - f' \frac{\partial u'_k}{\partial u_1} \tag{6}$$

Relationships of the kind just discussed are not peculiar to the first surface of a lens system, so that for any surface, i, of a system we may write

$$y_k = \frac{\partial y_k}{\partial y_i} y_i + \frac{\partial y_k}{\partial u_i} u_i \qquad (7)$$

$$u'_k = \frac{\partial u'_k}{\partial y_i} y_i + \frac{\partial u'_k}{\partial u_i} u_i \qquad (8)$$

and

$$\frac{\partial y_k}{\partial y_i} \frac{\partial u'_k}{\partial u_i} - \frac{\partial y_k}{\partial u_i} \frac{\partial u'_k}{\partial y_i} = N_i / N'_k \qquad (9)$$

We shall refer to these four derivatives for any surface i of the system as the fundamental paraxial transfer coefficients of that surface. Now the effect on any paraxial ray path of a small change in the optical construction of the system may be described in terms of changes of incidence points and inclination angles at the surface at which the change is made or at the succeeding surface. Hence, if the values of the fundamental transfer coefficients are known for each surface of the system we can easily deduce the effect on the path of paraxial rays in the final image space of any small changes made within the system. It is necessary therefore to investigate the relations which make it possible to calculate the values of the fundamental transfer coefficients at each surface, and then to develop from these the transfer coefficients for constructional changes within the system.

The Connecting Equations for the Fundamental Transfer Coefficients.

Suppose that the path of a paraxial ray has been traced through the system, the ray emerging from the last surface at a height y_k above the axis with an inclination angle u'_k . If some change is made at any surface i of the system the ray after refraction at this surface will follow a path through the remainder of the system which is different

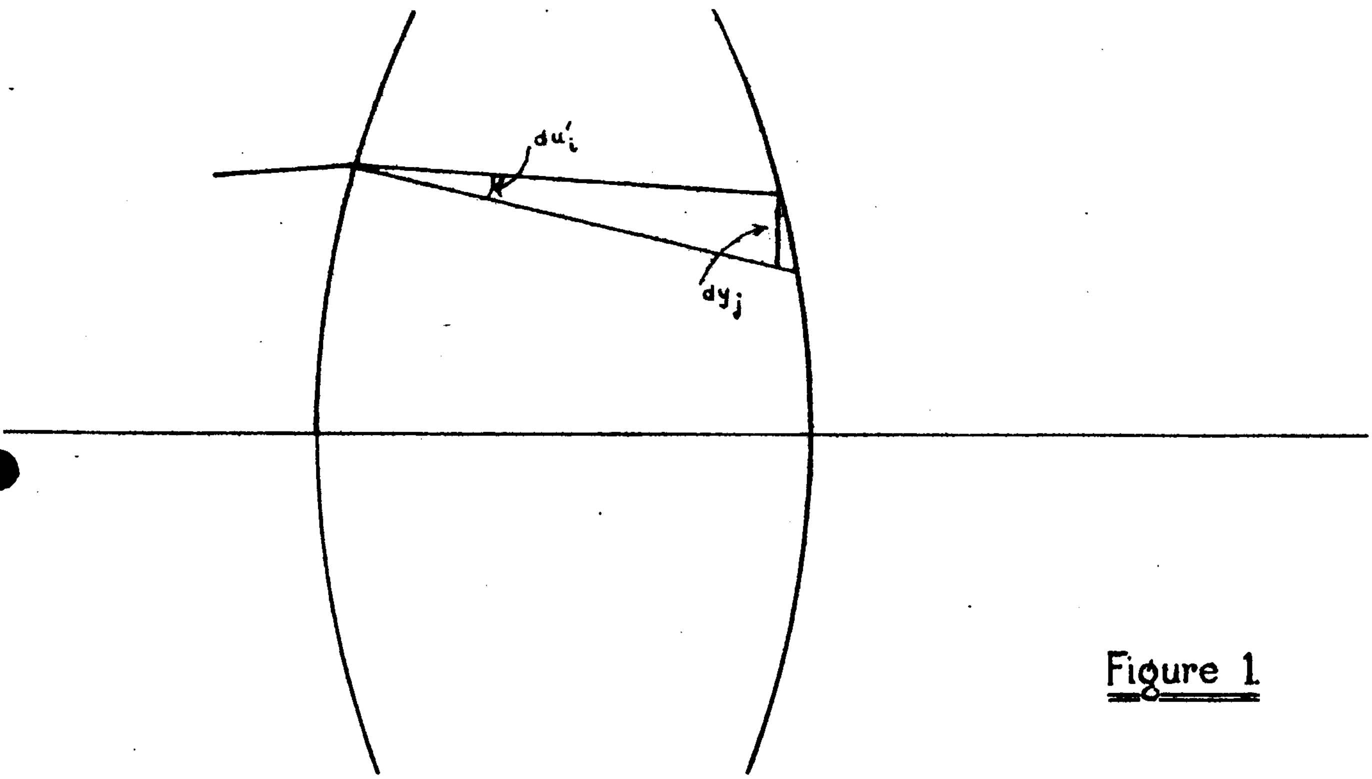

Figure 1

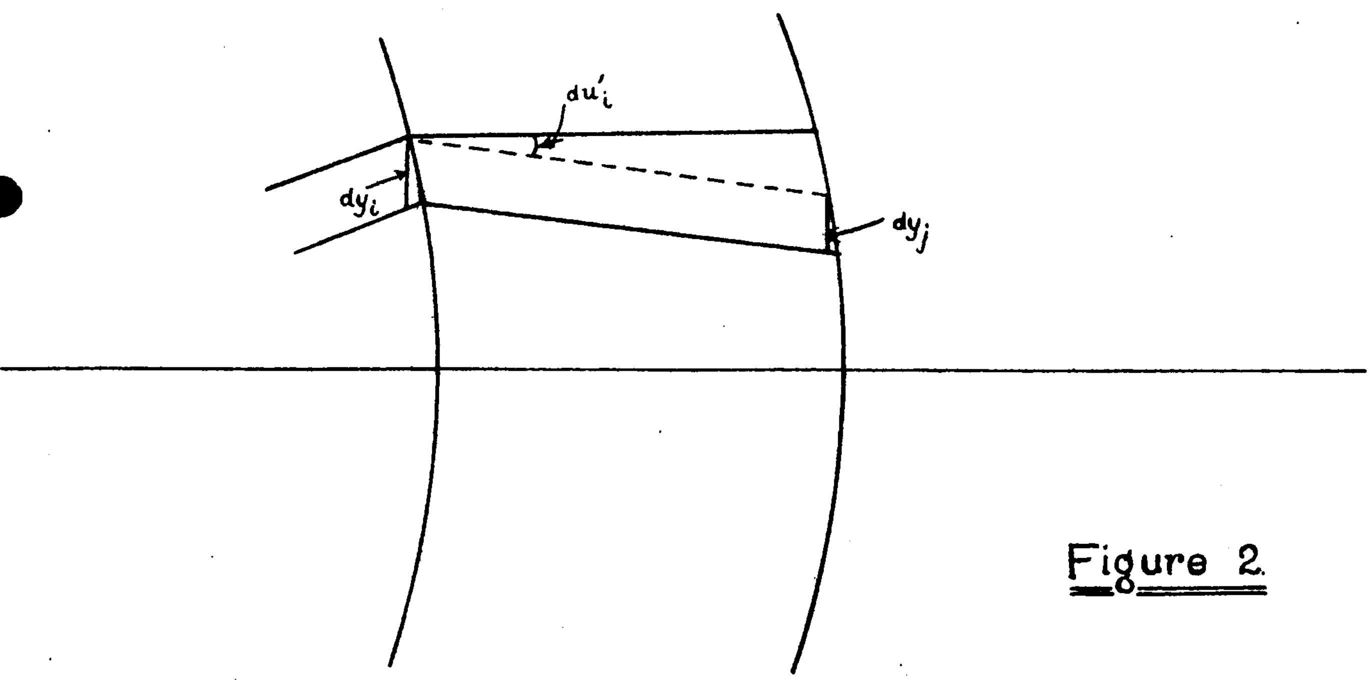

Figure 2

from the original traced path, and the ray will now leave the last surface of the system at a height $y_k + dy_k$ above the axis with an inclination angle $u'_k + du'_k$. The problem is to connect the quantities dy_k and du'_k with the small change from which they result.

For simplicity let us denote any two consecutive surfaces of the system by the subscripts $\underline{i}$ and $\underline{j}$, the axial separation between these two surfaces being $d'_i = -d_j$. Suppose now that the change made at surface $\underline{i}$ consists, in the first instance, of a change in the direction of the refracted ray through an angle du'_i, the incidence point remaining unchanged. Figure 1 shows that the ray under consideration will now meet surface $\underline{j}$ with an inclination angle differing from that of the original traced path by an amount $du_j = du'_i$, and at an incidence point displaced from that of the traced path by an amount $dy_j = -d'_i\, du'_i = d_j\, du'_i$. Expressing the resulting changes dy_k, du'_k, of the emergent ray in terms of these equivalent changes in the ray path at surfaces $\underline{i}$ and $\underline{j}$, we have

$$\frac{\partial y_k}{\partial u'_i}\, du'_i = dy_k = \frac{\partial y_k}{\partial y_j}\, dy_j + \frac{\partial y_k}{\partial u_j}\, du_j$$

$$= \frac{\partial y_k}{\partial y_j}\, d_j\, du'_i + \frac{\partial y_k}{\partial u_j}\, du'_i$$

whence
$$\frac{\partial y_k}{\partial u'_i} = \frac{\partial y_k}{\partial y_j}\, d_j + \frac{\partial y_k}{\partial u_j} \qquad (10)$$

and similarly
$$\frac{\partial u'_k}{\partial u'_i} = \frac{\partial u'_k}{\partial y_j}\, d_j + \frac{\partial u'_k}{\partial u_j} \qquad (11)$$

Anticipating the result of equation (22) we have also

$$\frac{\partial y_k}{\partial u_i} = \frac{\partial y_k}{\partial u'_i}\,\frac{\partial u'_i}{\partial u_i} = \frac{\partial y_k}{\partial u'_i}\, n_i \qquad (12)$$

and $$\frac{\partial u_k'}{\partial u_i} = \frac{\partial u_k'}{\partial u_i'} n_i \qquad (13)$$

In the second instance suppose that the change made at surface $\underline{i}$ consists of a displacement of the incidence point through dy_i , the inclination angle, u_i , remaining unchanged. The ray under consideration will undergo a direction change on refraction of amount $du_i' = (\partial u_i' / \partial y_i)\, dy_i$. It is easily seen from Figure 2 that the ray will arrive at surface $\underline{j}$ with an inclination angle differing from that of the traced path by an amount $du_j = du_i'$, and its incidence point will be displaced from that of the traced path by an amount $dy_j = dy_i + d_j\, du_i'$. Expressing the changes dy_k , du_k' , produced in the emergent ray in terms of these equivalent changes at the two surfaces, we obtain

$$\begin{aligned} \frac{\partial y_k}{\partial y_i} dy_i &= \frac{\partial y_k}{\partial y_j} dy_j + \frac{\partial y_k}{\partial u_j} du_j \\ &= \frac{\partial y_k}{\partial y_j} (d_j\, du_j + dy_i) + \frac{\partial y_k}{\partial u_j} du_j \\ &= \left(\frac{\partial y_k}{\partial y_j} d_j + \frac{\partial y_k}{\partial u_j} \right) du_j + \frac{\partial y_k}{\partial y_j} dy_i \\ &= \frac{\partial y_k}{\partial u_i'} \frac{\partial u_i'}{\partial y_i} dy_i + \frac{\partial y_k}{\partial y_j} dy_i \end{aligned}$$

whence $$\frac{\partial y_k}{\partial y_i} = \frac{\partial y_k}{\partial u_i'} \frac{\partial u_i'}{\partial y_i} + \frac{\partial y_k}{\partial y_j} \qquad (14)$$

and similarly

$$\frac{\partial u_k'}{\partial y_i} = \frac{\partial u_k'}{\partial u_i'} \frac{\partial u_i'}{\partial y_i} + \frac{\partial u_k'}{\partial y_j} \qquad (15)$$

If the fundamental transfer coefficients are known for surface $\underline{j}$ they may be computed also for surface i by means of equations (10) to (15). At the last surface of the system the fundamental transfer coefficients obviously have simple values for they reduce to derivatives associated with refractions at a single surface. Thus

$$\frac{\partial y_k}{\partial u'_k} = 0 \qquad\qquad \frac{\partial y_k}{\partial u_k} = 0$$

$$\frac{\partial u'_k}{\partial u'_k} = 1 \qquad\qquad \frac{\partial u'_k}{\partial u_k} = n_k$$

$$\frac{\partial y_k}{\partial y_k} = 1$$

$$\frac{\partial u'_k}{\partial y_k} = (1 - n_k) / r_k$$

the last relation being established in the next section. These quantities being known for surface k the fundamental transfer coefficients for surface $(k - 1)$ may be computed, and so the computation may be continued until the coefficients for each surface have been calculated.

Differential Changes at a Single Spherical Surface.

The standard paraxial ray-tracing equations give

$$(l - r)\, u = r\, i \tag{16}$$

$$N'\, i' = N\, i \tag{17}$$

$$u' + i' = u + i \tag{18}$$

Since $(u + i)$ is constant for a given incidence point on the surface we have immediately

$$\frac{\partial i}{\partial u} = -1 \tag{19}$$

Writing equation (16) in the form

$$yc - u = i$$

we obtain

$$\frac{\partial i}{\partial y} = c \tag{20}$$

and

$$\frac{\partial i}{\partial c} = y \tag{21}$$

Again, combining (17) and (18) and putting $N/N' = n$, we have

$$u' = u + (1 - n)\, i$$

and by partial differentiation of this relation, together with equations (19) to (21), we obtain

$$\frac{\partial u'}{\partial u} = n \tag{22}$$

$$\frac{\partial u'}{\partial y} = (1 - n)\, c \tag{23}$$

$$\frac{\partial u'}{\partial c} = (1 - n)\, y \tag{24}$$

$$\frac{\partial u'}{\partial n} = -\, i \tag{25}$$

<u>The Transfer coefficients for the Constructional Parameters.</u>

We consider next the effect on the path of a paraxial ray of small changes in the curvatures, thicknesses, and refractive indices of the components of the system. To describe the resultant effect of constructional changes made at surface $\underline{i}$ of the system it is necessary to compute the derivatives of y_k and u'_k with respect to the constructional parameters. First, suppose that a small change in curvature is made at surface $\underline{i}$. The effect on the path of a traced paraxial ray is that the ray after refraction will leave the surface with an inclination angle different from that of the traced path, and hence the effect of the curvature change may be described in terms of an equivalent change in u'_i. Hence we may write

$$\frac{\partial u'_k}{\partial c_i} = \frac{\partial u'_k}{\partial u'_i}\,\frac{\partial u'_i}{\partial c_i} \tag{26}$$

and

$$\frac{\partial y_k}{\partial c_i} = \frac{\partial y_k}{\partial u'_i}\,\frac{\partial u'_i}{\partial c_i} \tag{27}$$

In a similar manner, if a refractive index change is made at the surface $\underline{i}$ of the system the immediate effect on the ray path is a change in the inclination angle u'_i. Hence we have

$$\frac{\partial u'_k}{\partial n_i} = \frac{\partial u'_k}{\partial u'_i}\,\frac{\partial u'_i}{\partial n_i} \tag{28}$$

$$\frac{\partial y_k}{\partial n_i} = \frac{\partial y_k}{\partial u'_i}\,\frac{\partial u'_i}{\partial n_i} \tag{29}$$

In practice, of course, the common change is a glass change in which the refractive index of some singlet lens of the system is altered. This involves a refractive index change at two surfaces. To investigate this let us denote the general singlet lens component by the subscript $\underline{h}$, and its front and rear surfaces by the subscripts 1 and 2. The refractive index of the component will be N_h, and the media on either side of it will be N_{h-1} and N_{h+1} respectively. Then

$$\frac{\partial y_k}{\partial N_h} = \frac{\partial y_k}{\partial n_1}\,\frac{\partial n_1}{\partial N_h} + \frac{\partial y_k}{\partial n_2}\,\frac{\partial n_2}{\partial N_h}$$

$$= \frac{\partial y_k}{\partial n_1}\,\frac{\partial}{\partial N_h}\left(\frac{N_{h-1}}{N_h}\right) + \frac{\partial y_k}{\partial n_2}\,\frac{\partial}{\partial N_h}\left(\frac{N_h}{N_{h+1}}\right)$$

$$= -\,\frac{\partial y_k}{\partial n_1}\cdot\frac{N_{h-1}}{N_h^2} + \frac{\partial y_k}{\partial n_2}\cdot\frac{1}{N_{h+1}} \tag{30}$$

and similarly,

$$\frac{\partial u'_k}{\partial N_h} = -\,\frac{\partial u'_k}{\partial n_1}\cdot\frac{N_{h-1}}{N_h} + \frac{\partial u'_k}{\partial n_2}\cdot\frac{1}{N_{h+1}} \tag{31}$$

The axial separation between the surfaces $(i - 1)$ and $\underline{i}$ is specified by $d_i = - d'_{i-1}$, d_i therefore being an essentially negative quantity. In order to change the separation by an amount δd_i we move the surface $\underline{i}$ and all surfaces behind it through the distance δd_i. All paraxial rays originally incident on the surface $\underline{i}$ will now meet the surface at new incidence points as a result of the change. The displacement of the incidence point of any ray is given by $dy_i = u_i \, \delta d_i$. Hence the effect of an axial separation change may be described by the derivatives

$$\frac{\partial y_k}{\partial d_i} = \frac{\partial y_k}{\partial y_i} \frac{\partial y_i}{\partial d_i} = \frac{\partial y_k}{\partial y_i} u_i \tag{82}$$

$$\frac{\partial u'_k}{\partial d_i} = \frac{\partial u'_k}{\partial y_i} u_i \tag{83}$$

Thus the transfer coefficients defined in this section are sufficient for the description of the effects of the changes in the constructional parameters in terms of incidence point changes and direction changes of the emergent ray as it leaves the last surface of the system.

The Transfer Coefficients for the Positions of the Paraxial Image Plane, the Second Principal Plane, and for the Focal Length.

Suppose that a paraxial ray from an axial object point has been traced through the system, the emergent ray being specified by y_k and u'_k, and its final intersection length being l'_k. Often we wish to know how the position of the paraxial image plane, distant l'_k from the pole of the last surface, is affected by a constructional change made within the system. To investigate this consider first the effect of the small change, du'_i, in the direction of the ray after refraction at surface $\underline{i}$. As a result of the change the incidence point and inclination angle of the emergent ray will now be given by $y_k + dy_k$ and $u'_k + du'_k$, where

$$dy_k = \frac{\partial y_k}{\partial u'_i} du'_i \quad \text{and} \quad du'_k = \frac{\partial u'_k}{\partial u'_i} du'_i$$

and the new value of l'_k will be given by $(y_k + dy_k) / (u'_k + du'_k)$. For small changes, however, it is more convenient to intpoduce a differential transfer coefficient. Thus since $l'_k = y_k / u'_k$,

$$\frac{\partial l'_k}{\partial u'_i} = \left(\frac{\partial y_k}{\partial u'_i} - l'_k \frac{\partial u'_k}{\partial u'_i} \right) / u'_k \tag{34}$$

$$= C(u'_i) / u'_k \tag{35}$$

where we have introduced the symbol $C(u'_i)$ as a convenient abbreviation defined by

$$C(u'_i) = \frac{\partial y_k}{\partial u'_i} - l'_k \frac{\partial u'_k}{\partial u'_i} \tag{36}$$

Considering a change of incidence point at any surface we obtain similarly

$$\frac{\partial l'_k}{\partial y_i} = C(y_i) / u'_k \tag{37}$$

For changes in the constructional parameters it follows at once that

$$\frac{\partial l'_k}{\partial c_i} = \frac{\partial l'_k}{\partial u'_i} \frac{\partial u'_i}{\partial c_i}$$

$$= C(u') \frac{\partial u'_i}{\partial c_i} / u'_k$$

$$= C(c_i) / u'_k \tag{38}$$

and similarly

$$\frac{\partial l'_k}{\partial n_i} = c(u'_i)\frac{\partial u'_i}{\partial n_i} \;/\; u'_k$$

$$= c(n_i) \;/\; u'_k \tag{39}$$

$$\frac{\partial l'_k}{\partial N_h} = -\frac{\partial l'_k}{\partial n_1}\frac{N_{h-1}}{N_h^2} + \frac{\partial l'_k}{\partial n_2}\frac{1}{N_{h+1}} \tag{40}$$

$$\frac{\partial l'_k}{\partial d_i} = c(y_i)\frac{\partial y_i}{\partial d_i} \;/\; u'_k$$

$$= c(d_i) \;/\; u'_k \tag{41}$$

If the ray which has been traced through the system entered the system parallel to the principal axis, then the final intersection length, which we denote in this case by l'_f , locates the second focal plane of the system, and the preceding equations (38) to (41) specify the rate of change of the position of the focal plane with the constructional parameters which are frequently required. Further, the focal length of the system is given by the results of this trace as $f' = y_1 / u'_k$, and hence

$$\frac{\partial f'}{\partial c_i} = -\,(\,y_1 \;/\; u'^2_k)\;\frac{\partial u'_k}{\partial c_i}$$

$$= -\,(f'/u'_k)\;\frac{\partial u'_k}{\partial c_i} \tag{42}$$

Similarly we have

$$\frac{\partial f'}{\partial d_i} = -\,(f'/u'_k)\;\frac{\partial u'_k}{\partial d_i} \tag{43}$$

$$\frac{\partial f'}{\partial n_i} = -\,(f'/u'_k)\;\frac{\partial u'_k}{\partial n_i} \tag{44}$$

and finally

$$\frac{\partial f'}{\partial N_h} = - \frac{N_{h-1}}{N_h^2} \frac{\partial f'}{\partial n_1} + \frac{1}{N_{h+1}} \frac{\partial f'}{\partial n_2} \tag{45}$$

Equations (42) to (45) have a very important application when a system has to be manufactured in which a fairly strict tolerance is imposed on the focal length, for they specify completely the effect of the v variation of all the constructional parameters of the system on its focal length and thus permit tolerances to be established for each parameter to ensure production within the specified focal length tolerance.

If we denote the distance of the second principal plane from the pole of the last surface by l'_{pp} , then,

$$l'_{pp} = l'_f - f'$$

and

$$\frac{\partial l'_{pp}}{\partial c_i} = \frac{\partial l'_f}{\partial c_i} - \frac{\partial f'}{\partial c_i} \tag{46}$$

with corresponding expressions for derivatives with respect to the other parameters of the system. Problems arise in design in which this equation is very useful. Thus suppose that a projection lens has been designed in which the back focal length is shorter than desirable, bringing the back component of the system too close to the film gate. The required remedy is to increase the value of l'_{pp} relative to f' . The transfer coefficients of the type defined in equation (46) give a complete analysis of the tendencies of the system in this respect, and indicate which parameters may be varied with advantage to achieve the desired result.

Transfer Coefficients for the Linear Magnification.

We consider next the development of transfer coefficients which specify the effect of small alterations in the optical construction of a lens system on the magnification of an image produced by it. Thus, suppose that a paraxial ray from an axial object point has been traced through the system. The linear magnification is given by the Lagrange - Helmholtz law as

$$m' = N_1 u_1 / N_k' u_k'$$

For an alteration in curvature at surface i of the system we have

$$\frac{\partial m'}{\partial c_i} = - \frac{N_1 u_1}{N_k'} \frac{1}{u_k'^2} \frac{\partial u_k'}{\partial c_i}$$

$$= - (m'/u_k') \frac{\partial u_k'}{\partial c_i} \qquad (47)$$

with corresponding expressions for the variation of the magnification with respect to the other parameters of the system.

The Computation.

As a final matter we consider the computation involved in the use of the relations developed in the preceding sections. There is much that does not call for comment, but an example of the computation of the fundamental transfer coefficients, which forms the central core of the whole method, will afford a guide as to the total labour involved in the practical application of the method.

In the accompanying Table the complete computation of the fundamental transfer coefficients is given for a projection lens consisting of two doublets. In the notation of the first column the subscript + is used as an abbreviation for $(i + 1)$. The computing sheet is prepared

Typical Computation of Fundamental Transfer Coefficients.

Surface	1	2	3	4	5	6
d_+	-10.72	-3.55	-117.53	-14.72	-1.84	
$(\partial u'_k / \partial y_+) d_+$	-.07982	-.02713	-.87391	.03720	.00129	
$+ \ \partial u'_k / \partial u_+$	.20507	.25007	1.02548	1.51068	1.6504	
$\partial u'_k / \partial u'$	.12525	.22294	.15152	1.5479	1.6517	1.000
$(\partial y_k / \partial y_+) d_+$	- 9.74	- 2.62	-105.34	-14.750	-1.840	
$+ \ \partial y_k / \partial u_+$	-178.85	-191.82	- 10.89	- 1.683	0.000	
$\partial y_k / \partial u'$	-188.59	-194.44	-116.23	-16.433	-1.840	0.000
$\partial u' / \partial y$	.003333	-.000879	.001362	.006436	-.001107	-.000698
$(\partial u'_k/\partial u')(\partial u'/\partial y)$	.000418	-.000196	.000206	.009963	-.001829	-.000698
$+ \ \partial u'_k / \partial y_+$	.007446	.007642	.007436	-.002527	-.000698	
$\partial u'_k / \partial y$	.007864	.007446	.007642	.007436	-.002527	-.000698
$(\partial y_k/\partial u')(\partial u'/\partial y)$	-.62862	.17098	-.15831	-.10577	.00204	
$+ \ \partial y_k / \partial y_+$	.90894	.73796	.89627	1.00204	1.00	
$\partial y_k / \partial y$	.28032	.90894	.73796	.89627	1.00204	1.000
n	.65872	.91984	1.6504	.66247	.91463	1.6504
$\partial u'_k / \partial u$	.08250	.20507	.25007	1.0254	1.5107	1.6504
$\partial y_k / \partial u$	-124.23	-178.85	-191.82	-10.886	-1.6829	0.000

by entering in lines one, eight, and fifteen the values of d_+ , $\partial u' / \partial y$, and $\underline{n}$ from the ray trace or computation of the single surface coefficients. The transfer coefficients for the last surface are then written in, these having the simple values noted at the end of section 2 . For the computation of the fifth surface the value of d_+ is multiplied by each of the quantities $\partial u'_k / \partial y_+$ and $\partial y_k / \partial y_+$, and the products entered in lines two and five. Two additions complete the computation of $\partial u'_k / \partial u'$ and $\partial y_k / \partial u'$. In the next stage $\partial u' / \partial y$ is multiplied in turn by each of the quantities

$\partial u_k' / \partial u'$ and $\partial y_k / \partial u'$ and the products entered in lines nine and twelve. Two additions then complete the computation of the coefficients $\partial u_k' / \partial y$ and $\partial y_k / \partial y$. Finally $\underline{n}$ is multiplied in turn by $\partial u_k' / \partial u'$ and $\partial y_k / \partial u'$ and the products entered in sixteen and seventeen, thus completing the whole computation for that surface. In routine computation by machine the entries in lines three, six, ten, and thirteen are, of course, omitted. They were included in the above computation to make the example easier to follow. If a check is desired, and this is highly desirable because the fundamental coefficients form the basis for the remainder of the computations, equation (9) provides the necessary relation and requires only three operations at each surface in its use.

Alternative Transfer Theorems.

When only certain transfer coefficients such as $\partial l_k' / \partial c_i$, $\partial l_k' / \partial n_i$ are required, it is fairly obvious that shorter methods of computing these are available through the transfer theorems provided by the paraxial magnification laws for a centered system. The writer[9] has used these in another paper on the primary chromatic coefficients of a lens system. They have been avoided in the present paper as what has been aimed at is a complete systematic computation of all the transfer coefficients which may be needed in connection with paraxial rays.

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9. F. D. Cruickshank Paper submitted for publication along with the present one.

TRANSFER COEFFICIENTS FOR THE PRIMARY ABERRATIONS OF A LENS SYSTEM.

Introduction.

We propose to consider now the development of a differential correction method to accompany the primary aberration computing methods. We shall take the general expressions for the surface contributions to the primary aberrations as given by Conrady as the basis from which to derive expressions for the rates of change of these aberrations with the constructional parameters of the lens system. The only modification of Conrady's expressions is the use of transverse measures for each of the aberrations. This introduces a uniformity which is desirable and somewhat simplifies the computations involved in the method. The form of the expressions will be manipulated with a view to the ease of computation in the complete scheme, embracing both the calculation of the contributions from the surfaces and the calculation of the transfer coefficients for each aberration.

In the course of the development, a number of simple derivatives appear, some of which have been discussed in other papers, and for convenience these derivatives are collected immediately so that they are available for reference when needed. This will avoid digressions of the main argument. In this way we have

$$\frac{\partial i'}{\partial c} = n\frac{\partial i}{\partial c} = ny \tag{1}$$

$$\frac{\partial i'}{\partial y} = n\frac{\partial i}{\partial y} = nc \tag{2}$$

$$\frac{\partial i'}{\partial d} = \frac{\partial i'}{\partial y}\frac{\partial y}{\partial d} = ncu \tag{3}$$

The ratio $i_{pr} / i = i'_{pr} / i' = \theta$, occurs frequently in the expressions for the primary aberrations, so that it is convenient to include these derivatives in the present list. Fairly obviously

$$\frac{\partial\theta}{\partial c} = (y_{pr} - \theta y) / i \qquad (4)$$

$$\frac{\partial\theta}{\partial d} = c(u_{pr} - \theta u) / i \qquad (5)$$

The primary transfer coefficients for refractive index changes are a little more cumbersome than those for changes of curvature and axial separation, and the writer has not found much advantage in using them, as at the stage at which these primary coefficients are employed the glasses for a system have been selected, approximate achromatism has been obtained, and a glass change is not desirable until other manipulations fail to give the desired correct state. On this account the coefficients for the primary aberrations with respect to refractive index changes are not included in the present paper.

1. Transfer Coefficients for the Primary Spherical Aberration.

The total primary spherical aberration for a lens system is given in transverse measure by

$$\Sigma SC' = \Sigma\tfrac{1}{2} N'yi'(i' - u)(i - i') / N'_k u'_k$$

$$= \Sigma \frac{(1 - n)}{2n} N'yi'^2(i' - u) / N'_k u'_k \qquad (6)$$

$$= \Sigma\alpha / N'_k u'_k \qquad (7)$$

where for any surface

$$\alpha = \frac{(1 - n)}{2n} N'yi'^2(i' - u) \qquad (8)$$

and

$$n = N / N'$$

the summation including all the surfaces of the system. For changes of the primary aberration of the final image with respect to the curvature of any surface we have from (7)

$$\frac{\partial \Sigma SC'}{\partial c_i} = \frac{1}{N'_k\ u'_k}\ \frac{\partial \Sigma a_i}{\partial c_i} - \frac{1}{N'_k\ u'^2_k}\ \frac{\partial u'_k}{\partial c_i}\ \Sigma a_i$$

$$= \frac{1}{N'_k\ u'_k}\ \frac{\partial a_i}{\partial c_i} - \frac{\Sigma SC'}{u'_k}\ \frac{\partial u'_k}{\partial c_i} \qquad (9)$$

Similarly for changes of axial separation

$$\frac{\partial \Sigma SC'}{\partial d_i} = \frac{1}{N'_k\ u'_k}\ \frac{\partial a_i}{\partial d_i} - \frac{\Sigma SC'}{u'_k}\ \frac{\partial u'_k}{\partial d_i} \qquad (10)$$

Differentiation of (8) gives

$$\frac{\partial a}{\partial i'} = \frac{(1 - n)}{2n}\ N'y(3i'^2 - 2i'u)$$

$$= \frac{(1 - n)}{2n}\ N'yi'(3i' - 2u) \qquad (11)$$

Small curvature changes made at any surface involve only changes in the angle of incidence, whence

$$\frac{\partial a}{\partial c} = \frac{\partial a}{\partial i'}\ \frac{\partial i'}{\partial c} \qquad (12)$$

Since a small change of axial separation changes the height of the incidence point of any incident ray as well as the angle of incidence, we have

$$\frac{\partial a}{\partial d} = \frac{(1 - n)}{2n}\ N'i'^2(i' - u)\ \frac{\partial y}{\partial d} + \frac{(1 - n)}{2n}\ N'yi'(3i' - 2u)\ \frac{\partial i'}{\partial d}$$

$$= \frac{a}{y}\ \frac{\partial y}{\partial d} + \frac{\partial a}{\partial i'}\ \frac{\partial i'}{\partial d}$$

$$= \frac{au}{y} + \frac{\partial a}{\partial i'}\ \frac{\partial i'}{\partial d}$$

$$= \frac{a}{\ell} + \frac{\partial a}{\partial i'}\ \frac{\partial i'}{\partial d} \qquad (13)$$

2. Transfer Coefficients for the Primary Sagittal Coma.

The primary sagittal coma of the final image is given by

$$\Sigma CC' = \Sigma SC'_i \, \theta_i = \Sigma \alpha_i \theta_i \,/\, N'_k u'_k \tag{14}$$

For curvature changes at surface i we have then

$$\frac{\partial \Sigma CC'}{\partial c_i} = \frac{1}{N'_k u'_k}\left[\frac{\partial \alpha_i}{\partial c_i}\theta_i + \alpha_i \frac{\partial \theta_i}{\partial c_i}\right] - \frac{1}{N'_k u'^2_k}\Sigma \alpha_i \theta_i \frac{\partial u'_k}{\partial c_i}$$

$$= \frac{1}{N'_k u'_k}\frac{\partial \alpha_i}{\partial c_i}\theta_i + SC'_i \frac{\partial \theta_i}{\partial c_i} - \frac{\Sigma CC'}{u'_k}\frac{\partial u'_k}{\partial c_i} \tag{15}$$

and similarly

$$\frac{\partial \Sigma CC'}{\partial d_i} = \frac{1}{N'_k u'_k}\frac{\partial \alpha_i}{\partial d_i}\theta_i + SC'_i \frac{\partial \theta_i}{\partial d_i} - \frac{\Sigma CC'}{u'_k}\frac{\partial u'_k}{\partial d_i} \tag{16}$$

3. The Primary Astigmatism.

The primary astigmatism of the final image is given in transverse measure by

$$\Sigma AC' = \Sigma CC'_i \, \theta_i = \Sigma \alpha_i \theta_i^2 \,/\, N'_k u'_k$$

Differentiation gives at once

$$\frac{\partial \Sigma AC'}{\partial c_i} = \frac{1}{N'_k u'_k}\left[\frac{\partial \alpha_i}{\partial c_i}\theta_i^2 + 2\theta_i \alpha_i \frac{\partial \theta_i}{\partial c_i}\right] - \frac{\Sigma \alpha_i \theta_i^2}{N'_k u'^2_k}\frac{\partial u'_k}{\partial c_i} \tag{17}$$

$$= \frac{1}{N'_k u'_k}\frac{\partial \alpha_i}{\partial c_i}\theta_i^2 + 2\, CC'_i \frac{\partial \theta_i}{\partial c_i} - \frac{\Sigma AC'}{u'_k}\frac{\partial u'_k}{\partial c_i} \tag{18}$$

Similarly

$$\frac{\partial \Sigma AC'}{\partial d_i} = \frac{1}{N'_k u'_k}\frac{\partial \alpha_i}{\partial d_i}\theta_i^2 + 2\, CC'_i \frac{\partial \theta_i}{\partial d_i} - \frac{\Sigma AC'}{u'_k}\frac{\partial u'_k}{\partial d_i} \tag{19}$$

4. Transfer Coefficients for the Primary Chromatic Aberrations.

If we write

$$\nu = (N_v - N_r) \,/\, N$$

the primary longitudinal chromatic aberration of the final image in transverse measure is given by

$$\Sigma \ell chC' \quad = \quad \Sigma N'yi'(v_+ - v) \;/\; N_k' \, u_k' \tag{20}$$

where the subscript '+' denotes the next surface, i.e. the surface (i + 1) if the general surface is denoted by i. Hence we have

$$\frac{\partial \Sigma \ell chC'}{\partial c_i} = \frac{1}{N_k' \, u_k'} N'(v_+ - v)y \frac{\partial i'}{\partial c} - \frac{1}{N_k' \, u_k'^2} \frac{\partial u_k'}{\partial c_i} \Sigma N'yi'(v_+ - v)$$

$$= \frac{\ell chC'}{i'} \frac{\partial i'}{\partial c} - \frac{\Sigma \ell chC'}{u_k'} \frac{\partial u_k'}{\partial c_i} \tag{21}$$

and

$$\frac{\partial \Sigma \ell chC'}{\partial d} = \frac{\ell chC'}{i'} \frac{\partial i'}{\partial d} - \frac{\Sigma \ell chC'}{u_k'} \frac{\partial u_k'}{\partial d} \tag{22}$$

Again, the transverse chromatic aberration is given by

$$\Sigma tchC' = \Sigma \ell chC' \theta$$

$$= \Sigma N'yi(v_+ - v) \, \theta \;/\; N_k' \, u_k' \tag{23}$$

Differentiating with respect to c and d, we obtain

$$\frac{\partial \Sigma tchC'}{\partial c} = \frac{\ell chC'}{i'} \frac{\partial i'}{\partial c} \theta + \ell chC' \frac{\partial \theta}{\partial c} - \frac{\Sigma tchC'}{u_k'} \frac{\partial u_k'}{\partial c} \tag{24}$$

and

$$\frac{\partial \Sigma tchC'}{\partial d} = \frac{\ell chC'}{i'} \frac{\partial i'}{\partial d} \theta + \ell chC' \frac{\partial \theta}{\partial d} - \frac{\Sigma tchC'}{u_k'} \frac{\partial u_k'}{\partial d} \tag{25}$$

The Computation.

It may be thought from a cursory inspection of the foregoing equations that the computations required are such as to render the tool of little value in a primary aberration method. This, however, is not the case, as the equations contain many terms which carry over in the computation to later equations. Further, in many cases, the final term in the equations involving $\frac{\partial u_k'}{\partial c}$ or perhaps more particularly

$\partial u'_k / \partial d$, become very small and may frequently be neglected. On an accompanying sheet the lay-out of the computation of the primary transfer coefficients for curvature changes is given to afford an idea of the amount of work involved. In the way in which the writer works, the single surface coefficients are normally available from the general paraxial transfer coefficients of the system. Thus where routine computers are available the primary computing scheme extended in this way forms a useful addition to the methods normally used.

Computing Scheme - Transfer Coefficients for Primary Aberrations.

$2(i' - u) + i'$	
$\partial\alpha / \partial i'$	
$\partial\alpha / \partial c$	
$\partial\alpha / \partial c \,.\, 1 / N'_k u'_k$	
$- \Sigma SC' / u'_k \,.\, \partial u'_k / \partial c$	
$\partial\Sigma SC' / \partial c$	
$\partial\alpha / \partial c \,.\, 1 / N'_k u'_k \,.\, \theta$	
$SC' .\, \partial\theta / \partial c$	
$- \Sigma CC' / u'_k \,.\, \partial u'_k / \partial c$	
$\partial\Sigma CC' / \partial c$	
$\partial\alpha / \partial c \,.\, 1 / N'_k u'_k .\, \theta^2$	
$2\ CC'_i \quad \partial\theta / \partial c$	
$- \Sigma AC'_k / u'_k \,.\, \partial u'_k / \partial c$	
$\partial\Sigma AC' / \partial c$	
$\ell chC' / i'$	
$+ \ell chC' / i' \,.\, \partial i' / \partial c$	
$- \Sigma\ell chC' / u'_k \,.\, \partial u'_k / \partial c$	
$\partial\Sigma\ell chC' / \partial c$	
$\ell chC' / i' \,.\, \partial i' / \partial c .\theta$	
$+ \ell chC' \,.\, \partial\theta / \partial c$	
$- \Sigma tchC' / u'_k \,.\, \partial u'_k / \partial c$	
$\partial\Sigma tchC' / \partial c$	

A SERIES OF SMALLER CONTRIBUTIONS FOR USE IN THE DESIGN OF LENS SYSTEMS.

VI. THE CHROMATIC VARIATION OF THE TANGENTIAL ABERRATIONS.

BY F. D. CRUICKSHANK, B. Sc.

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The aberrations of an optical system are determined normally for incident light of some mean wave-length in the spectral range over which the system is expected to give satisfactory images. The full significance of the chromatic aberration of the system is really apparent only when, in addition to the usual analysis, the variation of the aberrations with the wave-length of the incident light has been examined. Hitherto the analysis of the dependence of the aberrations on the wave-length of the incident light has required repeated ray-tracings in different colours. It is of interest, therefore, to examine this matter in the light of the properties of the transfer coefficients which have been developed in the earlier papers of this series.

Suppose that the aberrations of the system for incident light of some mean wave-length, d, have been obtained by a ray-trace and the general transfer coefficients have been calculated from the trace. If the mean refractive indices of the glasses of the components a, b, ... h, .. of the system are changed by amounts δN_a, δN_b, ... δN_h, .. the change in the value of the aberration, A'_d, which may be any of the tangential aberrations of the system, resulting from the glass changes is given by

$$\delta A'_d = \sum \frac{\partial A'_d}{\partial N_h} \delta N_h$$

If, however, instead of considering glass changes we consider a change in the wave-length of the light which is incident on the system from the object, then the value of the aberration, A', for light of wave-length, x, will be given by

$$A'_r = A'_d + \sum \frac{\partial A'_d}{\partial N_h} (N_r - N_d)_h$$

Hence, using the computed values of the transfer coefficients, $\partial A'/\partial N_h$, for the various aberrations we can investigate the chromatic variation of the aberrations quite quickly.

As an example we consider a rough design of a wide angle photographic objective of the Ross Xpres type. In Table I the computed values of the transfer coefficients of the aberrations with respect to the refractive indices of the components are shown, the oblique aberrations being calculated from a pencil of about 22° obliquity.

TABLE I.

	LA'_m	TA'_s	X'_T	Coma'	Dist'
$\partial/\partial N_a$	150.13	58.202	-1[illegible]0.43	0.801	7.7[illegible]
$\partial/\partial N_b$	-140.51	-4[illegible].202	115.59	0.107	-7.[illegible]
$\partial/\partial N_c$	7.11	1.000	- 23.54	-1.337	1.308
$\partial/\partial N_d$	12.53	5.280	- 8.40	1.105	-1.328
$\partial/\partial N_e$	-115.64	-42.750	47.16	-0.051	5.580
$\partial/\partial N_f$	117.10	43.[illegible]	-35.34	-0.840	-0.180

The glass selected for components a and f was DBC [illegible]1555, for components b and e LF 549[illegible]7, and for components c and d HC 519604. The dispersions of these glasses are given in Table II. The ray-trace of the system having been carried out for sodium light we calculate next the aberration differences, $A' - A'_d$, for wave lengths from 7065 A.U. to 4341 A.U. Since the glasses in the front and rear halves of the lens are similar we may combine the coefficients for the components which have similar glasses as in the evaluation of the difference by the equation given above these coefficients will each be multiplied by the same dispersion values. In Table III the full calculation is set out for the case of the marginal spherical aberration, and finally in Table IV the chromatic differences for all the aberrations are collected, the detailed computations being omitted.

TABLE II.

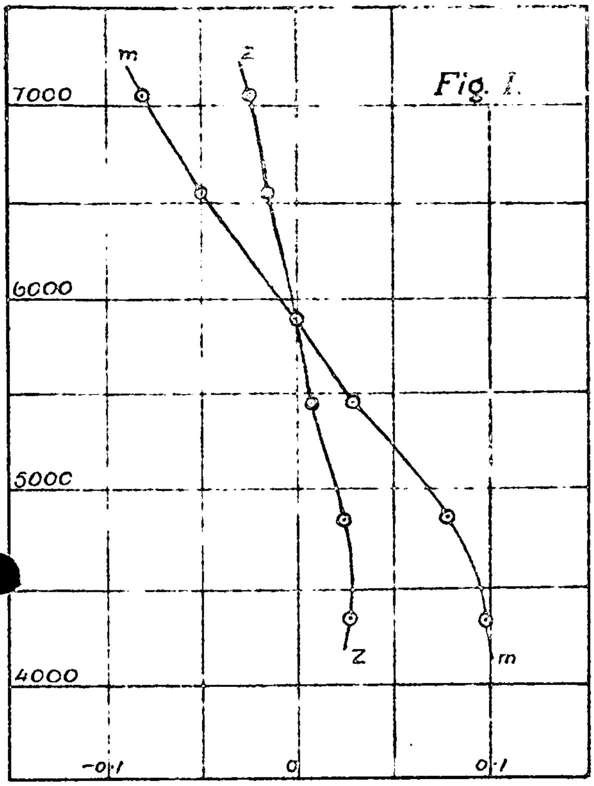
	DBC	LF	HC
$N_b - N_D$	−.00517	−.00530	−.00400
$N_C - N_D$	−.00523	−.00342	−.00250
$N_e - N_D$	.00274	.00201	.00211
$N_F - N_D$	.00785	.00335	.00665
$N_{G'} - N_D$	.01422	.01520	.01080

TABLE III.

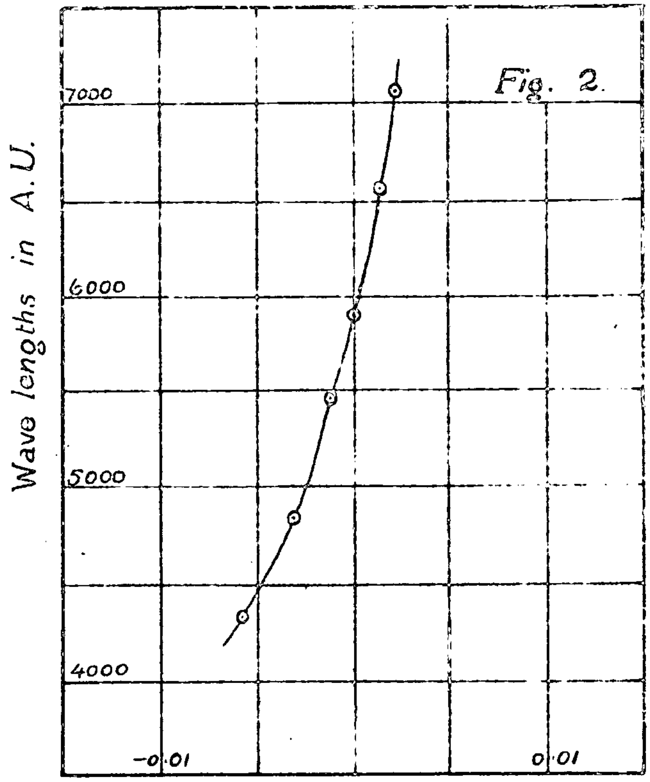
Comp.	Coeff.	Contrbn. b − D	Contrbn. C − D	Contrbn. e − D	Contrbn. F − D	Contrbn. G' − D
a,f	207.89	−1.5310	−0.3767	+0.7324	+2.0932	3.3303
b,e	−250.15	1.3806	0.8769	−0.7454	−2.1598	−3.0104
c,d	19.64	−0.0798	−0.0503	0.0415	0.1135	0.2149
	$TA'_{\lambda} - TA'_D$ =	−0.0311	−0.0510	0.0235	0.0770	0.0934

TABLE IV.

Range	TA'_m	TA'_s	K'_m	$Coma'_T$	Dist'
b − D	−0.0311	−0.0265	−0.0450	0.00205	−0.00045
C − D	−0.0510	−0.0165	−0.0201	0.00129	−0.00017
e − D	0.0235	0.0037	0.0339	−0.00100	0.00015
F − D	0.0770	0.0256	0.0996	−0.00510	0.00049
G' − D	0.0934	0.0275	0.2092	−0.00503	0.00047

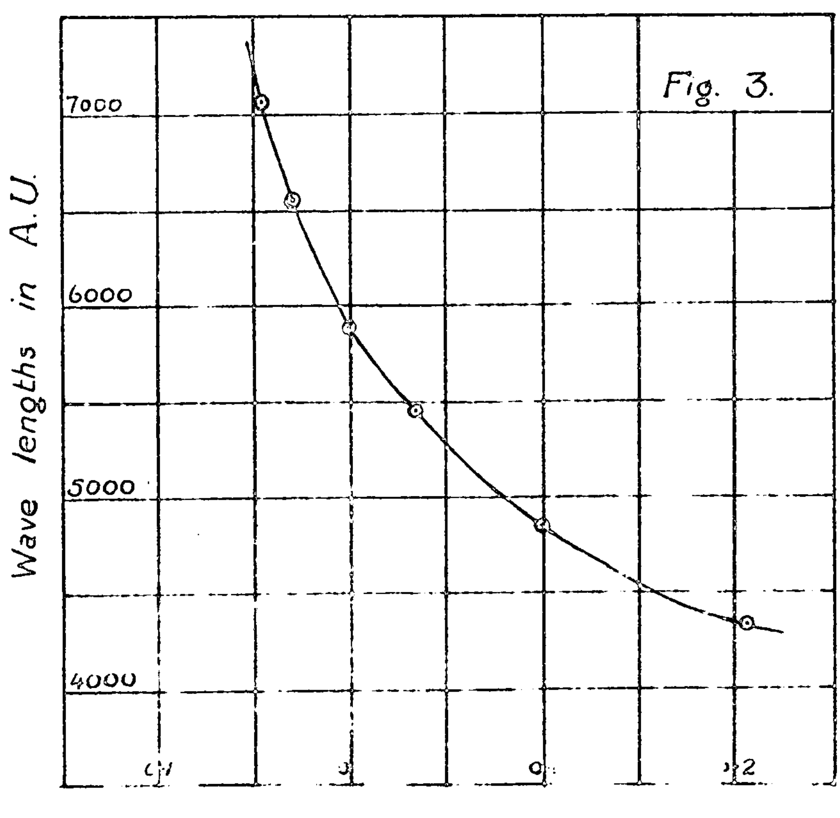


$LA' - LA'_D$



$Coma'_T - Coma'_{TD}$

(22° Pencil)



– 22° ncil

The accompanying graphs in Figures 1 - 8 show the variation of the aberrations plotted against the wave-length, and thus completely summarise the analysis of the dependence of the general correction state of the system on the wave-length of the incident light. As a check on the general validity of this analysis the differences of the spherical aberration for C and D light, and for F and D light, were determined by complete ray traces and compared with the values computed by the present method. The results are given in Table V, the discrepancies being no greater than the uncertainty of the last figure of the traces.

TABLE V.

	Marginal.		Zonal.	
	Trace	Coeffs.	Trace	Coeffs.
$LA'_C - LA'_D$	-0.033	-0.031	-0.013	-0.0135
$LA'_F - LA'_D$	0.077	0.0779	0.023	0.0236

The Analysis of the Secondary Spectrum.

To complete the discussion of the chromatic variation of the aberrations the matter of the secondary spectrum should be considered. It has been shown previously that the final intersection length, L'_x, for an axial ray of some colour x is related to the corresponding quantity, L'_d, for the traced ray for some mean wave-length, d, by

$$L'_x = L'_d + \sum \frac{\partial L'_d}{\partial N_h} (N_x - N_d)_h$$

and for a principal ray of an oblique pencil

$$H'_x = H'_d + \sum \frac{\partial H'_d}{\partial N_h} (N_x - N_d)_h$$

where H' measures the height at which the principal ray intersects a

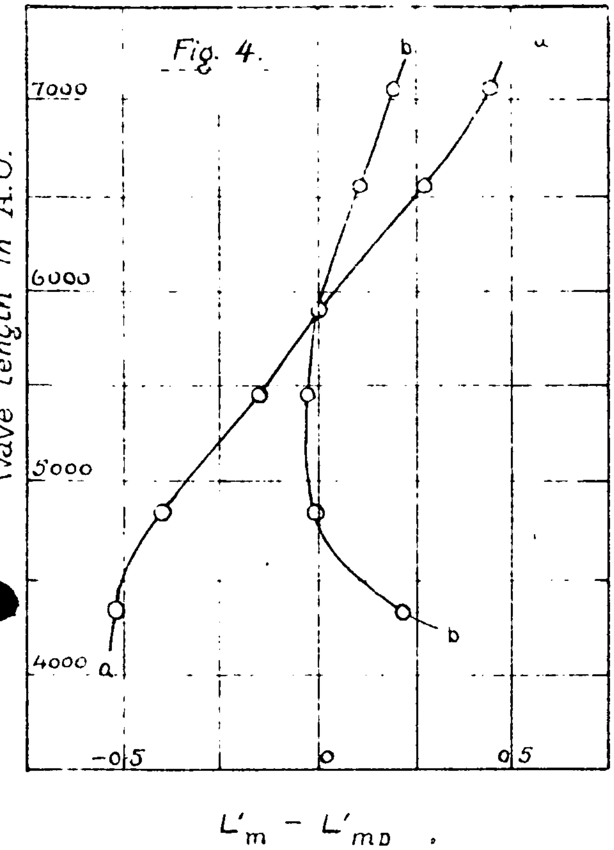


Fig. 4.
Wave length in A.U.
7000
6000
5000
4000
-0.5
0
0.5
a
b
$L'_m - L'_{mD}$

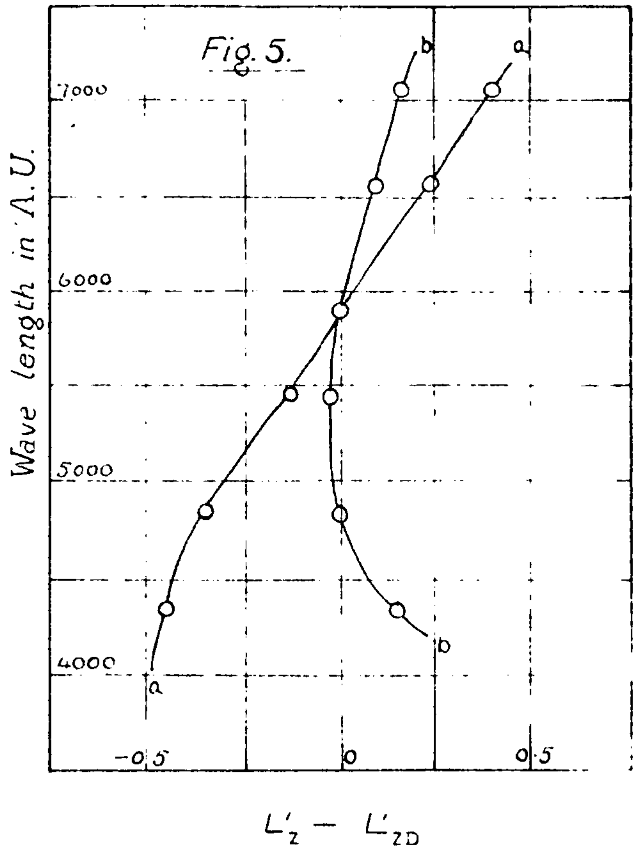


Fig. 5.
Wave length in A.U.
7000
6000
5000
4000
-0.5
0
0.5
a
b
$L'_z - L'_{zD}$

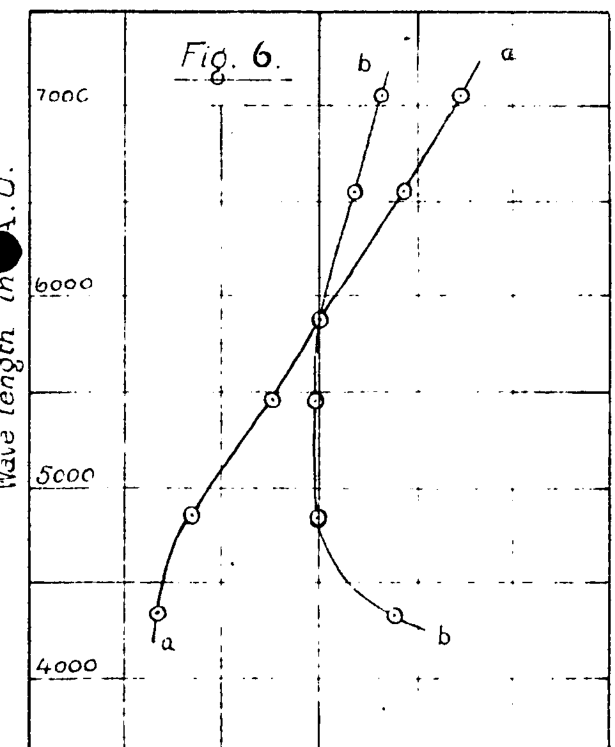


Fig. 6.
Wave length in A.U.
7000
6000
5000
4000
-0.5
0
a
b

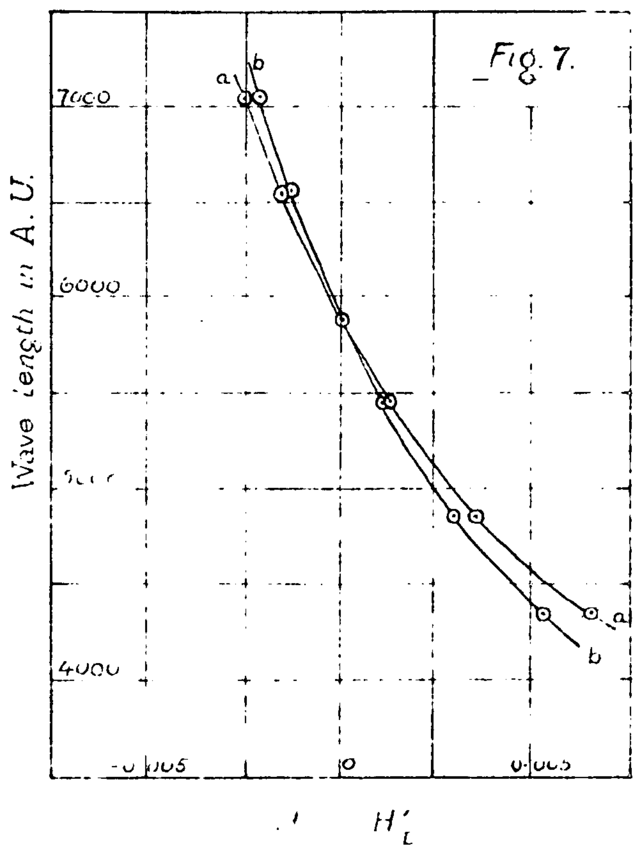


Fig. 7.
Wave length in A.U.
7000
6000
4000
0
a
b
H'_L

fixed image plane, conveniently the paraxial image plane. These transfer coefficients, $\partial L' / \partial N_\lambda$, and $\partial H' / \partial N_\lambda$, which may be termed appropriately the chromatic coefficients of the system, provide a means of analysing the importance of the secondary spectrum as it affects the axial and transverse chromatic aberrations of the system. It will also be obvious that they furnish a simple means of determining the effect on the colour correction of changes in the glasses of the system, provided these are not too drastic.

The continuation of our example will afford a clear insight into the use of these chromatic coefficients. In Table VI are collected the computed values of the chromatic coefficients for the marginal, zonal, and paraxial rays of the axial pencil, and the principal ray of an oblique pencil at 22°.

TABLE VI.

Comp.	$\partial L'_m/\partial N$	$\partial L'_z/\partial N$	$\partial l'/\partial N$	$\partial H'/\partial N$
a	-757.99	-640.16	-537.83	-39.917
b	768.59	677.25	623.03	30.509
c	-162.67	-157.51	-155.56	- 6.300
d	-127.07	-119.76	-114.53	8.881
e	574.99	502.11	459.35	-30.143
f	-521.15	-446.93	-403.93	38.215

TABLE VII.

Comp.	Chr. Coeff.	Contrbn. b - D	Contrbn. C - D	Contrbn. e - D	Contrbn. F - D	Contrbn. G' - D
a, f	-1037.14	5.6205	3.5659	-2.9763	-8.5540	-15.4591
b, c	1179.35	-6.3566	-4.0555	3.4519	9.8478	18.0520
c, d	- 277,27	1.1257	0.7023	-0.5950	-1.6719	- 3.0195
$L'_m - L'_{mD}$		0.3906	0.2423	-0.1519	-0.3535	-0.4466

In Table VII the contributions made by the pairs of components of similar glass to the zonal chromatic aberration, $L'_z - L'_{z0}$, are computed for the various spectral ranges, the last line of the Table giving the final value of the aberration. Omitting the corresponding detailed computation for the other three rays under consideration, we collect in Table VIII the values of the aberrations calculated for each ray. This provides a sampling of the axial colour correction in the paraxial, zonal, and marginal regions, and a single sampling of the transverse colour at one obliquity.

TABLE VIII.

Aberrn.	b − D	C − D	e − D	F − D	G' − D
$L'_m - L'_{m0}$	0.4443	0.2727	−0.1813	−0.4123	−0.5173
$L'_z - L'_{z0}$	0.3391	0.2423	−0.1519	−0.3535	−0.4400
$l' - l'_0$	0.3654	0.2257	−0.1252	−0.3349	−0.4192
$H' - H'_0$	−0.00255	−0.00153	0.00123	0.00354	0.00659

The curves marked aa in Figures 4 − 7 show these aberration values plotted against the wave-length and hence summarise completely the condition of the system as regards its secondary spectrum. The axial chromatic aberration is very undesirable and calls for attention. A study of the chromatic coefficients and the contributions to the aberrations shows that an improvement should be effected by increasing the V-number of the DBC components and also the HC components. By way of example we consider the result of changing to a DBC of the type 61[illegible]563, and also substitute a BSC of the type 517642 for the HC of components c and d. Ten minutes computation with the new dispersion values in place of the former ones gives a new set of aberration values. Omitting the details these are collected in Table IX, from which the curves bb in Figures 4 − 7 have been drawn. These curves show a very great improvement in the axial colour correction and a slight improvement in the already good transverse aberration. This example affords clear evidence of the general usefulness of these chromatic coefficients of the system.

TABLE IX.

Aberrn.	b – D	C – D	e – D	F – D	G' – D
$L'_m - L'_{mD}$	0.1233	0.1075	–0.0250	–0.0100	0.2193
$L'_z - L'_{zD}$	0.1627	0.0920	–0.0199	–0.0059	0.2097
$l' - l'_D$	0.1525	0.0932	–0.0196	–0.0069	0.1920
$H' - H'_D$	–0.00221	–0.00156	0.00116	0.00258	0.00525

As a final check on the method we investigate the reliability of the analysis of the curves as an analysis of the secondary spectrum. Ray-tracings were made for the three axial rays in colours C and F and the final intersection values for each compared with the values deduced by using the chromatic coefficients.

TABLE X.

Wave Length	Marginal		Zonal		Paraxial	
	Trace	Coeffs.	Trace	Coeffs.	Trace	Coeffs.
D	103.445	–	106.522	–	106.525	–
C	103.734	103.721	106.736	106.764	106.748	106.751
F	103.050	103.032	106.159	106.165	106.181	106.190

The first line in Table X gives the intersection lengths for D light for the three rays in the original trace. From these values and the aberration quantities in Table VIII, the predicted values of the intersection lengths of the rays for C and F light are obtained and entered in the Table. The values obtained from the complete traces are entered alongside the predicted values, from which it is seen that the agreement is within the limits of uncertainty of the traced values.

THE TRIGONOMETRICAL CORRECTION OF MICROSCOPE OBJECTIVES.

by F. D. CRUICKSHANK ,

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In the development of the design of a microscope objective it is usual to correct the system trigonometrically as fully as possible and then to apply the various wave theory tests in terms of path differences. This final trigonometrical adjustment is probably the most laborious part of the work, and also the most empirical, as usually undertaken, for the normal procedure is to change one datum at a time and make repeated ray tracings until the desired correction state is achieved. Thus in the notes of his lectures in 1919 Conrady says ' In arriving at a new design by the method of trigonometrical trials - the only one available for the deep curvatures of microscope objectives - it will be found necessary and also quickest to proceed systematically, changing one datum at a time so as to be able to interpolate in a simple and straightforward way for the desired correction. The temptation will often t be strong to superpose a second change, but students may be assured that the process does not pay.' The solution of the problem of the transfer coefficients of an optical system given recently by the writer makes available a new means of carrying out the trigonometrical correction, the new method being free of the empirical features of the old and giving a complete analysis of the potentialities for correction which are present in the system.

The Appropriate Transfer Coefficients.

Suppose that a rough design for an objective of either the Lister or Amici type has been developed in the usual way to a stage at which it is ready for trigonometrical correction. Apart from the chromatic aberration, it is usually sufficient to adjust the design so that the marginal spherical aberration is reduced to zero and the sine condition holds closely. Care must also be exercised during the correction process that the working distance of the objective is not reduced seriously, a fault which may be achieved easily in a high power objective. For these corrections to be undertaken it will be adequate to trace a paraxial and a marginal ray from an axial point determined by the tube length required, the trace being made towards the side of the shorter conjugate. The paraxial ray is traced for convenience with an initial inclination angle $u = \sin U_m$, and the departure from the sine condition may be measured by

$$e = \sin U'_{mk} - u'_k \qquad (1)$$

For any constructional parameter, c_i, which may be a curvature, thickness, or refractive index, we can calculate the transfer coefficients[2]

$$\frac{\partial U'_{mk}}{\partial c_i} = C(c_i)_m \operatorname{cosec} U'_{mk} \qquad (2)$$

$$\frac{\partial l'_k}{\partial c_i} = C(c_i)_{pk} / u'_k \qquad (3)$$

$$\frac{\partial}{\partial c_i}(\sin U'_{mk}) = \cos U'_{mk} \frac{\partial U'_{mk}}{\partial c_i} \qquad (4)$$

whence

$$\frac{\partial LA'_k}{\partial c_i} = \frac{\partial l'_k}{\partial c_i} - \frac{\partial L'_{mk}}{\partial c_i} \qquad (5)$$

$$\frac{\partial e}{\partial c_i} = \frac{\partial}{\partial c_i}(\sin U'_{mk}) - \frac{\partial u'_k}{\partial c_i} \qquad (6)$$

Equations (5) and (6) provide the necessary transfer coefficients for the adjustment of the spherical aberration and the sine condition, while equation (3) gives the rate of change of the working distance, and equation (4) the rate of change of the numerical aperture, with the parameter c_i. The calculation of these transfer coefficients provides the designer with a complete analysis of the correction potentialities of the system. This is best demonstrated by the consideration of an actual example.

Numerical Example - Lister Type Objective.

In the course of developing a design for a Lister Objective a stage was reached at which the system had the residuals shown in the first line of Table II. To examine the correction potentialities of the objective the transfer coefficients shown in Table I were calculated. The dimensions of the system were given in millimetres so that the curvature unit employed is the reciprocal millimetre. According to the first line of Table I, for example, a curvature change of 0.001 $mm.^{-1}$ at the first surface will cause a shift of the final intersection point of the marginal ray through 0.311 mm., a change of spherical aberration by 0.037 mm., and a change of the sine condition aberration quantity, ε, by 0.000112, etc. According to the tenth line of the Table a shift of the fourth surface together with all surfaces behind it - thereby altering the central airspace of the system - through a distance of one millimetre will introduce a spherical aberration change of amount 0.024 mm. and an ε-change of 0.00014. Table I thus gives the exact effect of a small change in curvature at any surface of the system and the effect of a small change in the thickness of any component or airspace. For the sake of brevity the transfer coefficients for refractive index changes have been omitted. A study of this Table enables the designer to select which changes may be made for the trigonometrical correction of the objective. Obviously, this may be done in a number of different ways. In this particular system it is easier to obtain the spherical correction than the ε-correction. The change which offers the greatest correction for the sine condition, a shift of the the fifth surface, cannot be used to advantage in the present case as the direction in which the change must be

TABLE I.

Surface		L'_{mk}	l'_k	$\sin U'_{mk}$	u'_k	$\Sigma A'_m$	e
1.	$\partial/\partial c$	-310.63	-273.27	0.43527	0.35547	37.38	0.11130
	$\partial/\partial d$	0.0155	0.0165	-0.00123	-0.00123	0.0003	0.00000
2.	$\partial/\partial c$	- 67.53	- 50.70	0.10335	0.00412	16.08	0.01473
	$\partial/\partial d$	0.2779	0.2320	0.00535	0.00541	-0.0459	-0.00003
3.	$\partial/\partial c$	312.59	294.19	-0.63237	-0.66384	-13.40	-0.01405
	$\partial/\partial d$	0.1418	0.1277	0.00242	0.00252	-0.0141	-0.00010
4.	$\partial/\partial c$	- 91.34	- 89.18	1.51293	1.49455	2.66	0.01861
	$\partial/\partial d$	0.2306	0.2070	0.00394	0.00403	-0.0236	-0.00014
5.	$\partial/\partial c$	- 20.56	- 14.83	0.35174	0.26033	5.70	0.03253
	$\partial/\partial d$	0.8756	0.8235	-0.00215	-0.00132	-0.0455	-0.00053
6.	$\partial/\partial c$	87.64	80.65	-1.59812	-1.54030	- 3.99	-0.05752
		0.6076	0.6169	0.00000	0.00000	0.0093	0.00000

TABLE II.

	L'_{mk}	l'_k	$\sin U'_{mk}$	u'_k	$\Sigma A'_m$	e
From original trace	11.339	11.393	0.21820	0.21769	0.057	0.00051
Predicted changes to first order	0.559	0.501	0.00953	0.00987	-0.057	-0.00054
Predicted changes corrected	0.534	0.478	0.00955	0.00987	-0.056	-0.00054
Predicted values new trace	11.873	11.874	0.22774	0.22756	0.001	0.00017
Actual values new trace	11.873	11.875	0.22769	0.22757	-0.003	0.00012

made to improve the correction results in the thinning down of a biconvex lens which is already close to its minimum thickness for reasonably easy manufacture. One simple change which would remove the spherical aberration and reduce the ε-value considerably is a positive d-change at surface four. A short calculation shows that if this surface were shifted forward through 2.42 mm. in the usual way the spherical aberration will be reduced to zero and ε will be reduced to one-third of its present value. The predicted results of this change are entered in the second line of Table II. Correcting these expected changes by the use of a second order correction term[3], for a displacement of 2.42 mm. is scarcely a differential change in a system of these dimensions, we obtain the third line in Table II. The fourth line of Table II combines the original values of the various quantities with the predicted changes and the last line gives the values of these same quantities as shown by a fresh trace of the altered system. The agreement with expectation is of a very high order, and demonstrates the usefulness of the new method of correction.

The agreement with prediction is not always as good as that given in the example, particularly when a large number of changes are made simultaneously. In general it is sufficiently good to require only a furbher small adjustment after the fresh trace is made. Looking at the matter geometrically, the differential transfer coefficient specifies the slope of the tangent to the curve which represents the relation between the aberration and the parameter of the system. If the slope of this curve is changing slowly at the point representing the present value of the aberration the predicted value of the aberration change following a small change in the parameter will be very close to the actual value. If, however, the slope of the curve is changing rapidly at this point the agreement of the actual change and the predicted change will not be as close.

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: Marginal 'M

System: Booth Teleph

Pencil: Axial

	Surface	1	2	3	4	5	6	7
	L		377.9345	153.5416	393.1704	211.0026	861.4683	875.0980
	r	161.78	432.35	420.12	361.80	98.39	1036.8	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.000	1.61517	1.000	1.65068	1.000	1.57338	1.57846
	N'	1.61517	1.000	1.65068	1.000	1.57338	1.57846	1.000
$[r_{-1}/r]$	a		.3741876	1.029111	1.161194	3.677203	.09484776	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β		1.305331	.0295392	2.181592	2.587255	.8946181	6.898323
	$\alpha \sin I'_{-1}$		.1038638	.5688191	.3916706	.0908766	.0309277	.0488085
	$+\beta \sin U$		.2383469	.0120447	.3766490	.4218938	.0381366	.2513022
	$\sin I$	.4483249	.3422108	.5567742	.0149717	.6127703	.0072089	.2024937
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6355744	.9967817	1.57846
	$\sin I'$	.2775714	.6527286	.3372999	.0247135	.3259052	.0071857	.3196282
$[Pl.\tan U]$	$\sin I'+\sin U'$	.4601664	.1449761	.1646283	.1877791	.3685341	.0354200	.2408960
$[Pl.\cot U']$	$r()$	74.44572	62.68042	69.16364	67.93870	36.26007	36.72346	36.77021
$[L \tan U \cot U'] [()/\sin U']$	L'	407.7095	153.7217	400.5502	416.6326	860.6983	861.9377	467.0230
	U		10-31.257	24-03.824	9-56.591	9-23.095	2-26.592	2-26.572
	$+I$		20-00.698	33-49.980	0-51.471	30-50.912	0-24.783	11-40.970
	$U+I$	26-38.176	9-29.441	9-46.156	10-48.062	21-27.817	2-01.809	14-07.482
	$-I'$	16-06.919	33-33.264	19-42.747	1-24.967	19-01.226	0-24.703	18-38.427
	U'	10-31.257	24-03.824	9-56.691	9-23.095	2-26.592	2-26.572	4-30.945
	$\sin U$		.1825950	.4077525	.1726716	.1630662	.0426289	.0426057
	$\cos I$	.8938707	.9396232	.8306639	.9998879	.8686268	.9999740	.9792836
	$\cos I'$	.9607050	.8333613	.9413972	.9996945	.9454024	.9999742	.9475430
	$\sin U'$	.1826950	.4077525	.1726716	.1630662	.0426289	.0426057	.0787332
	$\cos U'$	.9831882	.9130925	.9849794	.9866151	.9990910	.9990920	.9968958
	$\tan U'$				.1652784			.0789784
	$1-\cos(U+I)$	.1061293	.0136875	.0145009	.0177161	.0693500	.0006277	.0302331
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.07477	.8869101	1.133307	.9998066	1.101193	1.000	.9675879
$[n/()]$	$\partial U'/\partial U$	.5760580	1.82112	.5345514	1.650999	.6771689	.9967817	1.631336
	$1-\partial U'/\partial U$	.423942	.82112	.4654486	.650999	.4228311	.0032183	.631336
$[()/\cos I]$	e	.4742766	.8738822	.5603332	.6510720	.4925083	.003218384	.6446907
$[e/r]$	$\partial U'/\partial p$	.002931615	.00202124	.001333746	.001799536	.005005674	$.0^{5}3104151$	.004223603
$[eL \sin U]$	$\partial U'/\partial c$	34.39928	60.30649	35.08076	44.20086	16.94593	.1181902	24.11916
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.4666624	.4106392	.5914339	.01497628	.5423831	.007209086	.2137040
$[r(1-\cos(U+I))]$	X	17.1696	5.9178	6.0921	6.4097	6.8233	0.6508	4.6148
	$d'+X_{+1}-X$	6.6826	.0057	19.8818	192.397	17.0426	12.1960	
$[()/\cos U']$	D'	6.7969	.0062129	20.18499	195.0072	17.0581	12.2071	
	$\frac{1}{2}(I-U)$	13-19.088	15-16.977	28-56.902	4-32.560	20-07.003	1-25.687	4-37.229
	$\cos\frac{1}{2}(I-U)$	.9731060	.9647125	.8750563	.9968586	.9389939	.9996894	.9967501
	$\frac{1}{2}(I'-U')$	2-47.831	28-48.544	14-49.669	3-59.064	8-17.317	1-25.608	11-34.686
	$\cos\frac{1}{2}(I'-U')$	.9988085	.8762304	.9666992	.9975830	.9895544	.9996899	.9796520
	$L \sin U$	72.630	69.00986	62.60697	67.8894	34.4074	36.72345	37.41198
$[()/\cos\frac{1}{2}(I-U)]$	PA	74.53453	71.53412	71.54622	68.1033	36.64283	36.73486	37.53396
	$PA \cos\frac{1}{2}(I'-U')$	74.44573	62.68037	69.1637	67.9387	36.26007	36.72347	36.77022
$[()/\sin U']$	L'	407.7096	153.7216	400.6504	416.6326	850.5983	861.9380	467.0230
	$-l'$							462.315
	$L'-l'$							4.708
	H'_M							.37184

Computer:

Date:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Axial Pencil Ray 'M'

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.82112	·5345514	1.650999	·5771689	·9967817	1.631335	
D_+	6·796877	·006212962	20·18499	195.0072	17·05806	12.2071	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.402687	1.319287	2.358094	·8673290	1.574693	1.631335	
$(\partial U_k'/\partial p_+)D_+$	·038926	·000059	·109933	·5609538	·071963	·051558	
$\partial U_k'/\partial U'$	2.441613	1.319346	2.468027	1.428283	1.50273	1.579777	1.000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	437.2037	240.0587	415.5807	16.3219	11.7734		
$(\partial p_k'/\partial p_+)D_+$	11.6522	·0154	33.5038	235·8928	16·5058		
$\partial p_k'/\partial U'$	448·8559	240.0741	449.0845	251·7147	28·2792	11.81144	0.000
$\partial U'/\partial p$	·002931615	·002021238	·001335746	·001799536	·005005674	·53104151	·004223603
$\partial p'/\partial p$	1.07477	·8869101	1.133307	·9998066	1.101193	1.0000	·9675879
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·00715781	·00266671	·00329172	·00257025	·00752218	·00000490	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·00615523	·00839373	·00617230	·00287602	·00464560	·00422360	
$\partial U_k'/\partial p$	·00100264	·00572702	·00946402	·00544627	·00287659	·00421870	·004223603
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.3158727	·4852469	·598965	·4529697	·1415566	·0000367	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.8425319	2·199597	1.881103	1.206865	1.065541	·9675879	
$\partial p_k'/\partial p$	·5266592	1.714350	2.480068	1.659835	1.207098	·9676246	·9675879
$-S_k'.\partial U_k/\partial U'$	1143.612	617·9603	1155·984	668·9846	703.8544	739·9420	468·3838
$C(U')$	1592.468	858·0344	1605·069	920.6993	732.1336	751.7534	468·3838
$C(U')\sec U_k'$	1597·427	860·7063	1610.067	923.5664	734.4135	754.0944	469.8423
$-S_k'.\partial U_k'/\partial p$							
$C(p)$							
$C(p)\sec U_k'$							
$\partial U'/\partial c$							
$C(c)\sec U_k'$							
$-\partial l'/\partial c \tan U_k'$							
$\partial H'/\partial c$							
$\partial U'/\partial n$	·4666624	·4106392	·5914339	·01497628	·5423831	·007209086	·2137040
$C(n)\sec U_k'$	745.4591	·353·4397	952.2482	13·83159	398·3335	5.436331	100.4072
$-\partial l'/\partial n \tan U_k'$	658.3913	276·2721	741·9559	17.36757	359·5111	2.158011	93.19388
$\partial H'/\partial n$	87.0678	77·1676	210.2923	3.53598	38.8224	3.27832	7.21332
$\partial p/\partial d$							
$C(d)\sec U_k'$							
$-\partial l'/\partial d \tan U_k'$							
$\partial H'/\partial d$							
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: QM

System: Booth Telep

Pencil: Axial

Surface	1	2	3	4	5	6	7
Y= 67.0 L		380.535	158.2339	395.4364	215.2219	879.3266	896.9129
r	161.78	432.35	420.12	361.8	98.39	1036.8	152.64
d'	29.77	0.18	7.38	205.68	10.87	16.16	
N	1.00	1.61517	1.00	1.65068	1.00	1.57338	1.57846
N'	1.61517	1.00	1.65068	1.00	1.57338	1.57846	1.00
$[r_{-1}/r]$ a		·3741876	1.029111	1.161194	3.677203	·0948978	6.792453
$r_{-1}-d'_{-1}-r$							
$[()/r]$ β		1.305331	·0295392	2.181592	2.587255	·8946181	6.898323
$a \sin I'_{-1}$		·0959447	·5216278	·3592386	·0891266	·0285480	·0395260
$+\beta \sin U$		·2178749	·0109577	·3446562	·3841944	·0343869	·2266006
$\sin I$	·4141427	·3138197	·5106709	·0146834	·4733186	·0058379	·1870736
$[N/N']$ n	·6191299	1.61517	·6058109	1.65068	·6365744	·9967817	1.57846
$\sin I'$	·2564081	·5068722	·3093700	·0242376	·3008291	·0058191	·2952882
$[Pl.\tan U]$ $\sin I'+\sin U'$	·4233197	·1369176	·1514325	·1727326	·3392655	·0325987	·2222058
$[Pl.\cot U']$ $r()$	68.48466	58.76397	63.61982	62.49465	33.38033	33.79833	33.91749
$[L\tan U\cot U']$ $[()/\sin U']$ L'	410.3050	158.4128	402.8164	420.8536	868.4662	879.7570	464.0993
U		09.60830	21.77450	09.08724	08.53972	02.20279	02.20172
$+I$	24.46534	18.28951	30.70863	0.84133	28.24992	00.33448	10.78205
$U+I$	24.46534	8.68121	8.93403	9.92857	19.71020	01.86831	12.98377
$-I'$	14.85704	30.45571	18.02127	1.38885	17.50741	00.33341	17.17482
U'	09.60830	21.77450	9.08724	8.53972	02.20279	02.20172	04.19105
$\sin U$		·1669116	·3709546	·1579375	·1484950	·0384364	·0384178
$\cos I$	·9102119	·9494829	·8591762	·9998922	·8808913	·9999830	·9823459
$\cos I'$	·9665686	·8620212	·9509417	·9997063	·9536780	·9999831	·9554082
$\sin U'$	·1669116	·3709546	·1579375	·1484950	·0384364	·0384178	·0730824
$\cos U'$	·9859718	·9286510	·9874490	·9889131	·9992610	·9992618	·9973259
$\tan U'$							·0732784
$1-\cos(U+I)$	·0897881	·0114666	·0121322	·0149766	·0585896	·0005316	·0255663
$[\cos I'/\cos I]$ $\partial p'/\partial p$	1.061916	·9078849	1.106034	·9998141	1.082628	1.000	·9725782
$[n/()]$ $\partial U'/\partial U$	·5830310	1.779047	·5477326	1.650987	·5870663	·9967817	1.622965
$1-\partial U'/\partial U$	·4160690	·779047	·4522674	·650987	·4129337	·0032183	·622966
$[()/\cos I]$ e	·4581010	·8204961	·5260292	·610572	·4687681	·00321836	·6341605
$[e/r]$ $\partial U'/\partial p$	·00283163	·00189776	·00125209	·001799495	·00476439	·0⁵3104123	·004154615
$[eL\sin U]$ $\partial U'/\partial c$	30.69277	52.11439	30.87666	40.66128	14.98164	·1087744	21.82717
$[-\sin I/\cos I']$ $\partial U'/\partial n$	·4284669	·3640510	·6370160	·01468771	·4963085	·00583799	·1958049
$[r(1-\cos(U+I))]$ X	14.52592	4.953218	5.09698	5.418498	6.764621	·5511629	3.902440
$d'+X_{+1}-X$	10.29086	·036238	17.89548	194.4469	16.08346	12.80872	
$[()/\cos U']$ D'	10.43728	·0390222	18.12294	196.6269	16.09536	12.81818	
$\frac{1}{2}(I-U)$	12.23267	13.94890	26.24152	4.12296	18.39482	01.26864	4.29016
$\cos\frac{1}{2}(I-U)$	·9772954	·9705111	·896382	·9974121	·9489046	·9997549	·9971981
$\frac{1}{2}(I'-U')$	2.62387	26.11510	13.55426	3.57544	7.65231	01.26756	10.68290
$\cos\frac{1}{2}(I'-U')$	·9989516	·8979116	·9721484	·9980535	·9910944	·9997553	·9826681
$L\sin U$	67.00	63.51571	58.69759	62.45424	31.95938	33.79815	34.41900
$[()/\cos\frac{1}{2}(I-U)]$ PA	68.55655	65.44662	65.44218	62.61628	33.68029	33.80644	34.51571
$PA\cos\frac{1}{2}(I'-U')$	68.48467	58.76438	63.61951	62.49440	33.38035	33.79817	33.91749
$[()/\sin U']$ L'	410.3050	158.4139	402.8146	420.8519	868.4566	879.7529	464.0993
$-l'$							462.3150
$L'-l'$							1.7843
H'_{QM}							·1304

Computer:

TRANSFER COEFFICIENTS.

System Booth telephoto I

Date:

Axial Pencil Ray PM

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.779047	.5477326	1.650987	.6870663	.9967817	1.622965	
D_+	10.43728	.03902219	18.12294	196.6269	16.09535	12.81818	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.342742	1.316512	2.310033	.8793467	1.564668	1.622965	
$(\partial U_k'/\partial p_+)D_+$	.056625	.000340	.093534	.5198363	.066792	.058255	
$\partial U_k'/\partial U'$	2.399267	1.316852	2.403567	1.399183	1.497866	1.569710	1.0
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	430.9930	242.1684	412.4773	16.48551	12.42656		
$(\partial p_k'/\partial p_+)D_+$	17.5948	.0922	29.6515	233.3513	15.65461		
$\partial p_k'/\partial U'$	448.5878	242.2606	442.1288	249.8368	28.08117	12.46668	0.0
$\partial U'/\partial p$	$.0^{2}2831629$	.001897759	.001252093	.001799495	.004764388	$.0^{2}3104123$	
$\partial p'/\partial p$	1.061916	.9078849	1.106034	.9998141	1.082628	1.0	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	.00679383	$.0^{2}249907$	$.0^{2}300949$	$.0^{2}251782$	$.0^{2}713641$	00000467	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	.00575105	$.0^{2}791480$	$.0^{2}570835$	$.0^{2}264328$	$.0^{2}449264$	.00415462	
$\partial U_k'/\partial p$	.00104278	$.0^{2}541573$	$.0^{2}871784$	$.0^{2}516110$	$.0^{2}264377$	.00414975	.004154615
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.2702342	.459752	.558586	.449880	.1337896	.0000387	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.7901398	2.145516	1.809617	1.186551	1.052982	.9725782	
$\partial p_k'/\partial p$	.5199056	1.685764	2.363203	1.636131	1.186772	.9726169	.9725782
$-S_k'.\partial U_k/\partial U'$	1121.579	615.5855	1128.589	654.0726	700.2037	733.7884	467.4675
$C(U')$	1570.167	867.8461	1565.715	903.9094	728.2849	746.2551	467.4675
$C(U')\sec U_k'$	1574.377	860.1460	1569.916	906.3328	730.2374	748.2558	468.7208
$-S_k'.\partial U_k'/\partial p$	.4874658	2.531678	4.075307	2.412647	1.235877	1.939873	1.942147
$C(p)$	.0324398	4.217442	6.438510	4.048778	2.422649	.9672561	.9695688
$C(p)\sec U_k'$	.03252677	4.228749	6.455772	4.059633	2.429144	.9698493	.9721682
$\partial U'/\partial c$	30.69277	52.11439	80.87665	40.66128	14.98154	.1057744	21.82717
$C(c)\sec U_k'$	48321.99	44825.98	48473.75	36852.65	10940.086	81.39108	10230.85
$-\partial l'/\partial c\tan U_k'$	37640.36	32549.09	34359.68	33151.18	8487.046	76.93580	9060.009
$\partial H'/\partial c$	10681.63	12276.89	14114.07	3701.47	2458.034	4.45528	1170.841
$\partial U'/\partial n$	.4284669	.364051	.537016	.01468771	.4963085	.005837999	.1958049
$C(n)\sec U_k'$	674.5684	313.1370	843.0700	13.31196	362.4230	4.368317	91.77783
$-\partial l'/\partial n\tan U_k'$	610.8742	256.3331	688.4078	16.11413	333.5646	2.002263	86.46793
$\partial H'/\partial n$	63.6942	56.8039	154.6622	2.80216	28.8584	2.366054	5.30990
$\partial p/\partial d$		.1669116	.3709546	.1579375	.148495	.0384364	.0384178
$C(d)\sec U_k'$		.7058273	2.394798	.6411683	.3607157	.03727752	.03734856
$-\partial l'/\partial d\tan U_k'$		.6477502	1.607619	.5038500	.2560674	.02618429	.02625556
$\partial H'/\partial d$		.1580771	.787879	.1373183	.1046483	.01109323	.01109301
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							
$l' =$							462.3150
$-x' =$							3.9024
$l' - x' =$							466.2174
$O/\cos U_k' = S_k'$							467.4675
$\sec U_k' =$							1.002681
$\tan U_k' =$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: Q

System: Booth Telep

Pencil: Axial

	Surface	1	2	3	4	5	6	7
Y= 59.0	L		383.8743	163.9795	398.3944	220.7944	900.1832	916.7263
	r	161.78	432.35	420.12	361.8	98.39	1036.8	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.00	1.61517	1.00	1.65068	1.00	1.57338	1.57846
	N'	1.61517	1.00	1.65068	1.00	1.57338	1.57846	1.00
$[r_{-1}/r]$	a		·3741876	1.029111	1.161194	3.677203	·0948978	6.79245
	$r_{-1}—d'_{-1}—r$							
$[()/r]$	β		1.305331	·0295392	2.181592	2.587255	·8946181	5.898323
	$\alpha \sin I'_{-1}$		·0844886	·4551159	·3134971	·0843150	·0251182	·0293753
	$+\beta \sin U$		·1893163	·0094684	·2996065	·3321345	·0294568	·1941297
	$\sin I$	·3646928	·2738051	·4456476	·0138907	·4164525	·0043387	·1647546
$[N/N']$	n	·6191299	1.61517	·6058109	1.65068	·6355744	·9967817	1.57846
	$\sin I'$	·2257922	·4422418	·2699782	·0229291	·2646865	·0043247	·2600585
$[Pl.\tan U]$	$\sin I'+\sin U'$	·3708254	·1217049	·1326443	·1513024	·2976132	·0285880	·1964486
$[Pl.\cot U']$	$r()$	59.99213	52.61911	55.72652	54.74121	29.28216	29.64004	29.83327
$[L\tan U\cot U'][()/\sin U']$	L'	413.6441	164.1593	405.7740	426.4221	889.3135	900.5654	461.7446
	U		8.3392	18.6954	7.8936	7.3756	1.8869	1.8861
	$+I$	21.3887	15.8908	26.4648	0.7959	24.6108	0.2486	9.4830
	$U+I$	21.3887	7.5516	7.7694	8.6895	17.2352	1.6383	11.3691
	$—I'$	13.0495	26.2470	15.6630	1.3139	15.3483	0.2478	15.0735
	U'	8.3392	18.6954	7.8936	7.3756	1.8869	1.8861	3.7045
	$\sin U$		·1450332	·3205369	·1373339	·1283733	·0329267	·0329127
	$\cos I$	·9311277	·9617853	·8952083	·9999035	·9091576	·9999905	·9863346
	$\cos I'$	·9741753	·8968959	·9628663	·9997370	·9643346	·9999906	·9655929
	$\sin U'$	·1450332	·3205369	·1373339	·12837330	·0329267	·0329127	·0646099
	$\cos U'$	·9894268	·9472361	·9905248	·9917259	·9994578	·9994583	·9979106
	$\tan U'$				·1294443			·0647452
	$1—\cos(U+I)$	·0688723	·0086731	·0091798	·0114784	·0449035	·0004088	·0196223
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.046232	·9325323	1.075578	·9998335	1.060690	1.00	·9789709
$[n/()]$	$\partial U'/\partial U$	·5917711	1.732026	·5632422	1.650955	·5992084	·9967817	1.612367
	$1—\partial U'/\partial U$	·4082289	·732026	·4367578	·650955	·4007916	·0032183	·612367
$[()/\cos I]$	e	·4384242	·7611117	·487884	·6510178	·4408384	·003218331	·6208512
$[e/r]$	$\partial U'/\partial p$	·00271000	·00176041	·00161297	·00179939	·00448052	·0 ^{5} 310410	·004067421
$[eL\sin U]$	$\partial U'/\partial c$	26.86703	42.37453	25.64391	35.61918	12.49517	·09639152	18.73229
$[—\sin I/\cos I']$	$\partial U'/\partial n$	·3743606	·3052808	·4628344	·01389435	·4318548	·004338741	·1706263
$[r(1—\cos(U+I))]$	X	11.14216	3.749815	3.856618	4.152885	4.418055	·4238438	2.995148
	$d'+X_{+1}—X$	14.87803	·073197	15.3895	197.0591	14.86421	13.58870	
$[()/\cos U']$	D'	15.03702	·0772743	15.53671	198.7032	14.87227	13.59606	
	$\frac{1}{2}(I—U)$	10.6944	12.1150	22.5801	3.5488	15.9932	1.0678	3.79844
	$\cos\frac{1}{2}(I—U)$	·9826310	·9777283	·9233437	·9980824	·9612944	·9998264	·9978033
	$\frac{1}{2}(I'—U')$	2.3552	22.4712	11.7783	3.0308	6.7307	1.0670	9.38899
	$\cos\frac{1}{2}(I'—U')$	·9991553	·9240718	·9789448	·9986013	·9931080	·9998266	·9866035
	$L\sin U$	59.000	55.67452	52.56148	54.71306	28.34411	29.64006	30.17194
$[()/\cos\frac{1}{2}(I—U)]$	PA	60.04288	56.94273	56.92515	54.81818	29.48536	29.64521	30.23836
	$PA\cos\frac{1}{2}(I'—U')$	59.99216	52.61917	55.72658	54.74151	29.28215	29.64007	29.83327
$[()/\sin U']$	L'	413.6443	164.1595	405.7744	426.4244	889.3132	900.5663	461.7446
	$-l'$			$-l'$	73.15			462.3150
	$L'-l'$				353.2744			0.5704
	H'_Q			Y_o	45.7294			·03693

Computer: **TRANSFER COEFFICIENTS.** System Booth Telephoto

Date: Axial Pencil Ray Q

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·732026	·563242	1·650965	·599208	·996782	1·612367	
D_+	15·03702	·0772743	15·53671	198·7032	14·87227	13·59606	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·273620	1·312083	2·254463	·8938002	1·552055	1·612367	
$(\partial U_k'/\partial p_+)D_+$	·076047	·000611	·075056	·4717512	·060420	·055301	
$\partial U_k'/\partial U'$	2·349667	1·312694	2·329519	1·365551	1·491635	1·557066	1·00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	423·7677	244·4932	409·0867	16·6744	13·2673		
$(\partial p_k'/\partial p_+)D_+$	24·8562	0·1727	24·9952	231·1135	14·5601		
$\partial p_k'/\partial U'$	448·6239	244·6659	434·0819	247·7879	27·8274	13·3102	0·00
$\partial U'/\partial p$	·00271000	·00176041	·00161297	·00179939	·00448052	·0³31041	
$\partial p'/\partial p$	1·046232	·932532	1·075578	·9998335	1·06069	1·00	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·0063676	·00231087	·0027053	·00245715	·00668330	·0000048	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·0052911	·00736819	·00519601	·00237375	·00430915	·00406742	
$\partial U_k'/\partial p$	·0010765	·0050573	·00790127	·0048309	·00237415	·00406259	·00406742
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·215772	·430711	·504098	·445866	·124681	·000041	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·729424	2·083713	1·730369	1·162915	1·038428	·978971	
$\partial p_k'/\partial p$	·513652	1·653002	2·234467	1·608781	1·163109	·979012	·978971
$-S_k'.\partial U_k/\partial U'$	1095·613	612·089	1086·218	636·735	695·526	726·036	466·284
$C(U')$	1544·237	956·755	1520·300	884·523	723·354	739·346	466·284
$C(U')\sec U_k'$	1547·471	968·549	1523·484	886·375	724·868	740·894	467·261
$-S_k'.\partial U_k'/\partial p$	·501941	2·358149	3·684239	0·252573	1·107029	1·894322	1·896575
$C(p)$	·011711	4·011151	5·918706	3·861354	2·270138	·915310	·917604
$C(p)\sec U_k'$	·0117352	4·019550	5·931100	3·869440	2·274892	·917226	·919526
$\partial U'/\partial c$	25·86703	42·37450	26·64391	35·61918	12·49517	·0953915	18·73229
$C(c)\sec U_k'$	40028·48	56380·69	39068·09	31671·96	9057·351	70·6750	8752·865
$-\partial l'/\partial c \tan U_k'$	33267·17	28758·78	30358·53	29290·76	7498·737	67·9767	8004·980
$\partial H'/\partial c$	6771·31	7621·81	8709·56	2281·20	1558·614	2·6980	747·885
$\partial U'/\partial n$	·3743605	·3052808	·4628344	·0138944	·4318548	·00433874	·1706253
$C(n)\sec U_k'$	519·3120	262·0984	705·1208	12·3156	313·0378	3·214547	79·7265
$-\partial l'/\partial n \tan U_k'$	539·7385	226·4834	608·2433	14·2377	294·7215	1·769102	76·3988
$\partial H'/\partial n$	39·5735	35·6150	96·8775	1·9220	18·3165	1·44546	3·32769
$\partial p/\partial d$		·1460832	·320637	·1373339	·1283733	·032927	·0329127
$C(d)\sec U_k'$		·582968	1·901136	·531405	·292035	·0302012	·0302641
$-\partial l'/\partial d \tan U_k'$		·483966	1·420325	·445177	·226249	·0231352	·0231981
$\partial H'/\partial d$		·099003	·480811	·086228	·065787	·0070661	·0070660
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: 'Z'

System: Booth Teleph

Pencil: Axial

	Surface	1	2	3	4	5	6	7
Y = 51.28	L		386.6503	168.5370	400.8924	226.5457	915.7948	932.2947
	r	161.78	432.35	420.12	361.8	98.39	1036.80	162.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.00	1.61517	1.00	1.65068	1.00	1.57338	1.57846
	N'	1.61517	1.00	1.65068	1.00	1.57338	1.57846	1.00
$[r_{-1}/r]$	a		.374188	1.029111	1.161194	3.677203	.094898	6.79245
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β		1.305331	.0295392	2.181592	2.587255	.894618	6.898323
	$\alpha \sin I'_{-1}$		.0734335	.3926832	.2704662	.0774732	.0218185	.0221638
	$+\beta \sin U$		.1627509	.0081055	.2677027	.2842721	.0250920	.1653731
	$\sin I$	.3169737	.2361845	.3844777	.0127636	.3617453	.0032735	.1432089
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6355744	.9967817	1.57846
	$\sin I'$	.1962479	.3814781	.2329208	.0210685	.2299161	.0032630	.2260495
$[Pl.\tan U]$	$\sin I'+\sin U'$	.3209296	.1070793	.1147948	.1309425	.2579638	.0247743	.1698029
$[Pl.\cot U']$	$r()$	51.91999	46.29574	48.22759	47.3750	25.38106	25.68599	25.91871
$[L\tan U\cot U']$ $[()/\sin U']$	L'	416.4203	168.7170	408.2724	431.1757	904.9248	916.1365	460.8050
	U		07-09.7424	15-55.5713	6-47.0378	6-18.4832	1-36.4338	1-36.3977
	$+I$	18-28.8003	13-39.6873	22-36.6791	0-43.8790	21-12.4451	0-11.2534	8-14.0146
	$U+I$	18-28.8003	6-29.9449	6-41.1078	7-30.9168	14-53.9619	1-25.1804	9-50.4123
	$-I'$	11-19.0579	22-25.5162	13-28.1456	1-12.4336	13-17.5281	0-11.2173	13-03.8761
	U'	7-09.7424	15-55.5713	6-47.0378	6-18.4832	1-36.4338	1-36.3977	3-13.4638
	$\sin U$		.1246817	.2743988	.1181260	.1098740	.0280477	.0280373
	$\cos I$	.9484343	.9717082	.9231343	.9999185	.9322770	.9999946	.9896925
	$\cos I'$	.9805543	.9243779	.9724957	.9997780	.9732105	.9999947	.9741158
	$\sin U'$	.1246817	.2743988	.1181260	.1098740	.0280477	.0280373	.0562466
	$\cos U'$	.9921968	.9616160	.9929986	.9939455	.9996066	.9996069	.9984169
	$\tan U'$							.0563358
	$1-\cos(U+I)$	.0515657	.0064263	.0067992	.0085899	.0336211	.0003070	.0147118
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.033866	.9512917	1.053472	.999860	1.043907	1.000	.984261
$[n/()]$	$\partial U'/\partial U$	.598849	1.697870	.575061	1.650912	.608842	.996782	1.603700
	$1-\partial U'/\partial U$	.401151	.697870	.424939	.650912	.391158	.0032183	.603700
$[()/\cos I]$	e	.422961	.718189	.460322	.650965	.419573	.0032183	.609987
$[e/r]$	$\partial U'/\partial p$	.00261442	.00166113	.00109569	.0017992	.00426438	.0310409	.00399625
$[eL\sin U]$	$\partial U'/\partial c$	21.68944	34.62261	21.28820	30.82698	10.39769	.0826656	15.94448
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.3232597	.2555064	.3953516	.0127663	.3717030	.0032735	.1470142
$[r(1-\cos(U+I))]$	X	8.34230	2.778411	2.86648	3.107826	3.30798	3.182976	2.24561
	$d'+X_{+1}-X$	18.64929	.101931	13.34431	199.2142	13.85968	14.23269	
$[()/\cos U']$	D'	18.79596	.1059997	13.43840	200.4277	13.86513	14.23829	
	$\frac{1}{2}(I-U)$	9-14.400	10-24.7148	19-16.1251	3-01.5794	13-45.4641	0-53.8436	3-18.8085
	$\cos\frac{1}{2}(I-U)$	.9870244	.9835339	.9439811	.9986054	.9713100	.9998773	.9983282
	$\frac{1}{2}(I'-U')$	2-04.6577	19-10.5437	10-07.5917	2-33.0248	5-50.5471	0-53.8074	8-08.6699
	$\cos\frac{1}{2}(I'-U')$	.9993426	.9445156	.9844219	.9990098	.9948055	.9998775	.9899139
	$L\sin U$	51.28	48.20822	46.24635	47.35681	24.78161	25.68594	26.13903
$[()/\cos\frac{1}{2}(I-U)]$	PA	51.95414	49.01531	48.99076	47.42194	25.51360	25.68909	26.18280
	$PA\cos\frac{1}{2}(I'-U')$	51.91999	46.29572	48.22758	47.37498	25.38106	25.68594	25.91872
$[()/\sin U']$	L'	416.4203	168.7169	408.2724	431.1755	904.9248	916.1347	460.8051
	$-l'$							462.3150
	$L'-l'$							1.5099
	H'_2							0.08506

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

Axial Pencil Ray Zonal

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.69787	·575061	1.65091	·608842	·996782	1.60370	
D_+	$\overline{18.7960}$	$\overline{0.10600}$	$\overline{13.4384}$	$\overline{200.428}$	$\overline{13.8651}$	$\overline{14.2383}$	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.22208	1.30797	2.21289	0.905032	1.541822	1.603700	
$(\partial U_k'/\partial p_+)D_+$	·09004	·00078	·06160	0.435373	$\overline{.055342}$	$\overline{.056900}$	
$\partial U_k'/\partial U'$	2.31212	1.30875	2.27449	1.340405	1.486480	1.546800	1.0000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	$\overline{418.3416}$	$\overline{246.1649}$	$\overline{406.7218}$	$\overline{16.81415}$	$\overline{13.96909}$		
$(\partial p_k'/\partial p_+)D_+$	$\overline{30.6133}$	$\overline{0.2271}$	$\overline{21.3455}$	$\overline{229.5478}$	$\overline{13.64751}$	$\overline{14.01419}$	
$\partial p_k'/\partial U'$	$\overline{448.9549}$	$\overline{246.3920}$	$\overline{428.0673}$	$\overline{246.3619}$	$\overline{27.6166}$	$\overline{14.01419}$	0.0000
$\partial U'/\partial p$	·00261442	·00166113	$\overline{.00109569}$	$\overline{.00179924}$	$\overline{.00426438}$	$\overline{.000003104}$	·00399625
$\partial p'/\partial p$	1.033866	·9512917	1.053472	·9998595	1.043907	1.000	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·00604485	·00217399	$\overline{.00249214}$	$\overline{.00241171}$	$\overline{.00633892}$	$\overline{.0000048}$	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	$\overline{.00495251}$	$\overline{.00696427}$	$\overline{.00482872}$	$\overline{.00217191}$	·00416670	·00399625	
$\partial U_k'/\partial p$	·00109234	$\overline{.00479028}$	$\overline{.00732086}$	$\overline{.00458362}$	$\overline{.00217222}$	·00399145	·00399625
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	$\overline{1.1737571}$	$\overline{.409289}$	·469029	·4482264	·1177678	·00004350	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.6838751	2.038006	1.673328	1.145129	1.027522	·9842611	
$\partial p_k'/\partial p$	·5101180	1.628717	2.142357	1.588393	1.145290	·9843046	·9842611
$-S_k'.\partial U_k/\partial U'$	$\overline{1075.822}$	$\overline{608.9558}$	$\overline{1059.313}$	$\overline{623.6867}$	$\overline{691.6550}$	$\overline{719.7217}$	$\overline{465.2972}$
$C(U')$	$\overline{1624.777}$	$\overline{855.3478}$	$\overline{1486.380}$	$\overline{870.0486}$	$\overline{719.2716}$	$\overline{733.7359}$	$\overline{465.2972}$
$C(U')\sec U_k'$	$\overline{1627.195}$	$\overline{856.7044}$	$\overline{1488.737}$	$\overline{871.4285}$	$\overline{720.4124}$	$\overline{734.8996}$	$\overline{466.0352}$
$-S_k'.\partial U_k'/\partial p$							
$C(p)$							
$C(p)\sec U_k'$							
$\partial U'/\partial c$							
$C(c)\sec U_k'$							
$-\partial l'/\partial c \tan U_k'$							
$\partial H'/\partial c$							
$\partial U'/\partial n$	$\overline{.3232597}$	·2555064	·3953516	$\overline{.0127663}$	·371703	$\overline{.00327352}$	·1470142
$C(n)\sec U_k'$	493.6806	$\overline{218.8935}$	$\overline{588.5746}$	11.12494	$\overline{267.7795}$	2.405706	$\overline{68.51379}$
$-\partial l'/\partial n \tan U_k'$	$\overline{469.6348}$	197.0667	629.2419	$\overline{12.38840}$	256.4416	$\overline{1.539323}$	66.47579
$\partial H'/\partial n$	24.0458	$\overline{21.8268}$	$\overline{59.3327}$	$\overline{1.26346}$	$\overline{11.3379}$	·866383	$\overline{2.03800}$
$\partial p/\partial d$							
$C(d)\sec U_k'$							
$-\partial l'/\partial d \tan U_k'$							
$\partial H'/\partial d$							
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							
$C(U')$ cosec U_k'	$\overline{27108.78}$	$\overline{15207.10}$	$\overline{26426.13}$	$\overline{15468.46}$	$\overline{12787.82}$	$\overline{13044.98}$	$\overline{8272.449}$
$'/\partial n$ = $C(n)$ cosec U_k'	8763.176	$\overline{3885.511}$	$\overline{10447.61}$	197.4755	$\overline{4753.271}$	42.7030	$\overline{1216.167}$

200

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: QZ

System: Booth Telep

Pencil: Axial

	Surface	1	2	3	4	5	6	7
Y = 40.0	L		389.959	173.7380	403.9032	231.3400	932.4020	948.8666
	r	161.78	432.35	420.12	361.8	98.39	1036.8	162.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N							
	N'							
$[r_{-1}/r]$	α		.3741876	1.029111	1.161194	2.677203	.0948978	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β		1.305331	.0295392	2181592	2.587255	.8946181	6.898323
	$\alpha \sin I'_{-1}$		.0572805	.3035209	.2091454	.0642885	.0170087	.0146068
	$+\beta \sin U$		.1253226	.0062129	.1985539	.2177108	.0191659	.1263214
	$\sin I$	.2472494	.1826031	.2973080	.0105914	.2819993	.0021572	.1117164
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6355744	.9967817	1.57846
	$\sin I'$	.1530795	.2949350	.1801124	.0174830	.1792315	.0021503	.1763399
$[Pl.\tan U]$	$\sin I'+\sin U'$	.2490878	.0846069	.0890991	.1016304	.2006550	.0192662	.1324555
$[Pl.\cot U']$	$r()$	40.29742	36.57979	37.43231	36.76988	19.74246	19.97520	20.21801
$[L \tan U \cot U']$ $[()/\sin U']$	L'	419.7285	173.9178	411.2840	436.9699	921.5324	932.7014	466.7106
	U	.000000	5.50936	12.14158	5.22190	4.82700	1.22767	1.22717
	$+I$	14.31480	10.52142	17.29599	0.60685	16.37957	0.12360	6.41427
	$U+I$	14.31480	5.01206	5.15441	5.82875	11.55257	1.10397	7.64144
	$-I'$	8.80544	17.15364	10.37631	1.00175	10.3250	0.12320	10.15664
	U'	5.50936	12.14158	5.22190	4.82700	1.22757	1.22717	2.51520
	$\sin U$	.000000	.0960083	.2103281	.0910133	.0841474	.0214235	.0214165
	$\cos I$	.9689519	.9831867	.9547816	.9999439	.9594146	.9999977	9937401
	$\cos I'$	.9882139	.9555173	.9836461	.9998472	.9838069	.9999977	.9843293
	$\sin U'$	.0960083	.2103281	.0910133	.0841474	.0214235	.0214165	.0438844
	$\cos U'$	.9953805	.9776308	.9958497	.9964633	.9997705	.9997707	.9990366
	$\tan U'$							.04392672
	$1-\cos(U+I)$	.0310481	.0038237	.0040438	.0051702	.0202586	.0001856	.0088804
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.019879	.9718574	1.030232	.999033	1.025424	1.00000	.9905299
$[n/()]$	$\partial U'/\partial U$	.6070621	1.661941	.5880335	1.650840	.6198162	.9967817	1.593561
	$1-\partial U'/\partial U$	.3929379	.661941	.4119665	.650840	.3801838	.0032183	.593551
$[()/\cos I]$	e	.4055288	.6732607	.4314772	.6508765	.3962664	.003218307	.5972900
$[e/r]$	$\partial U'/\partial p$	.002506668	.001657212	.001027033	.001798995	$0^2$4027507	.$0^5$3104077	.$0^2$3913063
$[eL \sin U]$	$\partial U'/\partial c$	16.22115	25.20641	15.76703	23.92658	7.713993	.06428668	12.13777
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.2501983	.1911039	.3022610	.01059302	.2866409	.002157205	.1134949
$[r(1-\cos(U+I))]$	X	5.022962	1.653177	1.698881	1.870578	1.993244	.1924301	1.355504
	$d'+X_{+1}-X$	23.09386	.134296	10.94946	201.7662	12.67081	14.99693	
$[()/\cos U']$	D'	23.20104	.1373688	10.99509	202.4844	12.67372	15.00037	
	$\frac{1}{2}(I-U)$	7.15740	8.01539	14.71879	2.30758	10.60329	0.67559	2.59355
	$\cos \frac{1}{2}(I-U)$	.9922076	.9902307	.9671845	.9991891	.9829248	.9999305	.9989757
	$\frac{1}{2}(I'-U')$	1.64804	14.64761	7.79910	1.91261	4.54872	0.67519	6.33592
	$\cos \frac{1}{2}(I'-U')$	.9995864	.9674991	.9907476	.9994429	.9968503	.9999306	.9938920
	$L \sin U$	40.000	37.4393	36.54198	36.76056	19.46666	19.97531	20.32140
$[()/\cos \frac{1}{2}(I-U)]$	PA	40.31414	37.80866	37.78181	36.79039	19.80483	19.97670	20.34224
	$PA \cos \frac{1}{2}(I'-U')$	40.29747	36.57984	37.43224	36.76989	19.74244	19.97531	20.21799
$[()/\sin U']$	L'	419.7290	173.9180	411.2832	436.9700	921.5320	932.7066	460.7102
	$-l'$							462.315
	$l'-l$							1.6044
	$l'-l \tan U'_k = H'_{qz}$							.070476

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

Axial Pencil Ray QZ

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·661941	·5880335	1·65084	·6198162	·9967817	1·593551	
D_+	23·20104	·1373688	10·99509	202·4844	12·67372	15·00037	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·166766	1·302832	2·168108	·9176641	1·529914	1·593551	
$(\partial U_k'/\partial p_+)D_+$	·104489	·000924	·047467	·3957720	·049533	·058697	
$\partial U_k'/\partial U'$	2·271265	1·303756	2·215575	1·313336	1·480381	1·534854	1·00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	412·4894	247·9162	404·3780	16·9611	14·81049		
$(\partial p_k'/\partial p_+)D_+$	37·1864	0·2812	17·2242	227·9918	12·65428		
$\partial p_k'/\partial U'$	449·6758	248·1974	421·6022	244·9529	27·36477	14·85831	0·00
$\partial U'/\partial p$	·002506668	·001557212	·001027033	·001798995	·004027507	·0⁵3104077	
$\partial p'/\partial p$	1·019879	·9718574	1·030232	·9999033	1·025424	1·00	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	00569328	·00203022	·00227547	·00236268	·00596224	·0000048	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·00459316	·00653385	·00444758	·00195439	·00400766	·00391306	
$\partial U_k'/\partial p$	·00110012	·00450363	·00672305	·00431707	·00195458	·00390830	·00391306
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·1271879	0·386496	·432999	·440669	·110212	·000046	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·6346509	1·989285	1·613891	1·125863	1·015760	·9905299	
$\partial p_k'/\partial p$	·5074629	1·602789	2·046890	1·566532	1·125972	·9905760	·9905299
$—S_k'.\partial U_k/\partial U'$	1054·129	605·0961	1028·287	609·5424	687·0709	712·3528	464·1176
$C(U')$	1503·805	853·2935	1449·889	854·4953	714·4357	727·2111	464·1176
$C(U')\sec U_k'$	1505·255	854·1161	1451·287	855·3190	715·1244	727·9121	464·6650
$—S_k'.\partial U_k'/\partial p$	·5105851	2·090214	3·120286	2·003628	·907156	1·813911	1·816121
$C(p)$	·0031222	3·693003	5·167176	3·670160	2·033127	·823335	·8265911
$C(p)\sec U_k'$	·00312521	3·696563	5·172157	3·573602	2·035087	·8241287	·826387
$\partial U'/\partial c$	16·22115	25·20641	15·76703	23·92658	7·713983	·0642867	12·13777
$C(c)\sec U_k'$	24416·97	21529·2	22882·49	20464·86	5516·457	46·79506	5638·783
$—\partial l'/\partial c \tan U_k'$	22563·49	19511·64	20596·90	19872·46	5087·555	46·11913	5431·018
$\partial H'/\partial c$	1863·48	2017·66	2285·59	592·40	428·902	0·67592	207·765
$\partial U'/\partial n$	·2501983	·1911039	·302251	·0105930	·2866409	·00215721	·1134949
$C(n)\sec U_k'$	376·6122	163·2249	438·6529	9·060411	204·9839	1·570256	52·72576
$—\partial l'/\partial n \tan U_k'$	366·1882	153·6588	412·6667	9·659604	199·9551	1·200256	51·83316
$\partial H'/\partial n$	10·4240	9·5661	25·9872	·599193	6·0288	·370000	·89261
$\partial p/\partial d$		·0960083	·2103281	·0910183	·0841474	·0214235	·0214166
$C(d)\sec U_k'$		·3549007	1·087850	·3252453	·1712473	·01765572	·01769832
$—\partial l'/\partial d \tan U_k'$		·3283486	·9636266	·3020327	·1534995	·01569616	·01573888
$\partial H'/\partial d$		·0265521	·1242234	·0232126	·0177478	·00195956	·00195944
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

PARAXIAL RAY TRACE.

System: *Booth Telephoto*

	Surface	1	2	3	4	5	6	7
	y	1·0						
	l		394·9935	181·2435	408·5512	240·4246	$\overline{963{\cdot}9667}$	$\overline{970{\cdot}3646}$
	r	161·78	$\overline{432{\cdot}35}$	$\overline{420{\cdot}12}$	361·80	$\overline{98{\cdot}39}$	$\overline{1036{\cdot}8}$	$\overline{152{\cdot}64}$
	d'	29·77	0·18	7·38	205·63	10·87	16·16	
	N	1·00	1·61517	1·00	1·65068	1·00	1·57338	1·57846
	N'	1·61517	1·00	1·65068	1·00	1·57338	1·57846	1·00
$[r_{-1}/r]$	a		$\overline{\cdot 3741876}$	1·029111	$\overline{1{\cdot}161194}$	$\overline{3{\cdot}677203}$	·0948978	6·792463
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β		$\overline{1{\cdot}305331}$	·0295392	$\overline{2{\cdot}181592}$	$\overline{2{\cdot}587255}$	$\overline{\cdot 8946181}$	6·898323
	$a\,.\,i'_{-1}$		$\overline{\cdot 00143201}$	$\overline{\cdot 00748832}$	·00516124	$\overline{\cdot 00175182}$	$\overline{\cdot 00042490}$	·00028216
	$\beta\,.\,u$		$\overline{\cdot 00307308}$	·00015141	$\overline{\cdot 00487263}$	$\overline{\cdot 0052983}$	·00046656	$\overline{\cdot 00307533}$
	i	·00618123	$\overline{\cdot 00460509}$	$\overline{\cdot 00733690}$	·00028861	·00704465	·00004167	$\overline{\cdot 00279817}$
$[N/N']$	n	·6191299	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846
	i'	·00382698	$\overline{\cdot 00727649}$	$\overline{\cdot 00444477}$	·00047640	$\overline{\cdot 00447740}$	·00004153	$\overline{\cdot 00440891}$
	$r(u+i)$	·9999994	·9299157	·9289904	·9126066	·4918437	·4975126	·6059344
$[()/u']$	l'	424·7635	181·4240	415·9311	446·0543	$\overline{943{\cdot}0966}$	$\overline{954{\cdot}2043}$	462·315
	u		·00235425	·00512566	·00223362	·00204573	$\overline{\cdot 00062152}$	$\overline{\cdot 00052139}$
	$u+i$	·00618123	$\overline{\cdot 00216084}$	$\overline{\cdot 0022125}$	·00252213	$\overline{\cdot 00499892}$	$\overline{\cdot 00047985}$	$\overline{\cdot 00331456}$
	u'	·00235425	·00512565	·00223352	·00204573	$\overline{\cdot 00052152}$	$\overline{\cdot 00052139}$	·00109435
	lu	1·00	·9299134	·9289907	·9125073	·4918438	·4975127	·6059384
$[()/u']$	l'	424·7637	181·4235	415·9312	446·0546	$\overline{943{\cdot}0967}$	$\overline{954{\cdot}2046}$	462·3150
	$1-n$	·3808701	$\overline{\cdot 61517}$	·3941891	$\overline{\cdot 65068}$	·3644256	·0032183	$\overline{\cdot 67846}$
$[()/r]$	$\partial u'/\partial p$	$\cdot 0^{2}235425$	$\cdot 0^{2}142285$	$\overline{\cdot 0^{3}938277}$	$\overline{\cdot 0^{2}179845}$	$\overline{\cdot 0^{2}370389}$	$\overline{\cdot 0^{5}310407}$	$\cdot 0^{2}378970$
$[(1-n)lu]$	$\partial u'/\partial c$	·3808701	$\overline{\cdot 5720548}$	·3661980	$\overline{\cdot 5937502}$	·1792405	·00160115	$\overline{\cdot 2926661}$

Computer:

Date:

PARAXIAL TRANSFER COEFFICIENTS.

System *Booth Telephoto*

Surface	1	2	3	4	5	6	7
$\partial u_+'/\partial u_+$	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846	
d_+	29·77	0·18	7·38	205·63	10·87	16·16	
$(\partial u_k'/\partial u_+')\,(\partial u_+'/\partial u_+)$	2·092334	1·294959	2·108329	·9350520	1·512335	1·578460	
$(\partial u_k'/\partial p_+)d_+$	·122735	·001074	·029234	·3421971	·041143	·061242	
$\partial u_k'/\partial u'$	2·216049	1·296033	2·137563	1·277249	1·47192	1·517218	1·00
$(\partial p_k'/\partial u_+')\,(\partial u_+'/\partial u_+)$	404·6900	250·2092	401·6679	17·1469	16·1080		
$(\partial p_k'/\partial p_+)d_+$	46·6978	0·3465	11·3475	226·1879	10·8705	←	
$\partial p_k'/\partial u'$	451·3878	250·6557	413·0154	243·3348	26·9785	16·1600	0·00
$\partial u'/\partial p$	·00235425	·00142285	·000938277	·00179845	·00370389	$\cdot 0^{5}310407$	
$\partial p'/\partial p$							
$(\partial u_k'/\partial u')\,(\partial u'/\partial p)$	·00521713	·00184406	·00200663	·00229707	·00544913	·0000047	
$(\partial u_k'/\partial p_+)\,(\partial p'/\partial p)$	·00412278	·00596684	·00396121	·00166414	·00378499	·00378970	
$\partial u_k'/\partial p$	·00109435	·00412278	·00596684	·00396121	·00166414	·00378499	·0037897
$(\partial p_k'/\partial u')\,(\partial u'/\partial p)$	1·062678	·356504	·387623	·437626	·099925	·000060	
$(\partial p_k'/\partial p_+)\,(\partial p'/\partial p)$	1·568620	1·925124	1·537601	1·099975	1·000060	1·00	
$\partial p_k'/\partial p$	·5059416	1·568620	1·925124	1·537601	1·099975	1·000050	1·000
$-l'.\partial u_k'/\partial u'$	1024·513	599·1755	988·2274	590·4914	680·1541	701·4326	462·3150
$C(u')$	1475·901	849·7312	1401·243	933·8262	707·1326	717·5926	462·3150
$\partial l'/\partial u'$	1348655·	776471·0	1280434·	761937·4	646166·7	655724·9	422456·2
$-l'.\partial u_k'/\partial p$	·605934	1·906023	2·758560	1·831327	·769357	1·749858	1·752036
$C(p)$	·000007	3·474643	4·683684	3·368928	1·869332	·749808	·752036
$\partial l'/\partial p$	·0065792	3175·075	4279·877	3078·474	1708·166	685·1629	687·1988
$\partial u'/\partial c$	·380871	·5720548	·366198	·6937602	·1792405	·001601145	·2926651
$\partial l'/\partial c$	513662·4	444184·0	468892·4	452400·5	115819·2	1049·911	123638·2
$\partial u'/\partial n$	·00618123	·00450509	·0073369	·00028861	·00704465	·00004167	·00279317
$\partial l'/\partial n$	8336·347	3498·072	9394·416	219·9028	4552·018	27·32406	1179·992
$\partial p/\partial d$		·00235425	·00512565	·00223352	·00204573	·00052152	·00052139
$\partial l'/\partial d$		7·474920	21·93715	6·875833	3·494446	·3573262	·3582986
$\partial u_k'/\partial c$							
$\partial u_k'/\partial n$							
$\partial u_k'/\partial d$							
$-\partial l'/\partial c.1/u_k'$							
$\lambda'(c)$							
$-\partial l'/\partial d.1/u_k'$							
$\lambda'(d)$							
$1/u_k'$							
f/u_k'							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: a

System: Booth Telep

Pencil: 4°

	Surface	1	2	3	4	5	6	7
	L	1045·121	304·8829	125·8019	314·0521	83·4508	289·0116	274·0537
	r	161·78	432·35	420·12	361·80	98·39	1036·8	152·64
	d'	29·77	0·18	7·38	205·63	10·87	16·16	
	N	1·00	1·61517	1·00	1·65068	1·00	1·57338	1·57846
	N'	1·61517	1·00	1·65068	1·00	1·57338	1·57846	1·00
$[r_{-1}/r]$	a		·3741876	1·029111	1·161194	3·677203	·0948978	6·792463
	$r_{-1}—d'_{-1}—r$							
$[()/r]$	β		1·306331	·0295392	2·181692	2·687255	·8946181	6·898323
	$\alpha \sin I'_{-1}$		·0883030	·6259414	·4305402	·1682699	·0253788	·5720054
	$+\beta \sin U$		·2882735	·0139127	·4582622	·5890437	·0591060	·3880802
	$\sin I$	·3811574	·3765765	·6120287	·0277221	·4207737	·0844858	·1839250
$[N/N']$	n	·6191299	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846
	$\sin I'$	·2359859	·6082351	·3707737	·0457603	·2674330	·0842119	·2903183
$[Pl.\tan U]$	$\sin I' + \sin U'$	·4568291	·1372420	·1607151	·1819110	·2013657	·0184169	·1157927
$[Pl.\cot U']$	$r()$	73·90584	59·33658	67·51963	65·81540	19·81237	19·09464	17·67460
$[L\tan U \cot U']\,[()/\sin U']$	L'	334·6629	125·9818	321·4324	289·0808	299·8816	290·2142	101·2722
	U	4·00292	12·75866	28·09878	12·12579	13·16002	3·78814	3·77251
	$+I$	22·40539	22·12178	37·73633	1·58856	24·88345	4·84634	10·69846
	$U+I$	26·40831	9·36322	9·63755	10·53723	11·72343	1·05820	6·92595
	$—I'$	13·64975	37·46200	21·76834	2·62279	15·51157	4·83071	16·87700
	U'	12·75856	28·09878	12·12579	13·16002	3·78814	3·77251	10·05105
	$\sin U$	·0698073	·2208432	·4709931	·2100586	·2276713	·0660673	·0657950
	$\cos I$	·9245101	·9263866	·7908366	·9996157	·9071667	·9964249	·9829402
	$\cos I'$	·9717564	·7937570	·9287232	·9989524	·9635765	·9964479	·9569301
	$\sin U'$	·2208432	·4709931	·2100586	·2276713	·0660673	·0657950	·1745256
	$\cos U'$	·9753093	·8821369	·9776888	·9737880	·9978152	·9978332	·9846526
	$\tan U'$				·2338117			·1772469
	$1—\cos(U+I)$	·1043527	·0133232	·0141135	·0168637	·0208602	·0001706	·0070882
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1·051104	·8568322	1·174357	·9993364	1·062184	1·000023	·9736386
$[n/()]$	$\partial U'/\partial U$	·5890282	1·886048	·515866	1·651776	·5983656	·9967588	1·621364
	$1-\partial U'/\partial U$	·4109718	·886048	·484134	·651776	·4016344	·0032412	·621364
$[()/\cos I]$	e	·4445293	·9653775	·6121803	·6520266	·4427354	·00325283	·6321483
$[e/r]$	$\partial U'/\partial p$	·00274774	·00220973	·00145716	·00180274	·0044998	·0⁵313787	·00414143
$[eL \sin U]$	$\partial U'/\partial c$	32·43156	64·32683	36·2728	43·01376	8·411685	·06211023	11·39849
$[—\sin I/\cos I']$	$\partial U'/\partial n$	·3922365	·4744229	·6590001	·0277517	·4366791	·0847850	·1922032
$[r(1—\cos(U+I))]$	X	16·88218	5·760286	5·92936	6·101287	2·052435	·1768781	1·081943
	$d'+X_{+1}—X$	7·127534	·010922	19·41065	197·4763	12·74556	15·25494	
$[()/\cos U']$	D'	7·307973	·0123813	19·853361	202·8023	12·77347	15·28807	
	$\frac{1}{2}(I-U)$	9·20124	17·44018	32·91756	6·86718	19·02173	4·31724	7·18648
	$\cos\frac{1}{2}(I-U)$	·9871328	·9540304	·8394534	·9928469	·9453950	·9971625	·9921464
	$\frac{1}{2}(I'-U')$	0·44559	32·78039	16·94456	7·89140	9·64986	4·30160	13·46403
	$\cos\frac{1}{2}(I'—U')$	·9999698	·8407519	·9565872	·9905301	·9858505	·9971830	·9725163
	$L \sin U$	72·95708	67·33132	59·25183	65·96934	18·99935	19·09422	18·03136
$[()/\cos\frac{1}{2}(I—U)]$	PA	73·90807	70·57565	70·58382	66·44463	20·09673	19·14855	18·17409
	$PA \cos\frac{1}{2}(I'—U')$	73·90584	59·33661	67·51958	65·81541	19·81237	19·09461	17·67460
$[()/\sin U']$	L'	334·6530	125·9819	321·4321	289·0808	299·8816	290·2137	101·2722
	$-l'$				73·15			462·315
	$L'-l'$				215·9308			361·0428
	H'_a				50·4871			63·9934

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

4° Pencil Ray a

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.885048	.515866	1.651776	.598366	.996759	1.621364	
D_+	7.307973	.0123813	19.86361	202.8023	12.77347	15.28807	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.303016	1.221616	2.272114	.8976441	1.652999	1.621364	
$(\partial U_k'/\partial p_+)D_+$	.03742	.000113	.095972	.4779138	.052839	.063315	
$\partial U_k'/\partial U'$	2.340440	1.221728	2.368086	1.375558	1.500160	1.558049	1.000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	434.1699	230.2916	414.4863	16.3184	14.8353		
$(\partial p_k'/\partial p_+)D_+$	12.1806	.0314	31.9311	234.6153	12.4364		
$\partial p_k'/\partial U'$	446.3505	230.3230	446.4174	250.9337	27.2716	14.8835	0.000
$\partial U'/\partial p$	.00274774	.002209732	.00145716	.00180217	.00449980	.0⁵313737	
$\partial p'/\partial p$	1.051104	.8568322	1.174357	.9993364	1.062184	1.000023	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	.00643092	.00269969	.00345067	.00247899	.00675042	.0000049	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	.00538276	.00782073	.00567682	.00235499	.00439387	.00414163	
$\partial U_k'/\partial p$	.00104817	.00512104	.00912749	.00483398	.00235665	.00413664	.00414143
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.226455	.508952	.650500	.452226	.122717	.000047	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.7519349	2.175709	1.888748	1.156099	1.034150	.9735609	
$\partial p_k'/\partial p$	.5254798	1.666757	2.539248	1.608325	1.156867	.9736076	.9735385
$-S_k'.\partial U_k/\partial U'$	1101.457	574.9693	1114.468	647.3647	706.0048	733.2486	470.6197
$C(U')$	1547.808	805.2923	1560.885	898.2984	733.2764	748.1321	470.6197
$C(U')\sec U_k'$	1571.934	817.8444	1585.215	912.3002	744.7060	759.7932	477.9552
$-S_k'.\partial U_k'/\partial p$	.493290	2.410062	4.295577	2.274966	1.109039	1.946784	1.949040
$C(p)$	.032190	4.076819	6.834825	3.883291	2.265906	.973776	.975502
$C(p)\sec U_k'$	.0326920	4.140364	6.941359	3.943820	2.301225	.988345	.990707
$\partial U'/\partial c$	32.43156	64.32683	36.27280	43.01376	8.411685	.06211023	11.39849
$C(c)\sec U_k'$	50980.27	52609.34	57600.19	39241.46	6264.232	47.1909	5447.968
$-\partial l'/\partial c \tan U_k'$	91044.55	78729.79	83109.26	80186.13	20528.48	186.0924	21914.36
$\partial H'/\partial c$	40064.28	26120.45	25609.07	40944.67	14264.25	138.9015	16466.39
$\partial U'/\partial n$	.3922365	.4744229	.6590001	.0277612	.4366791	.0847850	.1922032
$C(n)\sec U_k'$	616.568	388.0041	1044.657	25.3174	325.1975	64.41904	91.86452
$-\partial l'/\partial n \tan U_k'$	1477.583	620.0189	1665.122	38.97687	806.8265	4.84308	209.1487
$\partial H'/\partial n$	861.015	232.0148	620.465	64.29427	481.6290	69.26212	117.2842
$\partial p/\partial d$	.0698073	.2208432	.4709931	.2100586	.2276713	.0660673	.065795
$C(d)\sec U_k'$	.00228214	.9143712	3.269332	.828483	.5239229	.0662973	.06518354
$-\partial l'/\partial d \tan U_k'$		1.324899	3.888290	1.218713	.6193762	.0633346	.06350696
$\partial H'/\partial d$	.00228214	.4105278	.618938	.390280	.0954633	.1286219	.1286905

$\partial U_k'/\partial c$

$\partial U_k'/\partial n$

$\partial U_k'/\partial d$

$\nu(c)$

$\gamma(c)$

$\pi(c)$

$\rho(c)$

$\nu(d)$

$\gamma(d)$

$\pi(d)$

$\rho(d)$

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: c

System: Booth Teleph

Pencil: 4°

Surface	1	2	3	4	5	6	7
L	777.1512	286.1003	129.7102	292.5020	54.2931	120.0212	104.2971
r	161.78	432.35	420.12	361.80	98.39	1036.8	152.64
d'	29.77	0.18	7.38	205.63	10.87	16.16	
N							
N'							
$[r_{-1}/r]$ a		·3741876	1.029111	1.161194	3.677203	·0948978	6.792453
$r_{-1}-d'_{-1}-r$							
$[()/r]$ β	3.803753	1.305331	·0295392	2.181592	2.687255	·8946181	5.898323
$a \sin I'_{-1}$		·0615156	·4767453	·3279708	·1916032	·0173197	·5916498
$+\beta \sin U$		·2253022	·0106229	·3596366	·4787591	·0700656	·4602892
$\sin I$	·2665297	·2868178	·4662226	·0315662	·2871559	·0873852	·1313593
$[N/N']$ n	·6191299	1.61517	·6058109	1.65068	·6355744	·9967817	1.67846
$\sin I'$	·1643974	·4632696	·2824427	·0521057	·1825089	·0871040	·2073454
$[Pl.\tan U]$ $\sin I'+\sin U'$	·3369990	·1070234	·1176380	·1329395	·1041900	·0090667	·0527309
$[Pl.\cot U']$ $r()$	54.51970	46.27157	49.42208	48.09751	10.25125	9.400356	8.048845
$[L\tan U\cot U']$ $[()/\sin U']$ L'	315.8702	129.8902	299.8827	259.9230	130.8911	120.4698	52.05750
U	4.00292	9.93912	20.86922	9.48589	10.66377	4.49194	4.47576
$+I$	15.39843	16.66754	27.78938	1.80891	16.68776	5.01320	7.54815
$U+I$	19.40135	6.72842	6.92016	7.67698	6.02399	0.52126	3.07239
$-I'$	9.46223	27.59764	16.40606	2.98679	10.51693	4.99702	11.96683
U'	9.93912	20.86922	9.48589	10.66377	4.49194	4.47576	8.89444
$\sin U$	·0698073	·1726016	·3562361	·1648047	·1850462	·0783189	·0780373
$\cos I$	·9641027	·9579851	·8846675	·9996017	·9578839	·9961746	·9913349
$\cos I'$	·9863942	·8862227	·9692842	·9986416	·9832042	·9961993	·9782678
$\sin U'$	·1726016	·3562361	·1648047	·1850462	·0783189	·0780373	·1546145
$\cos U'$	·9849917	·9343960	·9863262	·9827301	·9969283	·9969604	·9879749
$\tan U'$				·1882971			·1564964
$1-\cos(U+I)$	·0567861	·0068873	·0072860	·0089631	·0056230	·0000413	·0014374
$[\cos I'/\cos I]$ $\partial p'/\partial p$	1.023121	·925090	1.084344	·999139	1.026434	1.000025	·986819
$[n/()]$ $\partial U'/\partial U$	·605138	1.74596	·558689	1.652102	·619206	·996757	1.599544
$.1-\partial U'/\partial U$	·394862	·74596	·441311	·652102	·380794	·003243	·599544
$[()/\cos I]$ e	·409664	·778676	·498844	·652427	·397537	·00325545	·604785
$[e/r]$ $\partial U'/\partial p$	·00253161	·00180103	·00118738	·00180328	·00404042	·0⁵313990	·00396217
$[eL\sin U]$ $\partial U'/\partial c$	22.21919	38.45209	23.05031	31.45070	3.993927	·0306010	4.922384
$[-\sin I/\cos I']$ $\partial U'/\partial n$	·2691923	·3236408	·486011	·0316091	·2920613	·0877186	·1342774
$[r(1-\cos(U+I))]$ X	9.18669	2.97772	3.060574	3.24285	·5433096	·0428198	·2194047
$d'+X_{+1}-X$	17.60558	·097150	13.68342	201.8438	11.37049	16.98341	
$[()/\cos U']$ D'	17.87384	·103971	13.87312	206.3909	11.40652	16.03230	
$\frac{1}{2}(I-U)$	5.69776	13.30333	24.32930	5.64744	13.67576	4.75257	6.01196
$\cos\frac{1}{2}(I-U)$	·9950594	·9731655	·9111927	·9951462	·9716492	·9965618	·9945001
$\frac{1}{2}(I'-U')$	0.23844	24.23343	12.94597	6.82524	7.50394	4.73639	10.43064
$\cos\frac{1}{2}(I'-U')$	·9999914	·9118808	·9745817	·9929133	·9914358	·9965852	·9834748
$L\sin U$	54.25083	49.38137	46.20746	48.20570	10.04668	9.39993	8.139064
$[()/\cos\frac{1}{2}(I-U)]$ PA	54.52019	50.74303	50.71096	48.44082	10.33981	9.432358	8.184076
$PA\cos\frac{1}{2}(I'-U')$	54.51972	46.27159	49.42197	48.09753	10.25126	9.400148	8.048833
$[()/\sin U']$ L'	315.8703	129.8902	299.8820	259.9231	130.8912	120.4571	52.05743
$-l'$				73.15			462.315
$L'-l'$				186.7731			410.2576
H'_c			Y_0	35.1688			64.2038

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

4° Pencil Ray c

Date:

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·745960	·558689	1·652102	·619206	·996757	1·599544	
D_+	17·87384	·103971	13·87312	205·3909	11·40552	16·03230	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·185179	1·250800	2·178906	·920080	1·531040	1·599544	
$(\partial U_k'/\partial p_+)D_+$	·081090	·000763	·069907	·398789	·045137	·063523	
$\partial U_k'/\partial U'$	2·266269	1·251563	2·238813	1·318869	1·485903	1·536021	1·00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	420·0111	240·3319	408·4317	16·7345	15·7697		
$(\partial p_k'/\partial p_+)D_+$	28·7975	0·2298	21·7394	230·4849	11·2560		
$\partial p_k'/\partial U'$	448·8086	240·5617	430·1711	247·2194	27·0257	15·82098	0·00
$\partial U'/\partial p$	·00253161	·00180103	·00118738	·00180328	·00404042	·0⁵313990	
$\partial p'/\partial p$	1·023121	·925090	1·084344	·999139	1·026434	1·000025	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·00573731	·00225410	·00265832	·00237829	·00600367	·0000048	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·00464166	·00679087	·00468245	·00193994	·00406206	·00396227	
$\partial U_k'/\partial p$	·00109565	·00453677	·00734077	·00431823	·00194161	·00396745	·00396217
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·136208	·433259	·510777	·445806	·109195	·000050	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·648407	2·044414	1·699185	1·121211	1·012982	·986844	
$\partial p_k'/\partial p$	·512199	1·611155	2·209962	1·567017	1·122177	·986894	·986819
$-S_k'.\partial U_k'/\partial U'$	1060·986	586·9369	1048·132	617·4471	695·6464	719·1099	468·1641
$C(U')$	1509·795	826·4986	1478·303	864·6666	722·6721	734·9309	468·1641
$C(U')\sec U_k'$	1528·171	836·5579	1496·295	875·1904	731·4677	743·8757	473·8621
$-S_k'.\partial U_k'/\partial p$	·512944	2·123953	3·436685	2·021640	·908992	1·852736	1·854946
$C(p)$	·000745	3·735108	5·646647	3·588657	2·031169	·865842	·868127
$C(p)\sec U_k'$	·0007541	3·780568	5·715372	3·632335	2·055890	·876380	·878693
$\partial U'/\partial c$	22·21919	38·45209	23·05031	31·45070	3·993927	·0306010	4·922384
$C(c)\sec U_k'$	33954·72	32167·40	34490·06	27525·36	2921·43	22·7633	2332·531
$-\partial l'/\partial c \tan U_k'$	80386·32	69513·20	73379·97	70799·06	18125·29	164·3073	19348·93
$\partial H'/\partial c$	46431·60	37345·80	38889·91	43273·70	15203·86	141·5440	17016·40
$\partial U'/\partial n$	·2691923	·3236408	·4860109	·0316091	·2920613	·0877186	·1342774
$C(n)\sec U_k'$	411·3719	270·7443	727·2167	27·66398	213·6334	65·25173	63·6290
$-\partial l'/\partial n \tan U_k'$	1304·608	547·4357	1470·192	34·41400	712·3744	4·27612	184·6645
$\partial H'/\partial n$	893·2361	276·6914	742·9763	62·07798	498·7410	69·52785	121·0355
$\partial p/\partial d$	·0698073	·1726016	·3562361	·1648047	·1850452	·0783189	·0780373
$C(d)\sec U_k'$	·00005264	·652532	2·036022	·598626	·3804326	·068637	·0685708
$-\partial l'/\partial d \tan U_k'$		1·169798	3·433086	1·076043	·5468682	·0569203	·0560724
$\partial H'/\partial d$	·00005264	·517266	1·397063	·477417	·1664356	·1245674	·1246432
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: pr

System: Booth Teleph

Pencil: 4°

	Surface	1	2	3	4	5	6	7
	L	130.5484	111.1426	62.14046	106.8571	132.4797	128.5070	145.2335
	r	161.78	432.35	420.12	361.80	98.39	1036.80	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.00	1.61517	1.00	1.65068	1.00	1.57338	1.57846
	N'	1.61517	1.00	1.65068	1.00	1.57338	1.57846	1.00
$[r_{-1}/r]$	a		.3741876	1.029111	1.161194	3.677203	.0948978	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β	.1930498	1.305831	.0295392	2.181592	2.587256	.8946181	5.898323
	$\alpha \sin I'_{-1}$		.0031221	.1351599	.0926950	.2684402	.0019122	.6109581
	$+\beta \sin U$		.0844365	.0033908	.1369198	.2367375	.0921492	.6058392
	$\sin I$	.0134763	.0813144	.1317693	.0442249	.0317030	.0902370	.0051185
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6365744	.9967817	1.57846
	$\sin I'$	.0083436	.1313366	.0798273	.0730012	.0201496	.0899466	.0080793
$[Pl.\tan U]$	$\sin I'+\sin U'$	.0563423	.0165461	.0170659	.0185002	.1231535	.0127672	.0975193
$[Pl.\cot U']$	$r()$	9.115057	7.153706	7.169726	6.693372	12.11707	13.23703	14.89450
$[L\tan U\cot U'][()/\sin U']$	L'	140.9126	62.31967	114.2378	73.15049	117.6370	128.8729	140.9682
	U	04.00292	03.70882	06.59154	03.59833	05.24999	05.91218	05.89547
	$+I$	00.77216	04.66412	07.57185	02.53473	01.81675	05.17724	00.29327
	$U+I$	03.23076	00.95530	00.98031	01.06361	07.06674	00.73494	05.60220
	$-I'$	00.47806	07.54684	04.57864	04.18638	01.15457	05.16053	00.46291
	U'	03.70882	06.59154	03.59833	05.24999	05.91218	05.89547	06.06511
	$\sin U$	.0698073	.0646859	.1147905	.0627614	.0915014	.1030039	.1027138
	$\cos I$	.9999092	.9966885	.9912804	.9990216	.9994974	.9959203	.9999869
	$\cos I'$	.9999652	.9913378	.9968087	.9973318	.9997970	.9959466	.9999673
	$\sin U'$	.0646859	.1147905	.0627614	.0915014	.1030039	.1027138	.1056586
	$\cos U'$	.9979057	.9933898	.9980285	.9958049	.9946810	.9947109	.9944025
	$\tan U'$							.1062634
	$1-\cos(U+I)$	.0016894	.0001390	.0001464	.0001723	.0075965	.0000823	.0047763
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.000056	.9946315	1.005577	.9983085	1.00030	1.000026	.9999804
$[n/()]$	$\partial U'/\partial U$	.6190952	1.623888	.6024510	1.663477	.6353838	.9967558	1.578491
	$1-\partial U'/\partial U$	.3809048	.623888	.397549	.653477	.3646162	.0032442	.578491
$[()/\cos I]$	e	.3809394	.6259609	.401046	.654117	.3647995	.00325749	.5784986
$[e/r]$	$\partial U'/\partial p$	.00235468	.001447811	.00095460	.00180795	.00370769	$.0^{5}3141869$	.00378996
$[eL \sin U]$	$\partial U'/\partial c$	3.471589	4.500258	2.860716	4.386836	4.422129	.04311848	8.617859
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.01347677	.0820249	.1321912	.04434322	.03170944	.0906043	.00511867
$[r(1-\cos(U+I))]$	X	.2571331	.06009665	.06150557	.06233814	.7474196	.0853286	.7290544
	$d'+X_{+1}-X$	29.46277	.1785911	7.503844	204.8202	11.53209	15.51627	
$[()/\cos U']$	D'	29.51458	.1797795	7.618667	205.6831	11.59376	15.59877	
	$\frac{1}{2}(I-U)$	2.38754	4.18647	7.08169	3.06653	1.71662	5.54471	3.09437
	$\cos \frac{1}{2}(I-U)$	.9991319	.9973317	.9923714	.9985681	.9996512	.9953211	.9986420
	$\frac{1}{2}(I'-U')$	2.09344	7.06919	4.08765	4.71819	2.37880	5.52800	3.26401
	$\cos \frac{1}{2}(I'-U')$	.9993326	.9923983	.9974561	.9966113	.9991383	.9953490	.9983777
	$L \sin U$	9.113231	7.189359	7.133134	6.706501	12.12208	13.23672	14.89694
$[()/\cos \frac{1}{2}(I-U)]$	PA	9.121149	7.208594	7.187969	6.716118	12.12762	13.29895	14.91869
	$PA \cos \frac{1}{2}(I'-U')$	9.115062	7.153796	7.169683	6.693359	12.11707	13.23709	14.89449
$[()/\sin U']$	L'	140.9126	62.32046	114.2371	73.15036	117.6370	128.8736	140.9681
	$-l'$							462.315
	$L'-l'$							603.2831
	H'_{pr}							64.1009

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

4° Pencil Ray pr

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·623888	·602451	1·663477	·6353838	·9967558	1·578491	
D_+	29·51458	·1797795	7·618667	205·6831	11·59376	15·59877	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·096720	1·290087	2·111533	·9343681	1·514443	1·578491	
$(\partial U_k'/\partial p_+)D_+$	·122086	·001086	·029864	·3426578	·043886	·059119	
$\partial U_k'/\partial U'$	2·218806	1·291173	2·141397	1·277026	1·470657	1·519372	1·00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	406·1081	249·7343	402·9537	17·2458	15·5479		
$(\partial p_k'/\partial p_+)D_+$	46·3826	·3495	11·5767	226·456	11·5944		
$\partial p_k'/\partial U'$	452·4907	250·0838	414·5304	243·7008	27·1423	15·59846	0·00
$\partial U'/\partial p$	·00235468	·001447811	·000954599	·001807952	·003707689	·0^{5}8141869	
$\partial p'/\partial p$	1·000056	·9946315	1·005577	·9983085	1·00030	1·000026	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·00522457	·00186937	·00204417	·00230880	·00545237	·0000048	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·00413669	·00600583	·00399408	·00166313	·00378642	·00379005	-
$\partial U_k'/\partial p$	·00108788	·00413646	·00603825	·00397193	·00166595	·00378528	·00378995
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·0654685	·3620741	·3957102	·440699	·100635	·000049	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·5716020	1·933688	1·548314	1·099128	1·000355	1·000006	
$\partial p_k'/\partial p$	·5061335	1·571514	1·944024	1·539127	1·100990	1·000055	·9999804
$-S_k'.\partial U_k/\partial U'$	1033·188	601·2355	997·1428	594·6479	684·7657	707·4965	465·6506
$C(U')$	1485·679	851·3193	1411·673	838·3487	711·9080	723·0950	465·6506
$C(U')\sec U_k'$	1494·042	856·1114	1419·619	843·0678	715·9153	727·1653	468·2717
$-S_k'.\partial U_k'/\partial p$	·5065720	1·926145	2·811716	1·849532	·775751	1·762618	1·764794
$C(p)$	·0004385	3·497659	4·765739	3·389259	1·876741	·762563	·7648136
$C(p)\sec U_k'$	·00044097	3·517347	4·782509	3·408337	1·887305	·7668565	·7691187
$\partial U'/\partial c$	3·471589	4·500258	2·860715	4·386836	4·422129	·04311848	8·617869
$C(c)\sec U_k'$	5186·70	3852·72	4061·13	3698·40	3165·87	31·3543	4035·50
$-\partial l'/\partial c\tan U_k'$	54578·38	47196·06	49821·41	48069·09	12306·18	111·6566	13136·98
$\partial H'/\partial c$	49391·68	43343·34	45760·28	44370·69	15472·06	142·9109	17172·48
$\partial U'/\partial n$	·01347677	·0820249	·1321912	·0443432	·0317094	·09060426	·00611867
$C(n)\sec U_k'$	20·1349	70·2225	187·6611	37·38434	22·70127	66·88427	2·39693
$-\partial l'/\partial n\tan U_k'$	885·7652	371·6820	998·1886	23·36542	483·6674	2·90327	125·3782
$\partial H'/\partial n$	905·9001	301·4595	810·5275	60·74976	506·3687	68·78754	122·9813
$\partial p/\partial d$	·0698073	·0646859	·1147906	·0627614	·0915014	·1030039	·1027138
$C(d)\sec U_k'$	·000030783	·2275228	·5489866	·2439120	·1726910	·0789891	·0789991
$-\partial l'/\partial d\tan U_k'$		·7942357	2·330897	·7305806	·3712968	·0379671	·0380704
$\partial H'/\partial d$	·000030783	·5667129	1·781910	·5166686	·1986058	·1169562	·1170695
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

Ray: d

System: Booth telephot

Pencil: 4°

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

	Surface	1	2	3	4	5	6	7.
	L	516.0544	785.153	282.486	847.7316	6627.198	280.6061	297.4321
	r	161.78	432.35	420.12	361.8	98.39	1036.8	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.00	1.61517	1.00	1.65068	1.00	1.57338	1.57846
	N'	1.61517	1.00	1.65068	1.00	1.57338	1.57846	1.00.
$[r_{-1}/r]$	a		.3741876	1.029111	1.161194	3.677203	.09489776	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β	4.189853	1.305331	.02963918	2.181592	2.587255	894.6181	5.898323
	$\alpha \sin I'_{-1}$		.0677596	.2099483	.1451277	.3356731	.0210424	.6287590
	$+\beta \sin U$		.0585488	.0036439	.0898262	.0132048	.1139086	.7492569
	$\sin I$	.2924823	.1263083	.2063043	.0553014	.3488774	.0928662	.1204977
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6355744	.9967817	1.57846
	$\sin I'$	.1810845	.2040094	.1249814	.0912849	.2217375	.0925673	.1902008
$[Pl.\tan U]$	$\sin I'+\sin U'$	.2259381	.0806507	.0838008	.0963887	.3490640	.0344615	.2469678
$[Pl.\cot U']$	$r()$	36.55227	34.86933	36.20891	34.87343	34.34441	35.72968	37.69716
$[L\tan U\cot U']$ $[()/\sin U']$	L'	814.9239	282.6662	855.1124	6832.886	269.7350	281.2723	664.0682
	U	4.00892	2.57078	7.08698	2.35980	0.29243	7.31813	7.29793
	$+I$	17.00663	7.25631	11.90686	3.17016	20.41867	5.32862	6.92083
	$U+I$	13.00371	4.68553	4.81988	5.52996	20.12624	1.98661	14.21876
	$-I'$	10.43293	11.77151	7.17968	5.23753	12.81111	5.31182	10.96450
	U'	2.57078	7.08598	2.35980	0.29243	7.31613	7.29793	3.25426
	$\sin U$	.0698073	.0448536	.1233587	.0411746	.0051038	.1273265	.1270288
	$\cos I$	.9562709	.9919910	.9784879	.9984697	.9371683	.9956786	.9927136
	$\cos I'$	.9834675	.9789690	.9921691	.9958249	.9751064	.9957064	.9817452
	$\sin U'$	.0448536	.1233587	.0411746	.0051038	.1273265	.1270288	.0567670
	$\cos U'$	.9989936	.9923621	.9991519	.9999870	.9918608	.9918991	.9983874
	$\tan U'$				.0051039			
	$1-\cos(U+I)$	.0256445	.0033419	.0035362	.0046540	.0610632	.0006011	.0306351
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.028440	.986873	1.013972	.997351	1.040482	1.000028	.9889611
$[n/()]$	$\partial U'/\partial U$	.6020088	1.636654	.597463	1.655064	.610846	.996754	1.696095
	$1-\partial U'/\partial U$	.3979912	.636664	.402587	.655064	.389154	.003246	.696095
$[()/\cos I]$	e	.4161906	.641794	.411387	.656068	.415245	.00326009	.600470
$[e/r]$	$\partial U'/\partial p$	$.0^{2}257267$	$.0^{2}148443$	$.0^{3}979218$	$.0^{2}181334$	$.0^{2}422040$	$.0^{5}314438$	$.0^{2}393390$
$[eL\sin U]$	$\partial U'/\partial c$	14.99301	22.60202	14.33565	22.90006	14.04520	.1164780	22.68722
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.2973990	.1290218	.2079347	.0555333	.3577839	.0932666	.1227383
$[r(1-\cos(U+I))]$	X	4.148768	1.444870	1.485628	1.683817	6.008008	.6232205	4.676142
	$d'+X_{+1}-X$	24.17636	.139242	10.54946	197.9382	16.25479	12.10708	
$[()/\cos U']$	D'	24.20072	.140314	10.55840	197.9408	16.88818	12.20596	
	$\frac{1}{2}(I-U)$	10.50478	4.91355	9.49592	.40518	16.35555	6.32183	.18866
	$\cos\frac{1}{2}(I-U)$	.9832397	.9963251	.9862973	.999975	.9837112	.9939190	.9999946
	$\frac{1}{2}(I'-U')$	3.93108	.942875	4.76974	2.47255	2.74799	6.30463	3.85512
	$\cos\frac{1}{2}(I'-U')$	.9976472	.9864901	.9965369	.9990690	.9988501	.9939521	.9977373
	$L\sin U$	36.02436	35.21694	34.84711	34.90501	33.82389	36.72847	37.78244
$[()/\cos\frac{1}{2}(I-U)]$	PA	36.63843	35.34684	35.33124	34.90589	34.38396	35.94706	37.78264
	$PA\cos\frac{1}{2}(I'-U')$	36.55223	34.86931	35.2088	34.87339	34.34442	35.72966	37.69716
$[()/\sin U']$	L'	814.9230	282.6660	856.1116	6632.828	269.7851	281.2721	664.0680
	l'							462.315
	$L'-l'$							1126.383
	H'_d							64.04466

Computer:

TRANSFER COEFFICIENTS.

System: Booth Telephoto

Date:

4° Pencil Ray: d

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·636654	·597463	1·655064	·610846	·996754	1·596095	
D_+	24·20072	·140314	10·5584	197·9408	16·38818	12·20596	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·199877	1·343174	2·200015	·903236	1·543053	1·596095	
$(\partial U_k'/\partial p_+)D_+$	·114644	·000957	·048115	·426028	·064391	·048017	
$\partial U_k'/\partial U'$	2·314521	1·344131	2·248130	1·329263	1·478662	1·548078	1·00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	412·6167	251·8260	404·7189	17·2604	12·0319		
$(\partial p_k'/\partial p_+)D_+$	39·2716	0·2839	16·7733	227·2833	16·2082		
$\partial p_k'/\partial U'$	451·8883	252·1099	421·4922	244·5337	28·2401	12·0711	0·00
$\partial U'/\partial p$	·00267257	·00148443	·000979213	·00181334	·00422040	·0^{5}314438	
$\partial p'/\partial p$	1·028440	·986873	1·013972	·997351	1·040482	1·000028	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·0059548	·0019953	·0022014	·0024104	·0062405	·0000049	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·0048719	·0067325	·0046207	·0021466	·0040882	·00393401	
$\partial U_k'/\partial p$	·0010824	·0047372	·0068221	·0045570	·0021523	·0039291	·0039339
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·162514	·374239	·412731	·443423	·119185	·000038	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·668896	1·996984	1·610816	1·145197	1·029054	·988979	
$\partial p_k'/\partial p$	·506382	4·622745	2·023547	1·588620	1·148239	·989017	·988951
$-S_k'.\partial U_k/\partial U'$	1082·607	628·7111	1051·552	621·7567	691·6373	724·1064	467·7454
$C(U')$	1584·495	880·8210	1473·044	866·2904	719·8774	736·1776	467·7454
$C(U')\sec U_k'$	1636·973	882·2435	1475·423	867·6895	721·0400	737·3664	468·5008
$-S_k'.\partial U_k'/\partial p$	·506288	2·215804	3·191006	2·131516	1·006728	1·837818	1·840064
$C(p)$	·000094	3·838549	5·214553	3·720136	2·154967	·848801	·851113
$C(p)\sec U_k'$	·00009415	3·844748	6·222975	3·726144	2·158447	·850172	·852488
$\partial U'/\partial c$	14·99301	22·60202	14·33565	22·90006	14·04520	·1164780	22·68722
$C(c)\sec U_k'$	23043·85	19940·49	21152·40	19870·14	10127·15	86·88696	10628·98
$-\partial l'/\partial c\tan U_k'$	29206·18	25255·72	26660·61	25722·90	6585·33	59·69657	7029·91
$\partial H'/\partial c$	52260·03	44696·21	47813·01	45593·04	16712·48	145·5836	17658·89
$\partial U'/\partial n$	·2973990	·1290218	·2079347	·0555333	·3577839	·0932666	·1227383
$C(n)\sec U_k'$	457·0942	113·8286	306·7916	48·18566	257·9765	68·77166	57·50299
$-\partial l'/\partial n\tan U_k'$	473·9939	198·8958	534·1543	12·50339	258·8218	1·55361	67·09281
$\partial H'/\partial n$	931·0881	312·7244	840·9459	60·68906	516·7983	70·32527	124·5958
$\partial p/\partial d$	·0698073	·0448536	·1233587	·0411746	·0051038	·1273265	·1270288
$C(d)\sec U_k'$	·000006572	·172451	·644299	·153423	·0110163	·108249	·1082905
$-\partial l'/\partial d\tan U_k'$		·425014	1·247318	·390951	·1986897	·020317	·020372
$\partial H'/\partial d$	·000006572	·597465	1·891617	·544374	·209706	·128566	·128663
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: 6

System: Booth Telepho

Pencil: 4°

	Surface	1	2	3	4	5	6	7
	L	784·0244	562·6001	221·4151	695·5209	821·932	326·3963	342·2847
	r	161·78	432·35	420·12	361·80	98·39	1036·80	152·64
	d'	29·77	0·18	7·38	205·63	10·87	16·16	
	N	1·00	1·61517	1·00	1·65068	1·00	1·57338	1·57846
	N'	1·61517	1·00	1·65068	1·00	1·57338	1·57846	1·00
$[r_{-1}/r]$	a		·3741876	1·029111	1·161194	3·677203	·0948978	6·792453
	$r_{-1}—d'_{-1}—r$							
$[()/r]$	β	5·84624	1·305331	·0295392	2·181592	2·587255	·8946181	5·898323
	$\alpha \sin I'_{-1}$		·0945472	·3631334	·2506034	·3475203	·0289750	·6457116
	$+\beta \sin U$		·1239198	·0068912	·1933501	·1328775	·1243450	·8180207
	$\sin I$	·4081101	·2184670	·3562421	·0572532	·4803980	·0953700	·1723090
$[N/N']$	n	·6191299	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846
	$\sin I'$	·2526732	·3528613	·2158153	·0945067	·3053287	·0950631	·2719829
$[Pl.\tan U]$	$\sin I' + \sin U'$	·3476068	·1195702	·1271873	·1458652	·4443210	·0436239	·3088303
$[Pl.\cot U']$	$r()$	56·23583	51·69618	53·43393	52·77403	43·71674	45·22926	47·13986
$[L \tan U \cot U']$ $[()/\sin U']$	L'	592·3701	221·5961	602·9012	1027·562	314·5263	326·1247	1279·327
	U	04·00292	5·44750	13·49091	5·08468	2·94392	7·98954	7·97188
	$+I$	24·08617	12·61901	20·86959	3·28216	28·71140	5·47262	9·92210
	$U+I$	20·08325	7·17151	7·37868	8·36685	25·76748	2·51693	17·89398
	$—I'$	14·63576	20·66242	12·46336	5·42293	17·77794	5·45495	15·78230
	U'	5·44750	13·49091	5·08468	2·94392	7·98954	7·97188	2·11168
	$\sin U$	·0698073	·0949336	·2332911	·0886280	·0513585	·1389923	·1386870
	$\cos I$	·9129327	·9758444	·9343937	·9983597	·8770506	·9954419	·9850429
	$\cos I'$	·9675517	·9356756	·9764342	·9955242	·9522470	·9954713	·9623021
	$\sin U'$	·0949336	·2332911	·0886280	·0513585	·1389923	·1386870	·0368474
	$\cos U'$	·9954836	·9724069	·9960648	·9986803	·9902935	·9903363	·9993209
	$\tan U'$				·0514264			·0368724
	$1—\cos(U+I)$	·0608053	·0078232	·0082810	·0106483	·0994344	·0009647	·0483733
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1·059828	·9588369	1·044992	·9971598	1·085738	1·000030	·9769139
$[n/()]$	$\partial U'/\partial U$	·5841796	1·684510	·5797278	1·655382	·5853847	·9967518	1·616762
	$1 — \partial U'/\partial U$	·4158204	·684510	·4202722	·655382	·4146153	·0032482	·616762
$[()/\cos I]$	e	·4554776	·701454	·4497806	·6564688	·4727382	·003263073	·6251119
$[e/r]$	$\partial U'/\partial p$	·00281541	·00162242	·00107060	·001814425	·00480474	·0^{5}3147254	·00409534
$[eL \sin U]$	$\partial U'/\partial c$	24·92858	37·46441	23·23304	34·64778	19·95579	·1475809	29·67434
$[—\sin I/\cos I']$	$\partial U'/\partial n$	·4217967	·2334868	·3648398	·05751061	·5044889	·0958039	·1790592
$[r(1—\cos(U+I))]$	X	9·837081	3·382361	3·479014	3·860746	9·783361	1·000201	7·383701
	$d'+X_{+1}—X$	16·55056	·0833470	14·70976	191·9969	19·65315	9·7765	
$[()/\cos U']$	D'	16·62565	·08571206	14·76787	192·2496	19·84578	9·871899	
	$\frac{1}{2}(I — U)$	14·04456	9·03325	17·18025	0·90126	16·82766	6·73108	0·97511
	$\cos \frac{1}{2}(I — U)$	·9701073	·9875974	·9553802	·9998763	·9620864	·9931072	·9998562
	$\frac{1}{2}(I' — U')$	4·59413	17·07667	8·77402	1·23951	4·89420	6·71342	6·83531
	$\cos \frac{1}{2}(I'—U')$	·9967871	·9559127	·9882976	·9997660	·9963539	·9931433	·9928924
	$L \sin U$	54·73063	63·40965	61·65417	52·77983	42·21319	46·22758	47·47044
$[()/\cos \frac{1}{2}(I—U)]$	PA	56·41709	54·08039	54·06661	52·78636	43·87672	45·54149	47·47731
	$PA \cos \frac{1}{2}(I'—U')$	56·23583	51·69613	53·43390	52·77401	43·71674	45·22923	47·13986
$[()/\sin U']$	L'	592·3701	221·5950	602·9009	1027·561	314·5263	326·1245	1279·327
	$-l'$				73·15			462·315
	$L'-l'$				954·411			1741·642
	H'_b				Y_0 49·0819			64·2485

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

4° Pencil Ray b

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·684510	·6797278	1·655382	·5853847	·9967518	1·615762	
D_+	16·62565	·08571206	14·76787	192·2496	19·84678	9·871899	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·328966	1·381891	2·306402	·8716595	1·570216	1·615762	
$(\partial U_k'/\partial p_+)D_+$	·090571	·000687	·077288	·5216155	·081179	·040429	
$\partial U_k'/\partial U'$	2·419537	1·382578	2·383690	1·393275	1·489037	1·575333	1·000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	424·4679	251·7957	410·0250	16·977	9·61267		
$(\partial p_k'/\partial p_+)D_+$	28·0372	·1873	24·3093	230·7161	19·38880		
$\partial p_k'/\partial U'$	452·5051	251·9830	434·3343	247·6921	29·00147	9·643995	0·000
$\partial U'/\partial p$	·002815414	·001622422	·00107060	·001814425	·00480474	$\cdot 0^{3}3147254$	
$\partial p'/\partial p$	1·059828	·9588369	1·044992	·9971598	1·085738	1·000030	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·00681200	·00224312	·00255198	·00252799	·00715443	·0000060	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·00577358	·00769078	·00546897	·00270551	·00444121	·00409546	
$\partial U_k'/\partial p$	·00103842	·00544766	·00802095	·00523350	·00271322	·00409050	·00409534
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·2739892	·408823	·464998	·449419	·139344	·000030	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·7872727	2·095203	1·720153	1·196673	1·060737	·976943	
$\partial p_k'/\partial p$	·5132835	1·686380	2·185151	1·646092	1·200081	·9769736	·9769139
$-S_k'.\partial U_k/\partial U'$	1137·226	649·8364	1120·377	654·8642	699·8740	740·4347	470·0179
$C(U')$	1589·731	901·8194	1554·711	902·5563	728·8755	750·0787	470·0179
$C(U')\sec U_k'$	1590·812	902·4326	1555·768	903·1700	729·3711	750·6888	470·3375
$-S_k'.\partial U_k'/\partial p$	·488076	2·560498	3·769990	2·459839	1·275262	1·922608	1·924881
$C(p)$	·0252075	4·246878	5·965141	4·105931	2·475343	·9456344	·9479671
$C(p)\sec U_k'$	·0252246	4·249766	5·959190	4·108723	2·477026	·9462774	·9486117
$\partial U'/\partial c$	24·92858	37·46441	23·23304	34·64778	19·95579	·1475809	29·67434
$C(c)\sec U_k'$	39656·68	33809·10	36145·22	31292·84	14555·18	110·7726	13956·96
$-\partial l'/\partial c \tan U_k'$	18939·97	16378·13	17289·19	16681·09	4270·53	38·7127	4558·84
$\partial H'/\partial c$	58596·65	50187·23	53434·41	47973·93	18825·71	149·4853	18515·79
$\partial U'/\partial n$	·4217967	·2334858	·3648398	·05751061	·5044889	·09580387	·1790592
$C(n)\sec U_k'$	670·9993	210·7062	567·6061	51·94186	367·9696	71·90931	84·21826
$-\partial l'/\partial n \tan U_k'$	307·3811	128·9823	346·3947	8·10834	167·8438	1·00750	43·50914
$\partial H'/\partial n$	978·3804	339·6875	914·0008	60·05020	535·8034	72·91681	127·7274
$\partial p/\partial d$	·0698073	·0949336	·2332911	·088628	·0513585	·1389923	·1386870
$C(d)\sec U_k'$	·00176086	·4034456	1·390226	·3641479	·1272163	·1315253	·1315601
$-\partial l'/\partial d \tan U_k'$		·2756182	·808875	·2535285	·1288486	·0131755	·01321133
$\partial H'/\partial d$	·00176086	·6790638	2·199101	·6176764	·2560649	·1447008	·1447714
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: a'

System: Booth Telepho

Pencil: 7·5°

	Surface	1	2	3	4	5	6	7
	L	557·3631	267·8288	105·2925	263·6870	17·83160	21·87342	5·7875
	r	161·78	432·35	420·12	361·80	98·39	1036·8	152·64
	d'	29·77	0·180	7·38	205·63	10·87	16·16	
	N							
	N'							
$[r_{-1}/r]$	a		·3741876	1·029111	1·161194	3·677203	·09489776	6·792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β	2·445192	1·305331	·0295392	2·181692	2·687255	·8946181	5·898323
	$\alpha\sin I'_{-1}$		0750594	·6843868	·4703325	·4052436	·0205344	1·1224270
	$+\beta\sin U$		·3366787	·0157919	·5370955	·7466992	·1452453	·9544708
	$\sin I$	·3239916	·4117382	·6685949	·0667630	·3404655	·1657797	·1679668
$[N/N']$	n	·6191299	1·61517	·6058109	1·65068	·6356744	·9967817	1·57846
	$\sin I'$	·2005929	·6660272	·4050421	·1102043	·2163848	·1652462	·2651123
$[Pl.\tan U]$	$\sin I'+\sin U'$	·4685188	·1304186	·1588478	·1780159	·0540303	·0034255	·0060033
$[Pl.\cot U']$	$r()$	74·17917	56·38648	66·73614	64·40615	5·316041	3·561558	·9163437
$[L\tan U\cot U'][()/\sin U']$	L'	287·5988	105·4725	271·0670	223·4616	32·74342	21·94749	3·536518
	U	7-36·850	14-56·8217	32-19·0424	14-15·1454	16-45·0867	9-20·6152	9-18·7554
	$+I$	18-64·2696	24-18·8442	41-57·6229	3-49·6852	19-54·2775	9-32·5520	9-40·1409
	$U+I$	26-31·1196	9-22·0225	9-38·4806	10-25·4602	3-09·1918	0-11·9368	0-21·3855
	$-I'$	11-34·2979	41-41·0648	23-53·6259	6-19·6255	12-29·8070	9-30·6922	15-22·4175
	U'	14-56·8217	32-19·0424	14-15·1454	16-45·0867	9-20·6152	9-18·7554	15-01·0320
	$\sin U$	·1325015	·2579259	·5346086	·2461943	·2882202	·1623546	·1618207
	$\cos I$	·946060	·9113022	·7436268	·9977688	·9402606	·9861628	·9857944
	$\cos I'$	·9796747	·7468192	·9142981	·9939089	·9763081	·9860523	·9642175
	$\sin U'$	·2579259	·5346086	·2461943	·2882202	·1623545	·1618207	·2591090
	$\cos U'$	·9661647	·8450998	·9692205	·9575642	·9867325	·9868202	·9658481
	$\tan U'$				·3009931			·2682710
	$1-\cos(U+I)$	·1062110	·0133341	·01441246	·0165053	·0015140	·0000060	·0000194
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1·035631	·8195077	1·229512	·9961315	1·038338	1·000091	·9781122
$[n/()]$	$\partial U'/\partial U$	·5978864	1·970903	·4927247	1·65709	·6121074	·996691	1·613782
	$1-\partial U'/\partial U$	·4021136	·970903	·5072753	·65709	·3878926	·003309	·613782
$[()/\cos I]$	e	·4250403	1·065402	·6821638	·6685594	·4125373	·00335543	·6226268
$[e/r]$	$\partial U'/\partial p$	·002627273	·002464212	·001623736	·001820231	·004192878	·0⁵3236333	·004079054
$[eL\sin U]$	$\partial U'/\partial c$	31·38984	70·85001	38·39919	42·75262	2·120206	·01191596	·5931132
$[-\sin I/\cos I']$	$\partial U'/\partial n$	·3307135	·5513225	·7312668	·06717215	·3487173	·1680906	·1741892
$[r(1-\cos(U+I))]$	X							
	$d'+X_{+1}-X$							
$[()/\cos U']$	D'	7·228642	·0129819	19·8981	208·3509	11·16082	16·37913	
	$\frac{1}{2}(I-U)$							
	$\cos\frac{1}{2}(I-U)$							
	$\frac{1}{2}(I'-U')$							
	$\cos\frac{1}{2}(I'-U')$							
	$L\sin U$	73·85145	66·60073	56·29028	64·91824	6·139427	3·561248	·9365373
$[()/\cos\frac{1}{2}(I-U)]$	PA							
	$PA\cos\frac{1}{2}(I'-U')$							
$[()/\sin U']$	L'							

	223·4616		3·536518
$-l'_4$	73·150	$-l'$	462·315
	150·3116	$L'-l'$	458·779
Y_0	45·243	H'_a	123·0771

Computer:

Date:

TRANSFER COEFFICIENTS.

System Booth Telephoto

7.50 Pencil Ray 'a'

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·97090	·492725	1·65709	·612107	·996691	1·61378	
D_+	7·22854	·012982	19·8981	208·351	11·1608	16·3791	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·24587	1·13939	2·22333	·91594	1·54185	1·61378	
$(\partial U_k'/\partial p_+)D_+$	·03456	·00012	·08910	·42577	·04547	·06681	
$\partial U_k'/\partial U'$	2·28043	1·13951	2·31243	1·34171	1·49638	1·54697	1·000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	435·484	220·922	416·888	16·457	15·9676		
$(\partial p_k'/\partial p_+)D_+$	11·900	0·034	31·479	235·121	10·9181		
$\partial p_k'/\partial U'$	447·384	220·956	448·367	251·578	26·8857	16·0206	0·00
$\partial U'/\partial p$	·002627273	·008464212	·001623736	·001820831	·004192878	$.0^{5}32363$	
$\partial p'/\partial p$	1·03553	·819508	1·22951	·996132	1·03834	1·00009	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·0059913	·0028080	·0037548	·0024422	·0062741	·000005	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·0049508	·0075889	·0055056	·0020356	·0042306	·0040794	
$\partial U_k'/\partial p$	·0010405	·0047809	·0092603	·0044778	·0020435	·0040744	·0040791
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·175398	·544483	·72803	·45793	·11273	·000052	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·704699	2·19069	1·94515	1·12412	1·01576	·978201	
$\partial p_k'/\partial p$	·529301	1·64621	2·67318	1·58205	1·12649	·978253	·978112
$-S_k'.\partial U_k/\partial U'$	1091·56	545·445	1106·881	642·230	716·264	740·481	478·665
$C(U')$	1538·94	766·402	1555·25	893·808	743·150	756·502	478·665
$C(U')\sec U_k'$	1593·36	793·501	1610·24	925·412	769·427	783·251	495·590
$-S_k'.\partial U_k'/\partial p$							
$C(p)$							
$C(p)\sec U_k'$							
$\partial U'/\partial c$							
$C(c)\sec U_k'$							
$-\partial l'/\partial c \tan U_k'$							
$\partial H'/\partial c$							
$\partial U'/\partial n$	·3307135	·551323	·731266	·067172	·348717	·168091	·174189
$C(n)\sec U_k'$	526·946	437·475	1177·51	62·162	268·313	131·657	86·327
$-\partial l'/\partial n \tan U_k'$	2336·40	938·421	2520·25	88·993	1221·174	7·330	316·558
$\partial H'/\partial n$	1709·45	500·966	1342·74	121·155	952·861	138·987	230·231
$\partial p/\partial d$							
$C(d)\sec U_k'$							
$-\partial l'/\partial d \tan U_k'$							
$\partial H'/\partial d$							
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: C

System: Booth Teleph

Pencil: 7·5°

Surface	1	2	3	4	5	6	7
L	432·4311	234·3274	106·3205	237·4559	12·2665	29·12755	45·38165
r	161·78	432·35	420·12	361·80	98·39	1036·8	152·64
d'	29·77	0·18	7·38	205·63	10·87	16·16	
N	1·000	1·61517	1·000	1·65068	1·000	1·57338	1·57846
N'	1·61517	1·000	1·65068	1·000	1·57338	1·57846	1·000
$[r_{-1}/r]$ α		·3741876	1·029111	1·161194	3·677203	·0948978	6·792453
$r_{-1}-d'_{-1}-r$							
$[()/r]$ β	1·672958	1·305331	·0205393	2·181692	2·587255	·8946181	5·898323
$\alpha \sin I'_{-1}$		·0513543	·5561905	·3822492	·4338695	·0133803	1·1392119
$+\beta \sin U$		·2832589	·0128094	·4537279	·6557124	·1548785	1·0779408
$\sin I$	·2216694	·3346133	·5433813	·0714790	·2218426	·1682688	·1212709
$[N/N']$ n	·6191299	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846
$\sin I'$	·1372422	·5404574	·3291863	·1179890	·1409975	·1677173	·1914213
$[Pl.\tan U]$ $\sin I'+\sin U'$	·3542438	·1068181	·1212061	·1354504	·0321250	·0048641	·0506398
$[Pl.\cot U']$ $r()$	57·30956	46·18281	50·92111	49·00595	3·160779	5·043099	7·729659
$[L \tan U \cot U']$ $[()/\sin U']$ L'	264·0974	106·5005	244·8363	193·3636	18·25747	29·22157	31·93268
U	07·61417	12·53298	25·69874	12·00401	14·68113	9·96942	9·93794
$+I$	12·80711	19·54902	32·91412	4·09894	12·81728	9·68660	6·96546
$U+I$	20·42128	7·01604	7·21538	7·90507	1·86385	0·28282	2·97248
$-I'$	7·88830	32·71478	19·21939	6·77606	8·10557	9·65512	11·03574
U'	12·53298	25·69874	12·00401	14·68113	9·96942	9·93794	14·00822
$\sin U$	·1325015	·2170016	·4336393	·2079802	·2534394	·1731225	·1725814
$\cos I$	·9751219	·9423556	·8394859	·9974420	·9760825	·9857429	·9926194
$\cos I'$	·9905375	·8413713	·9442651	·9930149	·9900100	·9858352	·9815080
$\sin U'$	·2170016	·4336393	·2079802	·2534394	·1731225	·1725814	·2420611
$\cos U'$	·9761713	·9010866	·9781331	·9673513	·9849003	·9849953	·9702610
$\tan U'$				·2619931			·2494804
$1-\cos(U+I)$	·0628475	·0074880	·0079190	·0095027	·0005291	·0000122	·0013455
$[\cos I'/\cos I]$ $\partial p'/\partial p$	1·015809	·8928384	1·124814	·9955616	1·015309	1·000094	·988806
$[n/()]$ $\partial U'/\partial U$	·6094944	1·809028	·5385876	1·658039	·6259911	·996688	1·596329
$1-\partial U'/\partial U$	·3905056	·809028	·4614124	·658039	·3740089	·003312	·596329
$[()/\cos I]$ e	·4004685	·8585167	·5496369	·6597266	·3835664	·00335990	·600763
$[e/r]$ $\partial U'/\partial p$	·00247539	·0019857	·00130829	·001823457	·00389843	$\cdot 0^5 324065$	·00393582
$[eL \sin U]$ $\partial U'/\partial c$	22·94595	43·65508	25·34087	32·58134	1·192437	·01694276	4·705193
$[-\sin I/\cos I']$ $\partial U'/\partial n$	·223787	·3976999	·5754542	·0719818	·2240812	·1706764	·1235557
$[r(1-\cos(U+I))]$ X	10·16747	3·237437	3·32693	3·438077	·05205815	·01264896	·2053771
$d'+X_{+1}-X$	16·36509	·090507	14·14601	202·1399	10·90941	15·96727	
$[()/\cos U']$ D'	16·76457	·1004421	14·46123	208·9622	11·07666	16·2105	
$\frac{1}{2}(I-U)$							
$\cos \frac{1}{2}(I-U)$							
$\frac{1}{2}(I'-U')$							
$\cos \frac{1}{2}(I'-U')$							
$L \sin U$	57·29777	50·84942	46·10475	49·38613	3·108814	5·042634	7·832029
$[()/\cos \frac{1}{2}(I-U)]$ PA							
$PA \cos \frac{1}{2}(I'-U')$							
$[()/\sin U']$ L'				193·3635			31·93276
			$-l'_4$	73·15		$-l'$	462·315
				120·2135		$L'-l'$	494·2478
			Y_0	31·4951		H'_c	123·3051

Computer:

TRANSFER COEFFICIENTS.

System Booth telephoto I

Date:

7.5° Pencil Ray 'c'

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.809028	.5385976	1.658039	.62559911	.996688	1.596329	
D_+	16.76457	.1004421	14.46123	208.9622	11.07666	16.2105	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.166085	1.1966129	2.161562	.9289119	1.527451	1.596329	
$(\partial U_k'/\partial p_+)D_+$	.073733	.000762	.060199	.3747737	.043545	.063802	
$\partial U_k'/\partial U'$	2.239818	1.197375	2.221761	1.303686	1.483906	1.532527	1.00
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	423.992	234.1422	412.211	16.8581	15.97595		
$(\partial p_k'/\partial p_+)D_+$	26.983	.2331	22.5232	231.755	10.95427		
$\partial p_k'/\partial U'$	450.925	234.375	434.734	248.613	26.9302	16.02904	0.00
$\partial U'/\partial p$	$.0^{2}2475319$	$.0^{1}1985698$	$.0^{2}1308285$	.001823457	.00389843	$.0^{5}324065$	
$\partial p'/\partial p$	1.015809	.892838	1.124814	.995562	1.015309	1.000094	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	.0055444	.0023776	.0029067	.0023772	.0057849	.0000050	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	.0044677	.0067758	.0046823	.0017866	.0039914	.0039362	
$\partial U_k'/\partial p$	.0010767	.0043982	.0075890	.0041628	.0017935	.0039312	.00393582
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.116214	.465399	.568756	.453336	.104986	.000062	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.631958	2.071959	1.751887	1.104154	1.004091	.988899	
$\partial p_k'/\partial p$	.515744	1.60656	2.320643	1.557490	1.109077	.988951	.988806
$-S_k'.\partial U_k/\partial U'$	1067.714	670.785	1059.107	621.4631	707.3734	730.6509	476.6969
$C(U')$	1518.639	805.160	1493.841	870.0764	734.3036	746.5799	476.6969
$C(U')\sec U_k'$	1565.187	829.8393	1539.629	896.7451	756.8107	769.4633	491.3081
$-S_k'.\partial U_k'/\partial p$	.513279	2.096584	3.617667	1.984375	.854956	1.874000	1.876191
$C(p)$	.0024652	3.703144	5.938310	3.541865	1.964033	.886049	.887386
$C(p)\sec U_k'$	.0025408	3.816649	6.120320	3.650427	2.024233	.912177	.914584
$\partial U'/\partial c$	22.94595	43.65508	26.34087	32.58134	1.192437	.0169428	4.705193
$C(c)\sec U_k'$	35914.7	36226.7	39015.5	29217.2	902.45	13.0368	2311.70
$-\partial l'/\partial c\tan U_k'$	128148.7	110815.2	116979.6	112866.1	28894.62	261.9322	30845.31
$\partial H'/\partial c$	92234.0	74588.5	77964.0	83647.9	29797.07	274.9690	33157.01
$\partial U'/\partial n$	.223787	.3976999	.675454	.0719818	.224081	.170676	.123556
$C(n)\sec U_k'$	350.269	330.027	885.986	64.5493	169.587	131.3292	60.7039
$-\partial l'/\partial n\tan U_k'$	2079.755	872.7004	2343.723	54.8614	1135.639	6.8168	294.3849
$\partial H'/\partial n$	1729.487	542.6734	1457.737	119.4108	966.052	138.1460	233.6810
$\partial p/\partial d$	.132602	.2170016	.433639	.207980	.253439	.1731225	.172681
$C(d)\sec U_k'$	.0003367	.828219	2.654013	.759217	.5130204	.157918	.157840
$-\partial l'/\partial d\tan U_k'$		1.864846	5.472889	1.715386	.871796	.0891459	.089388
$\partial H'/\partial d$	.0003367	1.036627	2.818876	.956169	.358775	.247064	.247228
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

Ray: Pr

System: Booth Telepho

Pencil: 7·5°

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Surface	1	2	3	4	5	6	7
L	130·974	111·4585	61·34199	107·2460	132·4811	128·5327	145·0677
r	161·78	432·35	420·12	361·80	98·39	1036·80	152·64
d'	29·77	0.18	7·38	205·63	10.87	16·16	
N	1·000	1·61517	1·000	1·65068	1·000	1·57338	1·57846
N'	1·61517	1·000	1·65068	1·000	1·57338	1·57846	1·000
$[r_{-1}/r]$ α		·3741876	1·029111	1·161194	3·677203	·0948978	6·792453
$r_{-1}-d'_{-1}-r$							
$[()/r]$ β	·1904191	1·305331	·0295393	2·181592	2·587255	·8946181	5·898323
$\alpha \sin I'_{-1}$		·0058452	·2570903	·1763093	·5094346	·0036289	1·1579129
$+\beta \sin U$		·1605149	·0064602	·2602377	·4492678	·1746498	1·1482554
$\sin I$	·0252308	·1546697	·2506302	·0839282	·0601667	·1710209	·0096576
$[N/N']$ n	·6191299	1·61517	·6058109	1·65068	·6355744	·9967817	1·57846
$\sin I'$	·0156211	·2498179	·1518345	·1385386	·0382404	·1704705	·0152441
$[Pl.\tan U]$ $\sin I'+\sin U'$	·1073476	031120	·0325465	·0351079	·2334631	·0242044	·1849077
$[Pl.\cot U']$ $r()$	17·36669	13·45473	13·67344	12·70204	22·97043	25·09512	28·22431
$[L\tan U\cot U']$ $[()/\sin U']$ L'	141·2285	61·52199	114·6254	73·14884	117·6627	128·9078	141·0145
U	7-36-850	7-03·808	12-37·954	6-51·061	9-59·994	11-15·464	11-13-544
$+I$	1-26-746	8-53·858	14-30·888	4-48·864	3-26·963	9-50·831	0-33·201
$U+I$	6-10-104	1-50·050	1-52·934	2-02·197	13-26·957	1-24·633	10-40·343
$-I'$	0-53·704	14-28·004	8-43·995	7-57·797	2-11·493	9-48·911	0-52·407
U'	7-03·808	12-37·954	6-51·061	9-59·994	11-15·464	11-13·544	11-32·750
$\sin U$	·1325015	·1229687	·2186979	·1192880	·1736465	·1952227	·1946749
$\cos I$	·9996817	·9879663	·9680830	·9964718	·9981883	·9852674	·9999533
$\cos I'$	·9998780	·9682928	·9884059	·9903571	·9992685	·9853627	·9998838
$\sin U'$	·1229687	·2186979	·1192880	·1736465	·1952227	·1946749	·2001518
$\cos U'$	·9924106	·9757926	·9928597	·9848081	·9807589	·9808678	·9797649
$\tan U'$							·2042865
$1-\cos(U+I)$	·0057897	·0005124	·0005396	·0006316	·0274238	·0003031	·0172979
$[\cos I'/\cos I]$ $\partial p'/\partial p$	1·000196	·9800869	1·020993	·9938636	1·001082	1·000097	·9999305
$[n/()]$ $\partial U'/\partial U$	·6190086	1·647987	·5933646	1·660872	·6348875	·9966850	1·678570
$1-\partial U'/\partial U$	·3809914	·647987	·4066454	·660872	·3651125	·0033150	·678570
$[()/\cos I]$ e	·3811127	·6558797	·4200522	·6632119	·3657752	·00336457	·6785970
$[e/r]$ $\partial U'/\partial p$	·0023557	·00151701	$\cdot\overset{3}{0}9998386$	·00183309	·003717605	$\cdot\overset{3}{0}3245148$	·0037906
$[eL\sin U]$ $\partial U'/\partial c$	6·613925	8·989428	5·635151	8·484576	8·414615	·08442545	16·34018
$[-\sin I/\cos I']$ $\partial U'/\partial n$	·0252339	·1597344	·2535701	·08474539	·06021074	·1735614	·00965872
$[r(1-\cos(U+I))]$ X	·9366577	·2215361	·2266968	·2285129	2·698228	·3142541	2·640351
$d'+X_{+1}-X$	28·61181	·1748393	7·835210	202·7033	13·25397	13·83390	
$[()/\cos U']$ D'	28·83062	·1791767	7·891558	205·8303	13·51399	14·10374	
$\frac{1}{2}(I-U)$	4-31·798	7-58-833	13-34·421	5-49·963	3-16·516	10-33·148	5-53·373
$\cos\frac{1}{2}(I-U)$	·9968762	·9903153	·9720689	·9948228	·9983666	·9830877	·9947216
$\frac{1}{2}(I'-U')$	3-58-75·6	13-32·979	7-47·528	8-58·896	4-31·986	10-37·228	6-12·579
$\cos\frac{1}{2}(I'-U')$	·9975892	·9721673	·9907664	·9877386	·9968719	·9831898	·9941328
$L\sin U$	17·35425	13·70591	13·41536	12·79316	23·00488	25·0925	28·24104
$[()/\cos\frac{1}{2}(I-U)]$ PA	17·40863	13·83995	13·80094	12·85974	23·04262	25·52417	28·39090
$PA\cos\frac{1}{2}(I'-U')$	17·36666	13·115476	13·67351	12·70206	22·97044	26·09610	28·22432
$[()/\sin U']$ L'	141·2283	61·52208	114·6260	73·14895	117·6628	128·9077	141·0146
$-l'$							462·3150
$L'-l'$							603·3296
H'_{pr}							123·2516

Computer:

Date:

TRANSFER COEFFICIENTS.

System Booth Telephoto.

7.5° Pencil Ray 'pr'

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.647987	.593355	1.660872	.634888	.996685	1.578670	
D_+	28.830062	.179177	7.891558	205.8303	13.51399	14.10374	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.105743	1.276650	2.120012	.932579	1.520052	1.578570	
$(\partial U_k'/\partial p_+)D_+$	.120306	.001117	.031568	.343866	.051164	.053462	
$\partial U_k'/\partial U'$	2.226049	1.277767	2.151580	1.276445	1.468888	1.525108	1.000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	409.875	248.365	406.365	17.5045	14.0560		
$(\partial p_k'/\partial p_+)D_+$	45.531	0.358	12.196	227.1652	13.5150		
$\partial p_k'/\partial U'$	455.406	248.713	418.561	244.670	27.5710	14.1028	0.000
$\partial U'/\partial p$	.00235575	.00151701	.00099984	.00183309	.00371761	.0⁵32451	
$\partial p'/\partial p$	1.000196	.980087	1.020993	.993864	1.001082	1.000097	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	.00524401	.00193839	.00215123	.00233984	.00546075	.00000495	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	.00417369	.00611126	.00408420	.00166038	.00379012	.00379097	
$\partial U_k'/\partial p$	.00107032	.00417287	.00623543	.00400022	.00167063	.00378602	.003790599
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.072822	.377300	.418493	.448502	.102498	.000046	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.579574	1.956564	1.577824	1.096881	1.001155	1.000027	
$\partial p_k'/\partial p$	.506752	1.579264	1.996317	1.545382	1.103653	1.000073	.999931
$-S_k'.\partial U_k/\partial U'$	1056.390	606.375	1021.050	605.747	697.073	723.752	474.558
$C(U')$	1511.796	855.087	1439.611	850.417	724.644	737.855	474.558
$C(U')\sec U_k'$	1543.019	872.747	1469.343	867.981	739.610	753.094	484.359
$-S_k'.\partial U_k'/\partial p$	.507929	1.980269	2.959074	1.898337	.792811	1.796686	1.798859
$C(p)$	.001177	3.559533	4.955391	3.443719	1.896464	.796613	.798929
$C(p)\sec U_k'$	.0012018	3.633048	6.057735	3.514842	1.935632	.8130654	.8154288
$\partial U'/\partial c$	6.61393	8.989428	5.635151	8.484576	8.414615	.0844256	16.34018
$C(c)\sec U_k'$	10205.41	7845.50	8279.97	7364.45	6223.53	63.5803	7914.52
$-\partial l'/\partial c \tan U_k'$	104933.8	90740.35	95787.92	92418.86	23660.18	214.4816	25257.49
$\partial H'/\partial c$	94728.39	82894.85	87507.95	85054.41	29883.71	278.0619	23172.01
$\partial U'/\partial n$	.0252339	.159734	.2535701	.0847454	.0602107	.1735614	.00965872
$C(n)\sec U_k'$	38.936	189.4078	372.582	73.5574	44.6325	130.7081	4.6783
$-\partial l'/\partial n \tan U_k'$	1702.995	714.6054	1919.143	44.9230	929.9113	5.5819	241.0553
$\partial H'/\partial n$	1741.931	575.1976	1546.562	118.4803	974.4438	136.2900	236.3770
$\partial p/\partial d$	.1325015	.1229687	.218698	.1192880	.1736465	.1962227	.1946749
$C(d)\sec U_k'$	.0001592	.4467512	1.106116	.4192785	.3361157	.1587288	.1587435
$-\partial l'/\partial d \tan U_k'$		1.527018	4.481442	1.404633	.7138646	.0729966	.0731952
$\partial H'/\partial d$	.0001592	1.080267	3.375326	.985355	.377749	.231725	.231939
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

Ray: d

System: Booth Teleph

Pencil: 7.5°

		1	2	3	4	5	6	7
	Surface	1	2	3	4	5	6	7
	L	186.3492	1214.293	10661.10	1140.940	510.6241	210.9243	227.6456
	r	161.78	432.35	420.12	361.80	98.39	1036.80	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.000	1.61517	1.000	1.65068	1.000	1.57338	1.57846
	N'	1.61517	1.000	1.65068	1.000	1.57338	1.57846	1.000
$[r_{-1}/r]$	a		.3741876	1.029111	1.161194	3.677203	.0948978	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β	2.151868	1.305331	.0295393	2.181592	2.587255	.8946181	5.898323
	$\alpha \sin I'_{-1}$		.0660553	.0637705	.0448101	.5729049	.0213628	1.1749796
	$+\beta \sin U$		.0276899	.0000714	.0495747	.2187164	.1949044	1.2817616
	$\sin I$	.2851257	.0383654	.0636993	.0943848	.3641891	.1735416	.1067835
$[N/N']$	n	.6191299	1.61517	.6058109	1.65068	.6355744	.9967817	1.57846
	$\sin I'$	.1765298	.0619666	.0386897	.1557991	.2251135	.1729831	.1685535
$[Pl.\tan U]$	$\sin I'+\sin U'$	.1553169	.0595494	.0613138	.0712630	.4429767	.0443264	.3245959
$[Pl.\cot U']$	$r()$	25.12717	25.74618	25.75915	25.78295	43.58448	45.95761	49.54632
$[L\tan U\cot U'][()/\sin U']$	L'	1184.523	10651.24	1133.661	304.9934	200.0643	211.4846	317.6183
	U	7.61417	1.21560	0.13849	1.30211	4.84935	12.58366	12.55106
	$+I$	16.56637	2.19871	3.65217	5.41591	20.74376	9.99380	6.12993
	$U+I$	8.95220	3.41421	3.51368	4.11380	25.59311	2.58976	18.68099
	$-I'$	10.16770	3.56270	2.21157	8.96315	13.00955	9.96130	9.70373
	U'	1.21560	0.13849	1.30211	4.84935	12.58356	12.55106	8.97726
	$\sin U$	.1325015	.0212129	.0024172	.0227241	.0845361	.2178632	.2173095
	$\cos I$	.9584901	.9992638	.9979691	.9955358	.9351738	.9848266	.9942823
	$\cos I'$	.9842953	.9980782	.9992552	.9877888	.9743325	.9849248	.9856926
	$\sin U'$	.0212129	.0024172	.0227241	.0845361	.2178632	.2173095	.1560424
	$\cos U'$	.9997750	.9999971	.9997418	.9964204	.9759793	.9761028	.9877503
	$\tan U'$				.0848398			.1579776
	$1-\cos(U+I)$	.0121815	.0017749	.0018798	.0025765	.0981156	.0010213	.0526834
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.026923	.998814	1.001289	.992218	1.041873	1.000100	.991361
$[n/()]$	$\partial U'/\partial U$	.6028981	1.617089	.605031	1.663626	.610031	.996682	1.592215
	$1-\partial U'/\partial U$	.3971019	.617089	.394969	.663626	.389969	.003318	.592215
$[()/\cos I]$	e	.4142994	.617544	.396773	.666602	.4170021	.00336912	.595621
$[e/r]$	$\partial U'/\partial p$	.002560881	.00142834	.00094205	.00184246	.00423826	$.0^{5}32495$	.00390213
$[eL\sin U]$	$\partial U'/\partial c$	10.22969	15.90711	10.18950	17.28287	18.00038	.154820	29.46508
$[-\sin I/\cos I']$	$\partial U'/\partial n$	.289675	.0384393	.0637468	.0955516	.3635197	.176198	.1083336
$[r(1-\cos(U+I))]$	X	1.970723	.767378	.789742	.932178	9.653594	1.068884	8.041594
	$d'+X_{+1}-X$	27.03190	.157364	9.101919	195.0442	19.46471	9.17729	
$[()/\cos U']$	D'	27.03798	.157637	9.10427	195.7449	19.94377	9.40197	
	$\frac{1}{2}(I-U)$	12.09027	0.49160	1.89530	3.35900	7.94720	11.28868	3.21056
	$\cos\frac{1}{2}(I-U)$	.9778188	.9999632	.9994529	.9982820	.9903959	.9806634	.9984304
	$\frac{1}{2}(I'-U')$	5.69160	01.84560	0.45473	6.90625	0.21300	11.25618	0.36323
	$\cos\frac{1}{2}(I'-U')$	.9950701	.9994813	.9999685	.9927443	.9999981	.9807642	.9999797
	$L\sin U$	24.69155	25.75868	26.74584	26.92683	43.16617	45.95264	49.46956
$[()/\cos\frac{1}{2}(I-U)]$	PA	25.25106	25.75963	25.75993	25.97145	43.58476	46.85921	49.54731
	$PA\cos\frac{1}{2}(I'-U')$	25.12717	25.74627	25.75912	25.78301	43.68446	45.95784	49.54630
$[()/\sin U']$	L'	1184.523	10651.28	1133.660	304.9941	200.0643	211.4856	317.6182
	$-l'$							462.3150
	$L'-l'$							779.8332
	H'_{pr}							123.1962

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

7.5° Pencil Ray 'd'

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1·617089	·605031	1·663626	·610031	·996682	1·692215	
D_+	27·03798	·157637	9·10427	195·7449	19·94377	9·401971	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2·199767	1·359264	2·204674	·898363	1·550366	1·592215	
$(\partial U_k'/\partial p_+)D_+$	·129155	·001061	·041929	·426869	·077730	·036688	
$\partial U_k'/\partial U'$	2·328922	1·360325	2·246603	1·325222	1·472636	1·555527	1·000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	411·9622	254·4409	406·0039	17·7299	9·28982		
$(\partial p_k'/\partial p_+)D_+$	44·0400	0·3145	14·5380	226·3177	19·77405		
$\partial p_k'/\partial U'$	456·0022	254·7554	420·5419	244·0476	29·06387	9·32075	0·000
$\partial U'/\partial p$	·00256088	·0042834	·00094205	·00184246	·00423826	·0⁵32495	
$\partial p'/\partial p$	1·026923	·998814	1·001289	·992218	1·041873	1·000100	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	·0059641	·0019430	·0021164	·0024417	·0062414	·0000051	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	·0049054	·0067198	·0046114	·0021638	·0040607	·0039025	
$\partial U_k'/\partial p$	·0010587	·0047768	·0067278	·0046054	·0021807	·0038975	·00390213
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1·167767	·363878	·396170	·449648	·123180	·000030	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1·672674	1·992699	1·598896	1·147190	1·033007	·991460	
$\partial p_k'/\partial p$	·504906	1·628821	1·995066	1·596838	1·156187	·991490	·991361
$-S_k'.\partial U_k/\partial U'$	1109·009	647·773	1069·81	631·057	701·254	740·726	476·1898
$C(U')$	1565·01	902·528	1490·35	875·105	730·318	750·047	476·1898
$C(U')\sec U_k'$	1584·42	913·722	1508·83	885·958	739·376	759·349	482·096
$-S_k'.\partial U_k'/\partial p$	·504133	2·274663	3·203705	2·193064	1·038446	1·855935	1·858153
$C(p)$	·000774	3·903484	5·198771	3·789902	2·194633	·8644448	·866792
$C(p)\sec U_k'$	·0007833	3·951895	5·263246	3·836904	2·221851	·875166	·877542
$\partial U'/\partial c$	10·22969	15·90711	10·18960	17·28287	18·00038	·154820	29·46508
$C(c)\sec U_k'$	16208·13	14534·67	15374·26	15311·89	13309·04	117·6624	14204·98
$-\partial l'/\partial c \tan U_k'$	81147·15	70171·12	74074·50	71469·15	18296·84	165·8624	19532·07
$\partial H'/\partial c$	97355·3	84705·8	89448·8	86781·0	31605·9	283·4248	33737·05
$\partial U'/\partial n$	·289675	·0384393	·0637468	·0955516	·363520	·176198	·108334
$C(n)\sec U_k'$	458·967	35·1228	96·183	84·6547	268·7776	133·7956	52·2271
$-\partial l'/\partial n \tan U_k'$	1316·956	552·6170	1484·107	34·7397	719·1169	4·3166	186·4123
$\partial H'/\partial n$	1775·923	587·7398	1580·29	119·3944	987·8945	138·1122	238·6394
$\partial p/\partial d$	·132502	·0212129	·0024172	·0227241	·0845361	·2178632	·2173095
$C(d)\sec U_k'$	·0001038	·083831	·012722	·087190	·187827	·190666	·190698
$-\partial l'/\partial d \tan U_k'$		1·180870	3·465578	1·086228	·552044	·0564495	·0566032
$\partial H'/\partial d$	·0001038	1·097039	3·478300	·999038	·364217	·247116	·247302
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

Computer:

Date:

Ray: b

System: Booth Teleph

Pencil: 7.5°

RAY TRACE AND SINGLE SURFACE COEFFICIENTS.

	Surface	1	2	3	4	5	6	7
	L	317.8566	1697.336	413.7607	1995.327	1218.259	239.5130	256.2888
	r	161.78	432.35	420.12	361.8	98.39	1036.8	152.64
	d'	29.77	0.18	7.38	205.63	10.87	16.16	
	N	1.000	1.61517	1.000	1.65068	1.000	1.57338	1.57846
	N'	1.61517	1.000	1.65068	1.000	1.57338	1.57846	1.000
$[r_{-1}/r]$	a		·3741876	1.029111	1.161194	3.677203	·0948978	6.792453
	$r_{-1}-d'_{-1}-r$							
$[()/r]$	β	2.964746	1.305331	·0295392	2.181592	2.587255	·8946181	5.898323
	$\alpha \sin I'_{-1}$			·2058119	·1426581	·5838229	·0286912	1.1890563
	$+\beta \sin U$			·0030180	·0464747	·1081307	·2043119	1.3437571
	$\sin I$	·3928333	·1238198	·2027938	·0961835	·4756922	·1756207	·1546992
$[N/N']$	n	·6191299	1.61517	·6058109	1.65068	·6355744	·9967817	1.57846
	$\sin I'$	·2432148	·1999900	·1228547	·1587682	·3023378	·1750555	·2441865
$[Pl.\tan U]$	$\sin I'+\sin U'$	·2683516	·0978198	·1015516	·1169746	·5307167	·0527647	·3822249
$[Pl.\cot U']$	$r()$	43.41392	42.29239	42.66386	42.32141	52.21722	54.70644	58.34281
$[L\tan U\cot U'][()/\sin U']$	L'	1727.106	413.9406	2002.707	1012.629	228.6429	240.1299	422.6564
	U	7.61417	1.44039	6.86416	1.22067	2.39529	13.20165	13.16877
	$+I$	23.13092	7.11261	11.70038	6.51944	28.40443	10.11478	8.89935
	$U+I$	15.51675	6.67222	5.83622	6.74011	30.79972	3.08687	22.06812
	$-I'$	14.07636	11.53638	7.05689	9.13540	17.59807	10.08190	14.13377
	U'	1.44039	5.86416	1.22067	2.39529	13.20165	13.16877	7.93435
	$\sin U$	·1325015	·0251368	·1021702	·0213031	·0417936	·2283789	·2278202
	$\cos I$	·9196097	·9923047	·9792214	·9953636	·8796119	·9844579	·9879617
	$\cos I'$	·9699724	·9797979	·9924246	·9873159	·9532008	·9845585	·9697283
	$\sin U'$	·0251368	·1021702	·0213031	·0417936	·2283789	·2278202	·1380384
	$\cos U'$	·9996840	·9947669	·9997730	·9991263	·9735723	·9737632	·9904269
	$\tan U'$							·1393726
	$1-\cos(U+I)$	·0364477	·0048963	·0051834	·0069112	·1410376	·0014510	·0732622
$[\cos I'/\cos I]$	$\partial p'/\partial p$	1.054765	·9873962	1.013483	·9919148	1.083661	1.000102	·9815444
$[n/()]$	$\partial U'/\partial U$	·586984	1.635787	·597751	1.664135	·586507	·996680	1.608139
	$1-\partial U'/\partial U$	·413016	·635787	·402249	·664135	·413493	·003320	·608139
$[()/\cos I]$	e	·449121	·640718	·410784	·667229	·470086	·00337241	·615549
$[e/r]$	$\partial U'/\partial p$	·00277612	·00148194	·000977778	·00184419	·00477778	·0325271	·00403269
$[eL\sin U]$	$\partial U'/\partial c$	18.91541	27.33659	17.36549	28.36165	23.93464	·1844701	35.94055
$[-\sin I/\cos I']$	$\partial U'/\partial n$	·4049943	·1263728	·2043418	·0974192	·499047	·1783751	·1595284
$[r(1-\cos(U+I))]$	X	5.896509	2.116915	2.17765	2.500472	13.87669	1.504397	11.18274
	$d'+X_{+1}-X$	21.75658	·119266	12.05812	189.2528	23.24229	6.481657	
$[()/\cos U']$	D'	21.76346	·119892	12.06086	189.4183	23.87320	6.656707	
	$\frac{1}{2}(I-U)$	15.37255	4.27650	8.78227	2.14939	13.00457	11.65822	2.13471
	$\cos\frac{1}{2}(I-U)$		·9972157	·9882757	·9992964	·9743521	·9793704	·9993060
	$\frac{1}{2}(I'-U')$	6.31799	8.70027	4.13878	5.76635	2.19821	11.62533	3.09971
	$\cos\frac{1}{2}(I'-U')$		·9884932	·9973922	·9949417	·9992642	·9794863	·9985370
	$L\sin U$	42.11648	42.66560	42.27401	42.50666	50.91543	54.69972	58.38777
$[()/\cos\frac{1}{2}(I-U)]$	PA		42.78472	42.77652	42.53668	52.25568	55.85192	58.42832
	$PA\cos\frac{1}{2}(I'-U')$		42.29240	42.66397	42.32142	52.21723	54.70619	58.34284
$[()/\sin U']$	L'		413.9407	2002.712	1012.629	228.6430	240.1288	422.6566
	$-l'$							462.3150
	$L'-l'$							884.9716
	H'_b							123.3408

Computer:

TRANSFER COEFFICIENTS.

System Booth Telephoto

Date:

7.5° Pencil Ray b

Surface	1	2	3	4	5	6	7
$\partial U_+'/\partial U_+$	1.635787	.597751	1.664135	.686607	.996680	1.608139	
D_+	21.76346	.119892	12.06086	189.4183	23.8732	6.656707	
$(\partial U_k'/\partial U_+')(\partial U_+'/\partial U_+)$	2.309419	1.410897	2.297268	.867963	1.576045	1.608139	
$(\partial U_k'/\partial p_+)D_+$	.117955	.000912	.063073	.512496	.096160	.026844	
$\partial U_k'/\partial U'$	2.427374	1.411809	2.360341	1.380458	1.479885	1.581295	1.000
$(\partial p_k'/\partial U_+')(\partial U_+'/\partial U_+)$	420.4436	256.7773	409.6583	17.5645	6.5122		
$(\partial p_k'/\partial p_+)D_+$	36.6954	0.2510	199137	228.6044	23.4355		
$\partial p_k'/\partial U'$	457.1390	257.0283	429.5720	246.1689	29.9477	6.5339	0.000
$\partial U'/\partial p$	.00277612	.00148194	.00097778	.00184419	.00477778	.0^{5}325271	
$\partial p'/\partial p$	1.054765	.987396	1.013483	.9919148	1.083661	1.000102	
$(\partial U_k'/\partial U')(\partial U'/\partial p)$	.0067387	.0020922	.0023079	.0025458	.0070706	.0000051	
$(\partial U_k'/\partial p_+)(\partial p'/\partial p)$	.0057167	.0075121	.0053001	.0026838	.0043649	.0040331	
$\partial U_k'/\partial p$	.0010220	.0054199	.0076080	.0052296	.0027056	.00402796	.00403269
$(\partial p_k'/\partial U')(\partial U'/\partial p)$	1.2690745	.380901	.420026	.453983	.143083	.000021	
$(\partial p_k'/\partial p_+)(\partial p'/\partial p)$	1.778442	2.067004	1.673363	1.197118	1.063793	.981645	
$\partial p_k'/\partial p$	.509368	1.686103	2.093389	1.651101	1.206876	.981666	.981544
$-S_k'.\partial U_k/\partial U'$	1160.465	674.950	1128.419	659.962	707.495	755.977	478.074
$C(U')$	1617.604	931.978	1557.991	906.131	737.443	762.511	478.074
$C(U')\sec U_k'$	1633.240	940.987	1573.051	914.889	744.571	769.881	482.696
$-S_k'.\partial U_k'/\partial p$	.488592	2.591101	3.637180	2.500128	1.293492	1.925665	1.927924
$C(p)$	.020776	4.277204	5.730569	4.151229	2.500368	.943999	.946380
$C(p)\sec U_k'$	.020977	4.318647	5.785961	4.191355	2.624537	.953124	.955527
$\partial U'/\partial c$	18.9154	27.3366	17.3655	28.3616	23.9346	.184470	36.9406
$C(c)\sec U_k'$	30893.4	25728.4	27316.8	25947.8	17821.0	142.020	17348.3
$-\partial l'/\partial c \tan U_k'$	71590.5	61907.1	65350.8	63052.2	16142.0	146.329	17231.8
$\partial H'/\partial c$	102483.9	87630.5	92667.6	89000.0	33963.0	288.349	34580.1
$\partial U'/\partial n$	.404994	.126373	.204342	.0974192	.499047	.178375	.159528
$C(n)\sec U_k'$	661.453	118.915	321.440	89.1278	371.576	137.328	77.0036
$-\partial l'/\partial n \tan U_k'$	1161.858	487.535	1309.324	30.6484	634.427	3.808	164.4586
$\partial H'/\partial n$	1823.311	606.450	1630.764	119.7762	1006.003	141.136	241.4622
$\partial p/\partial d$	.132502	.025137	.102170	.021303	.041794	.228379	.227820
$C(d)\sec U_k'$	.0027794	.108555	.591153	.089289	.105510	.217673	.217688
$-\partial l'/\partial d \tan U_k'$		1.041799	3.057438	.958303	.487030	.049801	.049937
$\partial H'/\partial d$	.0027794	1.150354	3.648591	1.047592	.381520	.267475	.267625
$\partial U_k'/\partial c$							
$\partial U_k'/\partial n$							
$\partial U_k'/\partial d$							
$\nu(c)$							
$\gamma(c)$							
$\pi(c)$							
$\rho(c)$							
$\nu(d)$							
$\gamma(d)$							
$\pi(d)$							
$\rho(d)$							

CHROMATIC COEFFICIENTS. Booth Telephoto Lens.

	Pencil	Axial					4°				
	Ray	M	QM	Q	Z	QZ	a	c	pr	d	b
$0.\overline{383322}$	$(\partial H'/\partial n_1)_{L'}$	$\overline{285.751}$	$\overline{258.577}$	$\overline{228.963}$	$\overline{189.239}$	$\overline{144.364}$	$\overline{236.344}$		7.7181		257.209
	$(\partial H'/\partial n_2)_{L'}$	$\overline{353.440}$	$\overline{313.137}$	$\overline{262.098}$	$\overline{218.873}$	$\overline{163.225}$	$\overline{388.004}$		$\overline{70.2225}$		210.705
	$(\partial H'/\partial N_a)_{L'}$	$\overline{639.191}$	$\overline{571.714}$	$\overline{491.061}$	$\overline{408.132}$	$\overline{307.589}$	$\overline{624.348}$		$\overline{62.5044}$		467.914
$0.\overline{367007}$	$(\partial H'/\partial n_3)_{L'}$	349.482	309.413	258.784	216.011	160.989	383.396		68.8730		$\overline{208.315}$
	$(\partial H'/\partial n_4)_{L'}$	13.832	13.312	12.316	11.125	9.060	$\overline{25.317}$		$\overline{37.3840}$		$\overline{51.942}$
	$(\partial H'/\partial N_b)_{L'}$	363.313	322.725	271.100	227.136	170.049	358.079		31.489		$\overline{260.257}$
$0.\overline{403946}$	$(\partial H'/\partial n_5)_{L'}$	160.909	146.403	126.453	108.171	82.804	131.365		$\overline{9.170}$		$\overline{148.639}$
0.633529	$(\partial H'/\partial n_6)_{L'}$	3.444	2.767	2.037	1.524	0.995	$\overline{40.811}$		$\overline{41.740}$		$\overline{45.557}$
	$(\partial H'/\partial N_c)_{L'}$	164.353	149.170	128.490	109.695	83.799	90.554		$\overline{50.910}$		$\overline{194.196}$
$0.\overline{63149}$	$(\partial H'/\partial n_6)_{L'}$	$\overline{3.433}$	$\overline{2.759}$	$\overline{2.030}$	$\overline{1.519}$	$\overline{0.992}$	40.680		41.605		45.410
	$(\partial H'/\partial n_7)_{L'}$	$\overline{100.407}$	$\overline{91.777}$	$\overline{79.726}$	$\overline{68.514}$	$\overline{52.725}$	$\overline{91.865}$		$\overline{2.397}$		84.218
	$(\partial H'/\partial N_d)_{L'}$	$\overline{103.840}$	$\overline{94.536}$	$\overline{81.766}$	$\overline{70.033}$	$\overline{53.717}$	$\overline{51.186}$		39.208		129.628

	Pencil	7.5°					10°		
	Ray	a	c	pr	d	b	a	pr	b
$0.\overline{383322}$	$(\partial H'/\partial n_1)_{L'}$	$\overline{201.990}$	$\overline{134.266}$	14.925	175.932	253.493	$\overline{162.420}$	20.084	237.633
	$(\partial H'/\partial n_2)_{L'}$	$\overline{437.475}$	$\overline{330.027}$	$\overline{139.408}$	35.123	118.890	$\overline{448.069}$	$\overline{192.948}$	40.971
	$(\partial H'/\partial N_a)_{L'}$	$\overline{639.465}$	$\overline{464.292}$	$\overline{124.483}$	211.055	372.383	$\overline{610.489}$	$\overline{172.864}$	278.604
0.367007	$(\partial H'/\partial n_3)_{L'}$	432.156	325.163	136.740	$\overline{35.300}$	$\overline{118.090}$	442.163	189.273	$\overline{41.325}$
	$(\partial H'/\partial n_4)_{L'}$	$\overline{62.162}$	$\overline{64.549}$	$\overline{73.557}$	$\overline{84.655}$	$\overline{88.963}$	$\overline{90.208}$	$\overline{100.844}$	$\overline{116.169}$
	$(\partial H'/\partial N_b)_{L'}$	369.994	260.614	63.183	$\overline{119.955}$	$\overline{207.053}$	361.965	88.429	$\overline{157.494}$
0.403946	$(\partial H'/\partial n_6)_{L'}$	108.386	68.506	$\overline{17.989}$	$\overline{108.574}$	$\overline{149.979}$	81.959	$\overline{24.582}$	$\overline{144.674}$
0.633529	$(\partial H'/\partial n_6)_{L'}$	$\overline{83.409}$	$\overline{83.201}$	$\overline{82.807}$	$\overline{84.763}$	$\overline{86.821}$	$\overline{115.715}$	$\overline{114.519}$	$\overline{118.219}$
	$(\partial H'/\partial N_c)_{L'}$	24.977	$\overline{14.695}$	$\overline{100.796}$	$\overline{193.337}$	$\overline{236.800}$	$\overline{33.756}$	$\overline{139.101}$	$\overline{262.893}$
0.63149	$(\partial H'/\partial n_6)_{L'}$	83.140	82.933	82.641	84.491	86.641	116.342	114.151	117.838
	$(\partial H'/\partial n_7)_{L'}$	$\overline{86.327}$	$\overline{60.704}$	$\overline{4.678}$	52.227	76.966	$\overline{75.801}$	$\overline{6.360}$	68.257
	$(\partial H'/\partial N_d)_{L'}$	$\overline{3.186}$	22.229	77.863	136.718	168.507	39.541	107.791	186.095

Computer:

Date:

System: Booth Telephoi

PARAXIAL RAY TRACE.

Principal Paraxial 7·5° Pencil

	Surface	1	2	3	4	5	6	7
	y							
	l	130·974	111·4457	62·65087	107·1498	132·1378	128·3260	144·8489
	r	161·780	432·35	420·12	361·8	98·39	1036·8	152·64
	d'	29·77	0·18	7·38	205·63	10·87	16·16	
	N							
	N'							
$[r_{-1}/r]$	a		·3741876	1·029111	1·161194	3·677203	·0948978	6·792453
	$r_{-1}—d'_{-1}—r$							
$[()/r]$	β	·190419	1·305331	·0295392	2·181592	2·587255	·8946181	5·898323
	$a\ .\ i'_{-1}$		·0058625	·2576813	·1767265	·5109246	·0036072	1·1637163
	$\beta\ .\ u$		·1608877	·0064579	·2609003	·4511184	·1754853	1·1537326
	i	·0253052	·1550252	·2512233	·0841737	·0598061	·1718781	·0099841
$[N/N']$	n	·6191299	1·61517	·6058109	1·66068	·6355744	·9967817	1·57846
	i'	·0156672	·2503921	·1521938	·1389438	·0380112	·1713249	·0157595
	$r(u+i)$	17·40544	13·73615	13·69679	12·81423	23·03978	25·17205	28·33295
$[()/u']$	l'	141·2157	62·83082	114·5296	73·49219	117·4560	128·6892	140·6947
	u	·1328923	·1232543	·2186212	·1195917	·1743618	·1961567	·1956035
	$u+i$	·1075871	·0317709	·0326021	·0354180	·2341679	·0242786	·1856194
	u'	·1232543	·2186212	·1195917	·1743618	·1961567	·1956035	·2013789
	lu	17·40544	13·73616	13·69681	12·81423	23·03978	25·17200	28·33295
$[()/u']$	l'	141·2157	62·83087	114·5298	73·49219	117·4560	128·6889	140·6947
	$1-n$							
$[()/r]$	$\partial u'/\partial p$							
$[(1-n)lu]$	$\partial u'/\partial c$							

4°	10°
·0698605	·175835

BOOTH TELEPHOTO LENS

Chromatic Coefficients for Zonal and Paraxial Rays. Ray of Oblique Pencil.

		L'_z	ℓ'
− 0.3833218	$\partial/\partial n_1$	− 3359.116	− 3195.50
	$\partial/\partial n_2$	− 3885.511	− 3498.07
	$\partial/\partial N_a$	− 7244.627	− 6693.57
− 0.367007	$\partial/\partial n_3$	3834.346	3447.82
	$\partial/\partial n_4$	197.476	219.90
	$\partial/\partial N_b$	4031.822	3667.72
− 0.4039457	$\partial/\partial n_5$	1920.106	1838.77
0.6335289	$\partial/\partial n_6$	27.0536	17.31
	$\partial/\partial N_c$	1947.160	− 1856.08
− 0.63149	$\partial/\partial n_6$	− 26.9665	− 17.25
	$\partial/\partial n_7$	− 1216.167	− 1179.99
	$\partial/\partial N_d$	− 1243.133	− 1197.24

CYTOLOGICAL PAPERS.

1. The Chromosome Number of Eucalyptus globulus and E. Johnstoni.

2. The Male Meiotic Cycle in the Genus Eucalyptus.

3. Preliminary Note on the Development of Eucâlyptus globulus.